metrists, is still at work with several photogrammetric problems especially in optics and rectifying.

For investigations on the use of photogrammetric methods for forestry purposes, a special committee is at work in collaboration with other photogrammetric institutions.

Finally, there are plans to introduce photogrammetry in educational institutions in agronomy and forestry.

TWO PROBLEMS IN PHOTOGRAMMETRY

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DEVICES TO SECURE PLANE FILM SURFACES AND ELIMINATE LENS DISTORTION

IN THE year 1937, the Geographical Survey Office of Sweden—Rickets Allmänna Kartverk—established a special photogrammetric office. To begin with, the map survey of the major part of Sweden to the scale of 1:10,000 was to be based on the use of Zeiss wide-angle cameras RMK 20/3030 and stereo-instruments of the Multiplex type. Later on, precision stereo-instruments were also brought into use, i.e. Zeiss Stereoplanigraphs and Wild Autographs A5 and A6.

The first few years' experience showed, however, that the design of these cameras did not satisfy reasonable requirements of accuracy for aerial map plotting.

The first problem was how to keep the film surface geometrically true. The cameras RMK 20/3030 are built for film which, at the moment of exposure, is pressed by a slight excess air pressure against the supporting plate. The design for performing this operation is shown in Figure 1. The lifting up and pressing down of the plate is effected by an eccentric driven by the camera motor, the force being transmitted by a spring. The excess air pressure, acting on the opposite side of the supporting plate during the exposure, must obviously not be

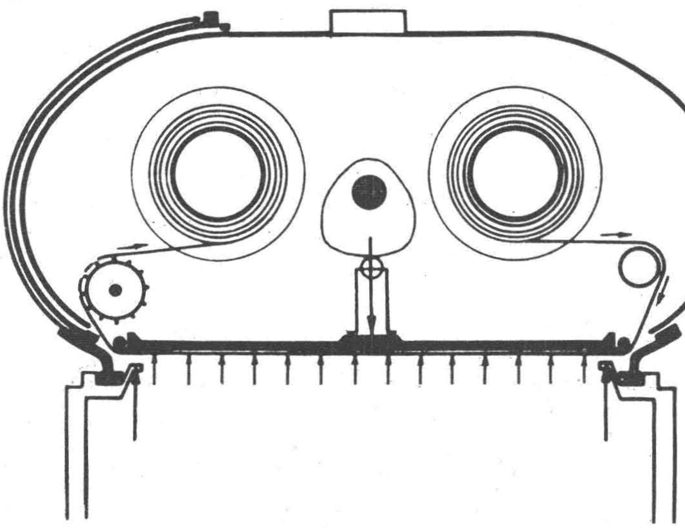

FIG. 1. Section through magazine of aerial camera.

so high as to cause the resultant of the air pressure to become equal to the force applied by the spring.

An excess air pressure of 30–50 mm. H_2O was recommended by Zeiss, but it soon became apparent that the film was not being flattened out by this low pressure so as to form a plane surface. Therefore, it was necessary to increase the pressure to the maximum, i.e. to about 90 mm. H_2O. Unfortunately, this pressure also proved to be insufficient for the purpose. About this time, it became clear that it was necessary to reconsider the whole question of making film surfaces plane and to rebuild the cameras according to a new design.

The second problem was the lens distortion. Gradually, it also became evident that the lens of the Topogon objective was impaired by a distortion of such a magnitude that, in practice, it was impossible to neglect its effect. The theoretical analysis of the corrections to be applied for eliminating the effects of lens distortion, however, was a very complicated task, as the form of the distortion curve was then unknown. Not until about 1940 could the true distortion curve be obtained, and its effect on the stereo-measurement be accurately investigated. This effect was found to be considerably larger than previously there had been reason to suppose. Thus, the height corrections necessary for eliminating the errors caused by the lens distortion amounted to no less than 10 m. in the centre of the stereo area. As a result of the theoretical analysis, corrections could be applied to the stereo-measurement. The height correction nomograms resulted in so-called correction-graphs varying in appearance according to the magnitude of the forward overlap. These correction-graphs are now in regular use for the thousands of pairs of pictures that are at present handled annually by the Geographical Survey Office.

