CONTRIBUTIONS TO THE THEORY AND MECHANICS OF PHOTO-INTERPRETATION FROM VERTICAL AND OBLIQUE PHOTOGRAPHS*

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EDITORIAL NOTE: Although shorter articles are generally preferred by readers, the Publications Committee presents this paper in full because the subject has received relatively little consideration in the past and because it seems that the treatment constitutes a singular contribution to the field of photogrammetry.

CONTENTS

I. PHOTO-INTERPRETATION, INTELLIGENCE, AND INFORMATION

WHEN and if Hollywood "does" the photo-interpreter, as they have "done" the newspaperman, the gangster, the treasury agent, and other work-a-day professionals, we may be assured that the emergent portrait of the photo-interpreter will be an interesting one.

A professional-looking individual, wearing the thick-lensed glasses conventionally associated with the scholar, will be seen seated at a large and expensive executive desk, reading the latest paper on Einstein's unified field theory. A knock at the door precedes the hurried entrance of a breathless messenger who deposits several aerial photographs in front of our hero. The latter whips out a pocket scale and microscope; after a few seconds of close study of areas on the photographs (which the lay observer might reasonably interpret as scratches, or fly-specks) he composes a Top Secret teletype to the appropriate Commanding General: "Have detected underground factories making Type 3AG bronchial tube assemblies for Type UW Schnorkels."

The preceding bit of fantasy will produce mirth in most photo-interpreters. Yet part of the portrait which emerges—that of an individual able to examine

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minute aggregations of silver grains on a photograph, and from this examination, to determine the nature of the corresponding ground area—is not exaggerated. It has been a constant source of amazement to the writer that photo-interpreters were able to derive accurate data from the photographs they had during World War II—especially when one considers the low quality of the photographs and the even lower quality of the analytical equipment used by the photo-interpreter.

That the photo-interpreter has performed a remarkably accurate job was borne out by the post-war studies of the U. S. Strategic Bombing Survey. The numerous reports of that group constitute a little-known and invaluable source of basic data about World War II, and contain much of relevance to the photo-interpreter. The Symposium on Military Photographic Interpretation, published in this journal (Vol. XIV, No. 4, pp. 453–521) describes in considerable detail the varied types of analysis conducted by the photo-interpreter.

In a generalized sense, anyone who looks at a photograph (whether aerial or ground), and who obtains data from the photograph—whether these data are engineering, military, or esthetic in nature—is a photo-interpreter. The particular type of photo-interpreter with which we are herein concerned is a military photo-interpreter, and he examines aerial photographs. These photographs may be negatives or positives, paper prints or transparencies, color or black or white; they may be made at dawn, noon, dusk or at night; they may vary in size from 16 mm. gun-camera frames to 9 inches by 18 inches or larger; they may be produced by cameras varying in focal length from one inch to 240 inches; they may be made at altitudes ranging from 50 feet to 50,000 feet. The only common denominators in this situation are that the photographs will almost always be of a situation or place not accessible to the photo-interpreter for pedestrian and personal observation, and that the photo-interpreter must analyze these photographs in a hurry.

Before we consider further the role and functioning of the photo-interpreter, it will be well to place photo-interpretation in proper military context. The ultimate objective in all this activity is Military Intelligence—the collection, evaluation, and dissemination of military information on which to base strategic and tactical decisions about strategic and tactical operations. Aerial reconnaissance is one of the most important agencies securing the basic and raw data. Aerial reconnaissance may be defined as the operation of securing information and data by airborne means. These data are not necessarily only photographic, for both older and newer forms of reconnaissance will continue to be used. Visual, weather, infra-red, electromagnetic, radar, magnetometer, and radiological reconnaissance (among other forms), will certainly be part of future reconnaissance activities.

We will concern ourselves in this discussion only with photographic reconnaissance. An unfortunate distinction was drawn in the past between what was called "reconnaissance" photography and what is known as "mapping and charting" photography. The term "reconnaissance" photography has been used to describe photographs taken for the non-mapping purposes—wherein detailed information about enemy activities was the primary goal. Of course the fact that some charts were made from non-charting photography, and that detailed information was often secured from photographs made for mapping and charting purposes, largely obviated the hard and fast distinction. Actually "aerial reconnaissance photography" covers both forms of aerial photography, for both forms eventually yield intelligence. Maps and charts, and the photography preceding them, are essentially geometrical intelligence—furnishing accurate data on distances, azimuths and the like. Sometime ago the writer proposed a pair of definitions, which, discounting the element of levity, clarify the matter:

> Mapping and charting photography gets information about the character of the terrain, and "intelligence" photography gets information about the characters on the terrain.

In both cases, the photography goes to the photo-interpreter who reports the information on the photograph. It is still only data and information, and does not become intelligence until another staff section evaluates this information, correlates it with other information, and produces intelligence.

