clearance. Over uneven ground, the elevation of the highest point in the circle is recorded, and unless careful inspection is made among the profile, the precision will suffer. This is not a serious matter since air photography is taken simultaneously, and it is possible to accept or reject points and to extract sufficient few points for map control.

For military purposes it will be necessary to map from higher altitudes than we have been using. To use APR at these altitudes a few refinements are necessary. These are gyro stabilization of the antenna and narrowing of the beamwidth. Neither is an item of tremendous cost.

The APR has speeded up topographic contouring by a factor respresented by the speed of an aircraft relative to that of a ground party. It can do the same for planimetric and all mapping.

This paper would be incomplete were mention not made of the energy exerted by Mr. *S. Jowitt* of the Mines and Technical Surveys. Not only did he assist with this paper, but his efforts throughout the development of APR have lent unquestionably to its success.

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THE APPLICATION OF STATISTICS TO PHOTOGRAMMETRY*

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RECENT research, primarily that by Professor Matthews of New Hampshire, has indicated that we need more specific and more precise information on a number of elementary (I use the word in its generic sense) and combinational problems before we can expect to get the best results from the use of aerial photographs. Broad generalizations as to film and filter combinations have been accepted in the past, and may result in the correct combination to use for certain types of images and for some particular purpose of photo interpretation; but for other types of images and other purposes, some different combination may be superior. When fine points of difference present themselves, and when by occular observation alone it is no longer possible to positively identify the differences that may classify one set of values as superior or inferior to another set, it is desirable to resort to a statistical analysis of the results.

I should indicate that I am a forester and as such may be looking for effects on aerial photographs that may be considered by many as minute. This is only mentioned because I do not wish you to think that the problems about to be mentioned are necessarily common or pertinent to all who use aerial photographs. It is only my ignorance of the problems in other fields that prompt this statement.

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Before proceeding further it may be desirable to define our terms. The title "Statistics in Photogrammetry" may be misleading and perhaps should read "Statistics in Photo Interpretation." For the photo interpreter there is a close relationship btween these terms, as one of the chief values of aerial photos for many kinds of work is centered about the ability to measure heights or differences in elevation on stereoscopic pairs. Any broad definition of Photo Interpretation is acceptable; so you may think of one that applies most directly to your own field.

Statistics, as described by Patterson, may be defined as that branch of applied mathematics which deals with facts and figures resulting from a series of observations, and which makes it possible to express the results simply without omitting any of the essential points. As a matter of fact we are not so much concerned with the statistic itself as with the *statistical methods* involved. This term is used with idea of contrasting it with *experimental methods.* The latter are the means by which we have reached most of our present conclusions on aerial photography and its interpretation. To contrast these two, expermental and statistical, we might think of an experiment in treating fence posts to prevent decay. Such an experiment can be reasonably well controlled. The same species of wood can be used; the same temperature and the same pressure can be maintained in the preservative chamber; the concentration of the preservative can be varied; presumably the preservative showing the deepest penetration would be the best. In this experiment it has been possible to exert a high degree of control. The wood preservative scientist can point out many factors, however, that have not been mentioned here that could cause a considerable amount of error.

Be that as it may, if we consider the above to be a controlled experiment, let us in contrast consider an agricultural experiment in which we are trying to find the optimum concentration of a given fertilizer for a particular crop species. The area can be planted in plots of the same size; the same amount of seed can be planted in each plot; the same drill can be used in the planting procedure; the plots can be planted, to all intents and purposes, at the same time; and similar controls exerted with respect to the harvesting. However we have no assurance that the quality of the soil is the same in the different plots or for that matter within a single plot, nor that each plot will get the same amount of moisture, the same amount of sun light, and that all of the other ecological factors of plant growth are the same. Since these factors can not be controlled, we can best resort to a statistical method for a means of evaluating the results.