DESIGN FOR PLANE FILM SURFACES

GENERAL CONSIDERATIONS

Any attempt to solve the problem of plane film surfaces along the lines, shown in Figure 1, must fail for the following reasons.

The stiff supporting plate is placed upon a frame which is also rigid. On the centre of the plate, a perpendicular force is acting, causing reaction pressure along the edges of the frame. The excess air pressure brought into play upon the film side of the plate reduces this reaction pressure. If the air pressure is gradually increased, the reaction pressure will eventually disappear, and the stress of the spring will balance the excess air pressure. For the camera RMK 20/3030, the upper limit was found to be about 100 mm. H_2O. If this air pressure is not sufficient to effect a satisfactory flattening out of the film, it cannot be further increased as this would produce a compression of the spring, and a lifting of the supporting plate from the frame. The result would be that, to maintain a state of equilibrium, the compressed air in the camera body would escape through the gap between the frame and the plate, thus reducing the excess pressure. As a consequence, the static condition would be changed into a dynamic one.

From a technical point of view, the mechanical lifting and resetting of the supporting plate in combination with the flattening of the film does not seem to be satisfactory. These two operations should be separated from each other. Further, the pressure of a considerable force on the centre of the plate must result in a deformation of the plate and thus, in a systematic deformation of the optical model.

LABORATORY TESTS

In the first instance, it was necessary to analyse the pressure conditions set up in the camera body and in the magazine. For this purpose the camera was driven by its regular motor, and manometers were connected to the camera and to the magazine.

In order to examine the buckling of the film, a glass plate was manufactured, engraved with sand-blasted grooves forming squares over its whole area. By placing this glass plate upon the film, it was possible to study the geometrical surface of the film during the moment of exposure. From the first tests it became evident that an excess air pressure of about 30–50 mm. H_2O, as recommended by Zeiss, was quite insufficient to flatten satisfactorily a film area of the size in question. Nor was it sufficient to increase the pressure to the permissible maximum of about 100 mm. H_2O. These tests showed clearly that quite another pressure must be used to ensure a plane surface.

NEW PRINCIPLES

The results of the investigations made it clear that a rebuilding of the camera was necessary. The fundamental considerations were:

(1) to rebuild the magazine for an excess or vacuum pressure of quite another magnitude, calculated to have an upper limit of about 1,000 mm. H_2O
(2) to produce this pressure
(3) to apply the pressure upon the film area in such a manner that the flattening process eliminates all buckling due to the elasticity of the film material
(4) to apply the pressure on the film area without deforming the surface of the supporting plate.

The results of the examination made it evident that the design for excess pressure in the camera body had so many snags that a change to a state of vacuum was to be preferred.

It was soon realized that, if the film could be pressed against the supporting plate in such a manner that vacuum suction commenced in the centre of the plate, and then, concentrically, extended to the outer parts of the plate, the risk of buckling would be eliminated. For this reason, the suction grooves on the plate surface must first be concentrically arranged, and, secondly, the connecting suction holes to the vacuum chamber through the plate should successively decrease in density per unit of area from the centre outwards. Further, it was considered essential that the total groove volume should be increased in comparison with types of supporting plates in general use; an opinion justified by the tests executed. This in turn led to the introduction of thin walls between the grooves, as Figure 2 shows, following the principle of "an infinitely great number of supports, infinitely thin." The width of the grooves was taken as 1 mm., owing to the elastic properties of the film. Figure 2 gives a comparison between the new supporting plate and the original Zeiss plate. It should be added that, in principle, the concentric arrangement of the grooves is in itself a guarantee for a theoretically correct flattening of the film. The vacuum chamber at the back of the supporting plate is arranged, by means of a plate cover braced against the plate, so as to avoid any deformations of the plate when vacuum is applied.