It is perhaps in order to discuss briefly the much abused term "intelligence." The most illuminating discussion yet seen by the writer is in Kent's *Strategic Intelligence* (2). In the preface Kent discusses the three parts of his book:

> "In part I, I consider intelligence as a kind of knowledge ('what intelligence have you turned up on the situation in Colombia') . . . . In part II, I consider intelligence as the type of organization which produces the knowledge ('Intelligence was able to give the operating people exactly what they wanted') . . . . Part III considers intelligence as the activity pursued by the intelligence organization ('The intelligence work behind that planning must have been intense')."*

It is perhaps trite to observe that an aerial photograph is of no significance if no one looks at it. Unless, after looking at the photograph, someone makes a positive decision, the photograph is still of no significance. This decision can be, for example, that there is no information on the photograph, or that no action should be taken; in either case, the decision is positive. Thus the photo-interpreter is the middle man between the operating people who produce the aerial photographs, and the people who evaluate the information abstracted from the photograph.

That the photo-interpreter has also been the forgotten man in the past is also true; there will be further discussion about this, and a program for alleviating the situation, in part II of this paper.

This program will be for equipment and techniques; however, the writer does not imply that a gadget can take the place of brains, for one cannot determine organizational strength by multiplying the number of people by the average I.Q. It is equally untrue on the other hand that the answer or solution to every problem is a new gadget, usually bigger and with more knobs and dials and gears than the previous gadget. There is, however, plenty of room in the photo-interpretation field for better instruments and techniques.

Up to now we have been discussing "information." Just what is this information, and how is it measured? This problem of defining and measuring the amount of information on a photograph has been given much attention in the last ten years. It is clear that a photographic print of a blank negative will show no detail, and it is equally clear that a "sharp" photograph will permit observation of more detail than one less "sharp," for smaller images can be resolved in the first photograph than in the second. The amount of information has been specified, for the last few years, in terms of resolving power exhibited in the photograph. Because, for analytical purposes, a number is preferable to a non-numerical evaluation, black and white line targets, varying in size and spacing by the sixth root of two ($2^{1/6} = 1.125$) are used in aerial testing of camera systems. These target elements are composed of two sets of three lines each, with the two sets oriented at 90° to each other, with the line spacing equal to the line width, and the lines white on a black background.

* Quoted by permission of the publisher, Princeton University Press.

The Air Force now has resolving power targets at several locations in the U.S., and these are in constant use. From photographs of these targets, the resolving-power in any angular zone of the photograph can be determined. If the resolution in both directions in a given zone of the plate is given by R_i and T_i lines/mm. (radial and tangential) and the plate area corresponding to this zone is A_i, then the average resolution in determined by the following formula, proposed by the writer in 1942.

$$\overline{R.P.} = \left[\frac{\sum R_i T_i A_i}{\sum A_i}\right]^{1/2}. \tag{1}$$

The summation is carried out over the whole plate, and therefore $\sum A_i$ = plate area. Eq. 1 is of course the discrete approximation to the more general double integral which could be written in its place, yielding

$$\overline{R.P.} = \left[\frac{\iint_A RT dx dy}{A}\right]^{1/2} \tag{2}$$

where R and T are the radial and tangential resolution point functions, and A is the plate area and the integration is carried out over the entire plate.

Although the writer feels able and willing to attach some significance to this average, many other workers in this field have criticized both the method of measurement and the method of averaging. It should be noted that in the field of resolution measurement, each worker seems to have an inner urge compelling the invention of a new type of target and a correspondingly new method of averaging data. None of these systems, including ours, finds universal acceptance, thus providing much fuel for symposia and discussions.

The writer finds much of merit in all these systems—the British low-contrast Cobb target, Dr. Howlett's low contrast annular target (Canada), and others. Their proponents have argued persuasively, well, and at length about their targets and measurement systems, and the writer is willing to concede the validity of much of their criticism of the U.S.A.F. system. Eventually the Air Force will either find a new system or establish the correlation between the various systems.

The aerial photographer or photo-interpreter can, and does, ask embarrassing questions. He may say "All this talk of resolving-power and lines/mm. leaves me cold. What I want to know is this. If my particular camera system will resolve say, 30 lines/mm., what detail can I see in the photograph of airplanes sitting on the ground?" This difficult question cannot now be answered with confidence; however, a program designed to answer such questions is being started. Various types of military material, such as trucks, aircraft, guns are being placed in the immediate vicinity of some resolution targets. The line targets will then be imaged close to the photograph of the assorted military *objets d'art*, and the correlation of lines/mm., with a given focal length lens, and ground detail will be established.

In spite of the numerous and not easily answered objections to the high contrast line target, it has served what may be its most important purpose—that of choosing the better of the two lenses or two camera systems. Independent of the validity of the number calculated by Eq. (1) there is enough evidence at hand to substantiate the statement that the serial grading of lenses by their photograph-making ability (as determined by experience) will not differ from the serial grading of the same lenses using the calculated average resolving-power.

Studies conducted by the N.D.R.C. during the last war, and similar studies pursued after the war, indicate that resolution is only part of the story. Microscopic detail contrast is also important, and is associated with the "cleanness" of a photograph. It is well known that photographs exhibiting high resolution and good tonal separations in the small details, can be sorted out from poorer quality photographs without any optical aid except the eye, i.e., without actually examining the microscopic detail.