The basic problems of photo interpretation are very similar to our agricultural experiment. We should try to control as many factors as possible but we cannot control all of them. Furthermore in analyzing the results we should not dare to make generalizations beyond the scope of the physical experiment. These problems which concern us now and for which we do not choose to make generalizations are an indication of the growth of the science and the art of aerial photogrammetry. These problems are many and diversified. To mention a few that apply to forestry; what is the effect of using each of a number of film and filter combinations on stands of different species such as longleaf pine, short leaf and loblolly pine, stands of mixed hardwoods of the south, of northern hardwoods, of white pine in the Inland Empire, of white pine in its usual mixture in the northeast, of spruce and balsam, of western yellow pine in the many parts of its range. Another problem is whether positive prints or positive transparencies are better suited to height determinations obtained from parallax measurements. Another problem is what magnification and what type of instrument can be used most effectively for tree height determination and also for tree counts.

These are some of the problems and the answers are hidden in statistical analyses of a large number of basic experiments. What instruments shall we use for interpretation; can we separate one forest type from another and still maintain sufficient resolution on the print to obtain a good estimate of height and furthermore obtain an accurate count of the trees? Perhaps this problem can be solved only by using more than one camera in the plane with different film and filter combinations in each, or possibly by means of two separate flights one of which might be devoted to tri-camera or trimetrogon photography.

We have defined statistics in rather vague terms. To show what we can expect from this science and how it is applied, let us consider a distribution of measurements such as 1, 3, 5. These may be inches or miles or thousands of feet of flying height or anything else you may wish them to symbolize. The mean of this distribution of 1, 3, 5, is three and the mean is a very well known statistic (it is one way of describing our distribution). However, if we should take another distribution 2, 3, 4, we would have the same mean but the distribution most assuredly is different—there is a smaller range of values, from the smallest to the largest figure. If someone should ask which of these two distributions is the stronger you would not hestitate in saying the 2, 3, 4.

The statistics that measure the amount of scatter in a distribution are many but that one which is most useful to us is the variance. The variance is the square of the standard deviation with which many of you are familiar. One of the statistical methods which can be used to excellent advantage in our work is known as the analysis of variance and it works in general along these lines. Let us assume that we wished to determine which of three crews of chainmen is the most consistent in its work. Let them measure the length of this room to the nearest thousandth of a foot, each crew measuring the length five times. Each crew would presumably have a different mean length and a different distribution of values from which the mean was calculated. There would be three means. If we obtain the variance (the amount of scatter) for all fifteen measurements, we would have the total variance. Also by very simple arithmetical manipulation we can obtain a variance for the three pooled sets of observations; this is often referred to as the within variation because it measures the scatter within the three groups of five measurements taken together. We can go one step further and obtain the variance of the three means. This latter is a measure of scatter of the three mean lengths of the room. If you are willing to agree that the three means would probably not be the same, the question that arises is "are they different because of some fundamental differences in the crews and the equipment used in the measurement of the room, or are the differences about what you would expect as the result of chance happenings." In other words, as the statistician would say, "are the differences significant or not." We can test this by dividing the variance of the means by the pooled variance of the groups. This latter variance represents for the most part the chance happenings, and if the quotient is sufficiently larger than one, it indicates that there is a difference between the means, and therefore between crews with their respective equipment. How can we pin this discrepancy on the crews? We have asked that they measure the same line (one of the controlled parts of the experiment). We can ask that they use the same equipment (a second control). If the lighting is not changed, and if the temperature and the humidity remain constant, we would then be reasonably sure that any significant difference in the means of the sets of measurements is due to the crews themselves.

There is one more point that should be made here. When we set up this experiment, we indicated that we wished to know which crew is the most *consistent*—not the most *accurate*. What we obtained from the analysis of variance is an estimate of whether there is any significant difference in the consistency of the crews. There is a decided difference between *accuracy* and *consistency*.

Suppose each crew used a different tape, but that the tape used by the crew that had the highest record for consistency (the smallest variance) was one foot short. The record for consistency still holds, but the accuracy of the result is anything but satisfactory. This is mentioned because some work has been done and more is being done, which is aimed directly at the accuracy of results. It is my contention that in most cases the factor or factors which cause inaccurate work can be sought out, eliminated or compensated. These are the errors that the civil engineer classes as constant errors.