The next question was a regulating device for the opening and shutting of the suction conduit. For this reason a regulator was designed as shown in Figure 3 the outer cover being removed. On the inner part, fastened on the magazine, the suction conduit is seen, just exposed by the opening, cut out in the thin

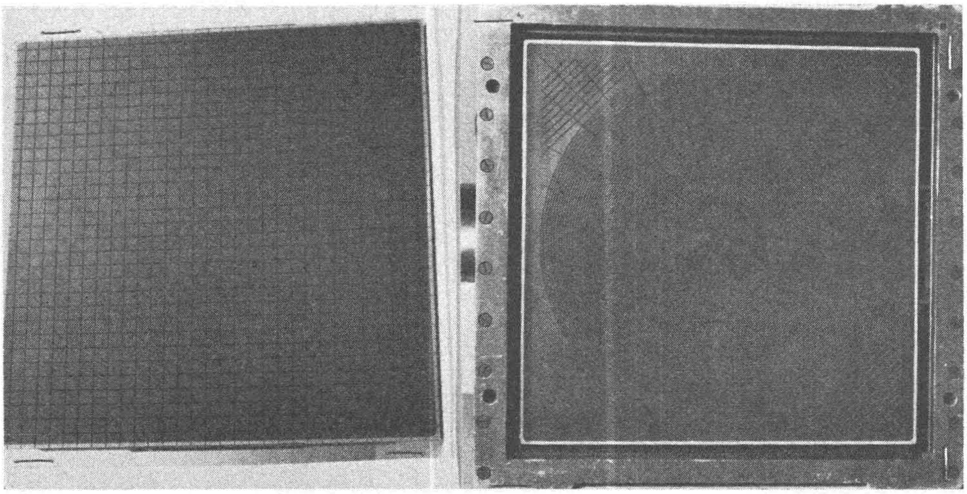

FIG. 2. Film supporting plates of the aerial camera: Left: Zeiss design. Right: new design.

regulating membrane. This membrane is connected to the driving-axis of the magazine and regulates the suction by means of the opening in the membrane. The design is very sensitive and ensures that the film has a plane surface at the exact moment of exposure.

Only the problem of the vacuum pump now remained. This was easily solved by fitting a suitable pump directly onto the shaft of the aircraft motor.

Finally, it must be also mentioned that the camera has been rebuilt in other ways. In particular, an indicator registering the contact between the supporting

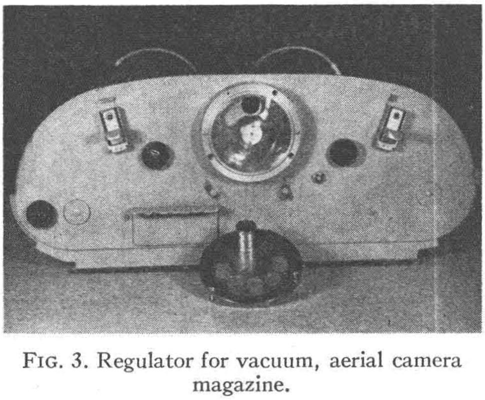

FIG. 3. Regulator for vacuum, aerial camera magazine.

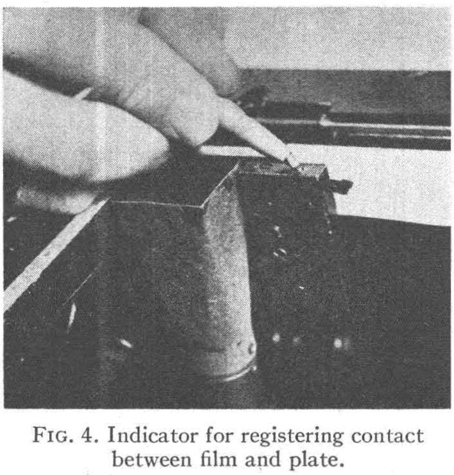

FIG. 4. Indicator for registering contact between film and plate.

plate and the frame is an important device. The indicator box is seen in Figure 4. On the right there appears a metal tongue made to protrude by pressing a small button, visible on the upper side of the box. The indicator is fixed on the inner side of the frame, and in such a position that the button is sticking up 0.1 to 0.2 mm. above its plane. The tongue, of course, will be registered on the film during exposure. By two such indicators, applied in two diagonal corners of the frame, it is obviously possible to obtain a check on the position of the supporting plate at the moment of exposure.

LENS DISTORTION COMPENSATION

When dealing with a pair of stereoscopic pictures marred by certain lens distortion, in instruments having lenses which are practically speaking free from such distortions, it must be accepted as a matter of course that errors will arise when reconstructing the pencil of rays. For example, this will occur when wide-angle pictures taken by the camera RMK 20/3030 are to be plotted in a Multiplex instrument. This feature is illustrated diagrammatically in Figures 5 and 6. The corresponding rays s_I and s_{II} of the pencil of rays in a pair of pictures I and II which, when the photographs were taken, originated from a point on the object, no longer intersect at a point, but cross in space, when the pencil of rays in the stereo-instrument is reconstructed. Thus, in the intersecting plane illustrated in the figure, a discrepancy will result in the form of a parallax Δp which may be divided into a horizontal component Δp_h and a vertical component Δp_v.

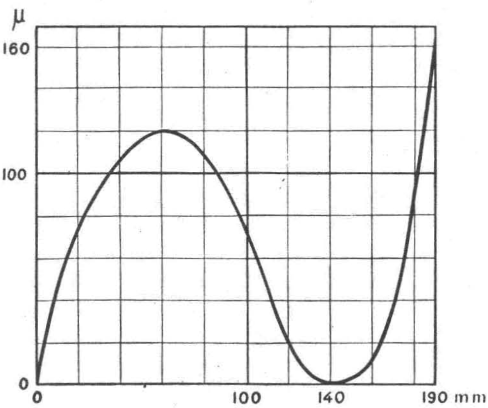

FIG. 5. Distortion curve for Zeiss Topogon lens f/6.3/20.

Of these, the horizontal component may obviously be directly dealt with by a correction for height at the point in question. The vertical parallax, on the other hand, results in a blurred image of the point being given, which is manifested as a lack of definition scarcely visible to the human eye.

Whereas horizontal parallax at the actual plotting of a pair of pictures substantially affects the height measurement, the vertical parallax causes discrepancies in the orientation of the stereogram, and particularly in relative orientation.

A mathematical treatment of the problem would have to be based upon the formula for lens distortion, and it would be a very troublesome and tedious business. The present problem is really a technical one which on practical grounds should be solved by graphical methods.

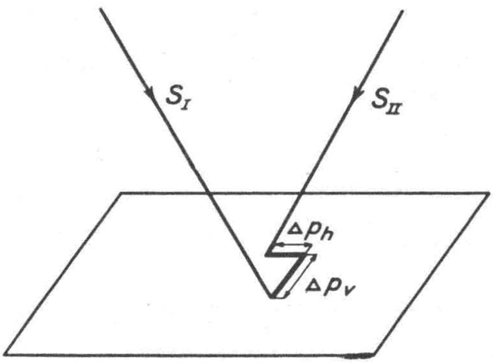

FIG. 6. Horizontal and vertical parallaxes caused by the lens distortion of a pair of pencils of rays with the stereo-area.

HEIGHT CORRECTION AND VERTICAL PARALLAX NOMOGRAMS

Graphical treatment of the problem may be applied in connection with the purely mathematical deduction. To render their practical application possible, the parallaxes deduced are converted, in relation to the ordinary map-plotting data for Sweden, to the scale of 1:10,000. These data are: an altitude of flight above the ground of 4,000 m. and a camera-constant of 200 mm., i.e. aerial photographs to a scale of 1:20,000. The investigation has been made for various values of the overlap, viz. 55, 60, 65, 70 and 75%.