What has all this to do with photo interpretation? If we are comparing different emulsions by means of parallax measurements for height, we may find from statistical analysis that one emulsion is decidedly superior to the others with respect to consistency. If we are concerned only with consistency, our problem is solved even though the photos may have been taken in California and analyzed in Cambridge. Ultimately, however, we will wish to obtain accuracy, and if it is found that the consistent emulsion shows an error of some given per cent of some given number of feet of tree height due to lack of resolution, this constant can be applied as a correction factor. If we have the actual heights of the different objects measured in the field, the test of accuracy and consistency can be made simultaneously. However, even though the accuracy test is made directly, the consistency test is probably the more important; if a given degree of accuracy can not be consistently maintained, all the measurements are open to question.

One other statistic that is simple in application and often of inestimable value should be mentioned. This is Chi-square. A simple example of its use is found in the classical birth rate problem. If it be reasonable to expect an equal number of male and female births, the total would be divided 50 per cent to each. If out of the first two thousand births for 1950 in Cambridge, eleven hundred were male and nine hundred were female, the question would arise "is this a chance occurrence or is there something about Cambridge that favors male births?" Chi-square is computed for these statistics by squaring the differences between the actual and the expected values, dividing each of the two squares by the expected value and adding. This is Chi-square. No differences would result in zero Chi-square, and large differences would result in a large Chi-square. Reference to a suitable table would indicate the significance or lack of significance of the results. It is not my thought to indulge in any mental gymnastics; I only wish to emphasize the term *expected value*. The use of this statistic is dependent upon obtaining an expected value with which to compare the actual value. In the birth rate problem we had a 50-50 expectancy; we could have had any other ratio, but we must have some figure in order to obtain a difference.

Chi-square is said by some authorities on statistics to apply particularly to enumeration data in contrast to measurement data, to which the analysis of variance is more applicable. It is easier for me to think of Chi-square as most applicable to yes and no questions. As a matter of record it is often used in the analysis of questionnaires.

You can see that this statistic lends itself to ready use when two sets of values are to be compared. The object of such an experiment would be to find if one set is significantly different from the other, or whether the differences that exist are due to the ordinary variation that would develop in sampling. As an example of its use, some time ago a member of the Society developed a new method of finding areas of land from aerial photographs. He wished to compare the results obtained by this new method with those obtained by three other methods, the radial plot method, the parallax method, and a method developed by the United States Forest Service. Twenty-one separate areas were analyzed under each of these methods, and it was found that his method compared very favorably with the radial plot method which had been taken as a standard or expected value. It was also found that his method differed significantly (a high value of Chi-square) from the other two methods. This is a simple example and should not be thought of as a model as there are other ways of handling a problem such as this when it is properly planned for statistical analysis. Photogrammetrists have probably noted several points which leave a great deal to be desired. The first is whether we were justified in using the radial plot method as a base or expected value. A ground survey giving the actual area would have been far superior, but the areas were in the state of Washington and the work was done in connection with a problem in the State of New York Next, the results obtained by this one man might not be the same as those obtained by other men—in other words there might have been personal bias even though entirely unintentional. This could be overcome by using a number of different operators. Again there might have been a within variation for the twenty-one measurements that was so great as to make any one or all of the sets quite unreliable. This latter could be checked by having each operator repeat his measurements say five or ten times on each of the twenty-one areas. The reason for calling attention to the weak points in the area problem is to show how necessary it is to plan an experiment carefully so as to obtain results that are capable of developing the maximum amount of information when treated statistically.

In summary there are these few points. First, there are a number of unsolved basic problems related to aerial photogrammetry that exist at the present time and that will multiply as this science is used in other fields. These problems have not been solved because the differences in the values obtained are too fine to be readily distinguishable by casual observation, but they are capable of solution by means of statistical analysis.

Second, the methods of statistics are not difficult mathematically. They are, however, often difficult of interpretation. To reduce this difficulty as much as possible, it is desirable to avoid confounding the experiment by varying one factor under a given set of conditions and varying another factor under a different set of conditions. It is best to subject each variable to all of the same variable conditions. Control as many factors as possible, and above all plan the statistical procedure before starting the experiment. If this be not done the chances are that a mass of expensive data will be collected that are not capable of analysis.