Height correction nomograms, referring to the overlap 65% are shown in Figures 7 and 8.

The height correction nomograms, which are symmetrical in respect both to the base and to the perpendicular through its centre, present the following characteristics. At the central portion of the stereo-area, a depression is formed which rises towards the zero-plane in the proximity of the principal point of orientation. Outside of this depression, the curves show larger and larger nega-

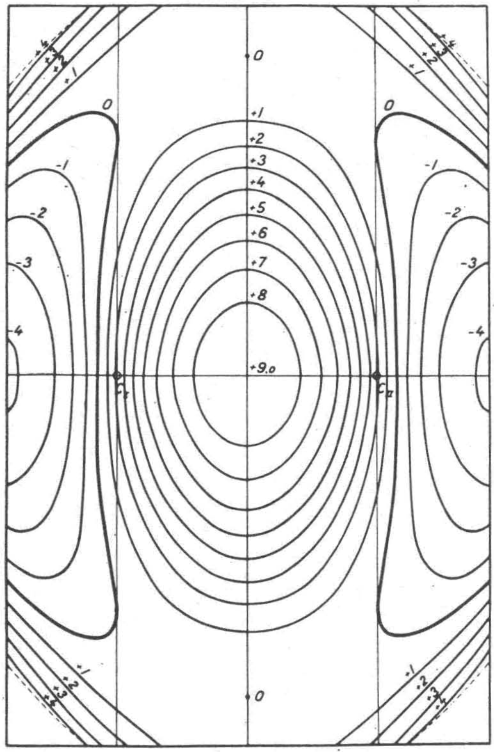

FIG. 7. Height corrections, indicated in meters, caused by the lens distortion of the wide-angle Topogon objective, f/6.3/20, with a forward overlap of 65%.

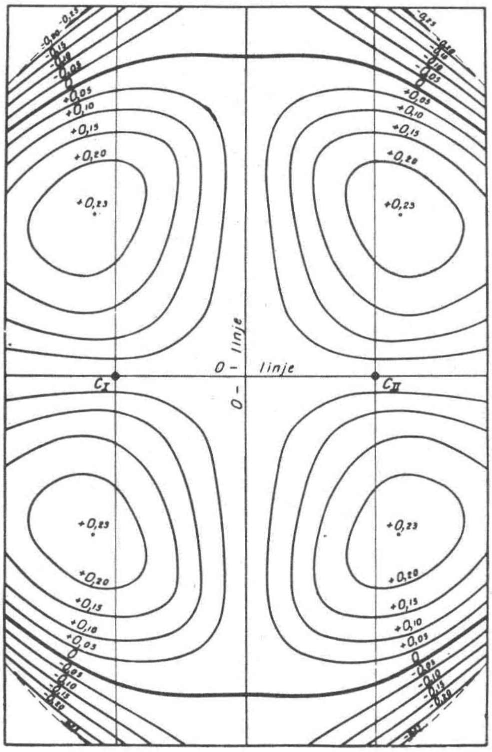

FIG. 8. Vertical parallaxes, indicated in millimeters, caused by the lens distortion of the wide-angle Topogon objective, f/6.3/20, with a forward overlap of 65%.

tive values with increasing proximity toward the margin. At the corners of the stereo-area, positive curves again appear. The investigation is carried out as far as 190 mm. from the principal point, the limit of the lens distortion curve. As was to be expected, the highest values for a curve of this type are obtained at a forward overlap of 75%. The correction amounts to no less than +11.1 m. at the centre of depression, to −6.3 m. at the lateral parts and to +6.9 m. in the corners. Furthermore, it is to be noted that the first value is approximately proportional to the forward overlap, i.e, the corrections are 11.1, 10.0, 9.0, 8.0 and 7.0 for the forward overlaps of 75, 70, 65, 60 and 55%, respectively.

The vertical parallax nomograms plotted to the scale of 1:10,000, also have two symmetrical axes which at the same time are zero-lines. The vertical parallaxes are zero along curves which follow a gently curving line at the upper and lower portions of the nomograms. These represent, of course, the most interesting curves of the nomograms, and on these curves the points of orientation in accordance with the "six-point system" must be chosen, theoretically, at those points where the axes of the image intersect the zero-curves. These curves, therefore, are of special importance to the reciprocal orientation of the stereogram. If, in that orientation, these points are incorrectly selected, residual parallaxes will occur, and as a result the entire measuring process will be erroneous. If it should be difficult to find suitable objects in the picture at precisely those six-points, other adjacent objects should be employed which must, however, be necessarily selected along the zero-curve, and not in a direction perpendicularly towards it.

The vertical axes of the image through the principal points I and II are intersected by the zero-curve at a distance from the outside edges, varying between 38 and 18 mm. in the original picture of dimensions 300 × 300 mm., corresponding to forward overlaps of 55 and 75% respectively.

Between the zero-curves and the symmetrical axes, a maximum parallax is brought about which, in this case, is reasonably constant. Thus, it varies only from +0.24 to +0.20 mm. for the forward overlaps of 55 and 75% respectively. Outside the zero-curves the parallaxes increase towards the corner. The investigation is carried out as far as 190 mm. from the principal point, the limit of lens distortion curve.

THE CORRECT IMAGE

In photogrammetric stereo-measurement, attempts have been made hitherto to redirect the pencil of rays distorted by the camera objective by subjecting the rays to a compensating refraction in the stereo-instrument. This can be done either by means of a lens especially designed for that purpose or by means of a compensating plate serving the same purpose. This method suffers, however, from the inconvenience that the camera and the stereo-instrument have to be fitted exactly to each other from the optical point of view, with the consequence that a particular stereo-instrument can be only used for pictures which have been produced in a certain type of camera.

All of the above mentioned methods are based on the use of films or plates marred by lens distortion, and aim to correct or compensate their imperfection, by means of distorted images in the stereo-instruments. In contrast, the authors have examined the idea of compensating the distortion in the camera itself, resulting in a correct image. The object is to provide an arrangement for eliminating the effect of lens distortion on a film, and thus to ensure the production of negatives free from distortion which can be reproduced or stereo-treated in normal instruments, without any special precautions. Further reprojecting operations, which would otherwise result in optical losses and, consequently, in impaired pictures, are entirely avoided. The arrangement signifies that the pencil of rays will strike the film at exactly the correct radial distance, thus satisfying the formula

$$r = (c + \Delta c) \tan w \tag{1}$$

The influence of the lens distortion will be apparent from Figure 9. A certain pencil of rays which theoretically would strike the image plane at point P_i', will be deflected, due to lens distortion, so as to fall on point (P_i'). Since the supporting plate is shaped in such a manner that point (P_i') will be displaced to the position indicated at P_i, the distance c for the said point will be increased to $(c+x_i)$. Consequently, the deflected ray of light will strike the film at the same distance $(r_i+\epsilon_i)$ from the optical axis as would the theoretically correct ray. Using the symbols shown in Figure 9, it follows that

$$\tan w_i \approx \tan w_i' = \frac{\epsilon_i}{x_i} = \frac{r_i}{c}$$

and generally

$$x = c \cdot \frac{\epsilon}{r} \tag{2}$$

The supporting plate for the film can be shaped in such a manner that the image of any point, in spite of the lens distortion, will be located at the correct distance from the optical axis. As a result, the pencil of rays will strike the film at exactly the same radial distance as if the lens were free from distortion. It is obvious that, for each pencil of rays which is distorted inwardly towards the centre of the image, the corresponding point of the supporting plate has to be lowered, and vice versa.

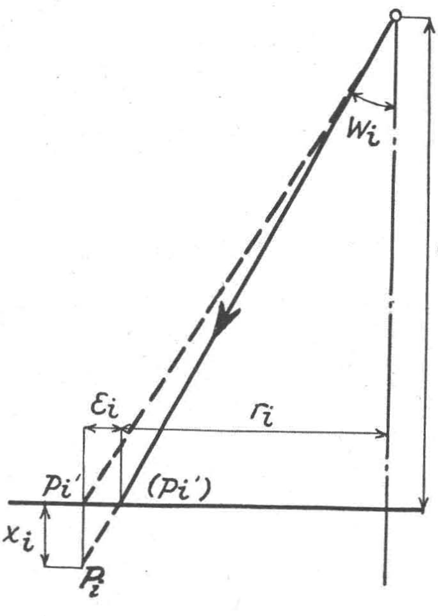

FIG. 9. Diagram showing the distorted and the correct pencils of rays.

According to the formula (2) a mathematical treatment has been made for the camera RMK 20/3030 with Topogon objective. The result is diagrammatically shown in Figure 10, where the curve represents the shape of the film, which is necessary to eliminate the effect of lens distortion entirely.

Of course, certain objections may be raised against the design. Thus, the curving of the film in two dimensions will cause a strain in the film and, further, the definition of the image may be influenced by the variation of the camera-constant. A minute examination of negatives taken has, however, proved that, in practice, no trace of imperfection can be found due to the causes mentioned.

Tests were made by using a magazine belonging to the camera RMK 20/3030, and fitted with a supporting plate shaped according to the principles mentioned. Aerial tests were made: the same strips being taken by using both a film magazine of the ordinary type and of the new type. In this manner the results could be directly compared. In order to get a sufficiently level ground surface, an area was chosen in the archipelago of Stockholm with a great many small islands. Along the shores of these islets it was possible to observe a great number of details evenly distributed over the whole stereo area, and these were used as check points.

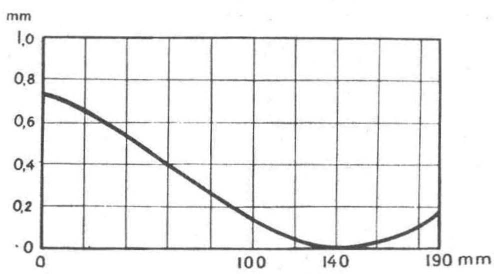

FIG. 10. Section through the curved film-supporting plate of the aerial camera.

Since the lens distortion has a more marked influence on the z-coordinates than the x- and y-coordinates, a stereo-measurement of the heights only of the check points is required in order to compare the result obtained by the use of the ordinary supporting plate and of the curved one.

The photographs, taken by the ordinary supporting plate and designated "ordinary" pictures, were plotted in the autograph A6 of Wild, at first without compensating plates and then with such plates. In the first case, correction values were obtained by use of the theoretical diagrams. These values are shown in Figure 11 within brackets. Then the same measurement was repeated with compensating plates. In actual fact, all the check points have zero level, the divergences of 1 m. found being equal to the mean error of the instrument. The measurement of the "curved" pictures, of course, was made without compensating plates. The result is shown in Figure 11. Here, for a still greater number of check points, zero level has been obtained.

The result of the measurements shows that the curved plate exactly compensates the lens distortion.

The use of curved supporting plates for eliminating lens distortion will make it possible to use more luminous wide-angle objectives with larger apertures, which are cheaper to manufacture than those complicated objectives with particularly small lens distortion, and still obtain perfectly correct images.

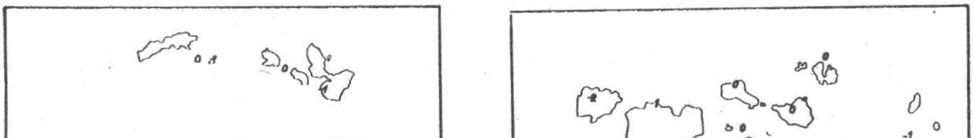

FIG. 11. Result of measurements in the archipelago of Stockholm. Left: "Ordinary" pictures. Right: "Curved" pictures.