This matter is really quite sophisticated, and further detailed discussion in this paper will lead us too far afield.

A fresh approach may be to examine the actual nature of photo-interpretation. The writer feels that Professor Norbert Wiener's brilliant and provocative *Cybernetics* (3) can offer much help in the understanding of the psychophysics of photo-interpretation, and in the formulation of a new measure of information. This book is subtitled *Control and Communication in the Animal and the Machine*. The jacket carries the following interesting message (to which should be appended "and photo-interpreters"): "A study of vital importance to psychologists, physiologists, electrical-engineers, radio engineers, sociologists, philosophers, mathematicians, anthropologists, psychiatrists, and physicists." In Chapter VI (Gestalt and Universals, p. 156 ff.) Wiener discusses the problem of recognition of objects by their forms (our problem—what makes a photo-interpreter think that a little gray blob on a piece of flat paper is the image of a medium tank). Thad Jones (4) presents an interesting and relevant discussion on this recognition problem, describing the mechanics of recognition, by Mediterranean Theatre photo-interpreters, of loose scattered grain in photographs taken from 20,000′. Wiener (3 p. 18) describes the development of a statistical theory of information in which the unit of information is transmitted as a single decision between equally probable alternatives. This idea occurred to several people at the same time, according to Wiener: the English statistician R. A. Fisher, working in mathematical statistics; Dr. Shannon of the Bell Telephone Laboratories, in connection with information coding problems; and Wiener, in his work on noise and message in electrical filters. The analogy to our problem—of identification of large ground objects from small plane images of them (or in the case of stereo-viewing, from distorted spatial images) lies in the consideration of the mechanics of recognition of an outline; and Wiener (3 p. 156 ff. in particular p. 159.) shows that " . . . three-fourths of the fibers in the optic nerve respond only to the flashing 'on' of illumination. We thus find that the eye receives its most intense impression at boundaries, and that every visual image in fact has something of the nature of a line drawing."

From a consideration of these stimuli, or decisions, Wiener shows that the amount of information, as he uses this concept, is related to the notion of entropy in classical statistical mechanics and thermodynamics. He finds that as the amount of information in a system is a measure of its degree of organization, so entropy of a system is a measure of the degree or disorganization of the system; hence amount of information is the negative of entropy (3 p. 18). In Wiener's analysis, the information carried by a precise message in the absence of noise is infinite; in the presence of a noise, the information carried is finite, and approaches 0 as the noise increases. The analogy to the photo-interpretation problem of measuring the information in a photograph, would involve calling directional blurrings (motion, vibration) distortion. Poor contrast, fog, haze, and grain effects, are, in this analogy, noise.

Our problem is complicated by other considerations, when we wish to evaluate "useful" information. For example, suppose an object can be identified correctly 95% of the time when the resolution with a given altitude and focal length combination is say, 20 lines/mm., and cannot be identified more than say 5% of the time when, under the same conditions, the resolution is 10 lines/mm. Is the 20 lines/mm. twice as good as the 10 lines/mm.? Certainly not in this case; the relative value ought perhaps to be some function of the probabilities, approximately 20 to 1. Were the probability of identification at the 10 lines/mm. level close to zero, the relative value of two resolution levels would approach infinity—for the particular identification job at hand.

Some very interesting and fundamental work along this line is being conducted by the very able group under Dr. D. E. Macdonald, of the Boston University Optical Research Laboratory, which, under contract to the Photographic Laboratory, is doing much other work basic to an understanding of, and progress in the field of photo-interpretation.

Eventually, as the powerful methods of modern statistics, psychophysics and the related sciences (including cybernetics), are focussed on these engrossing problems, we may expect genuine advances. The writer has been presenting speculative considerations only; perhaps someone can convert the speculative into the substantive.

PART II—A PROGRAM FOR IMPROVING PHOTO-INTERPRETATION

INTRODUCTION

Everyone is willing to concede the utility, value, and importance of aerial reconnaissance. The ultimate aim of aerial reconnaissance, the sole reason for its existence and the only justification for spending millions of dollars on research and development, is the eventual production of intelligence data. The long neglected photo-interpreter stands squarely in the middle between the aerial reconnaissance operation on the one hand and the Intelligence Staff on the other hand.

In general, when one wants to achieve a better solution to a long standing technical problem, two avenues of approach are open. He can perform better experiments, getting more and better data, and he can use more powerful methods of analysis. The analogous reconnaissance problem can be defined as a job of getting more information onto the negative and print, and getting more information off of the negative and print. A third possibility, of course, is hiring more and/or better analysts; the analogous Air Force problem lies in the field of psychological selection and testing of interpreters, and their subsequent training. This particular problem is under thorough study and investigation by the Boston University Optical Research Laboratory under Duncan E. Macdonald. No attempt will be made in this paper to discuss their work on this problem. Four main lines of attack on this overall problem are being pursued by the Air Force through its Photographic Laboratory at Wright-Patterson Air Force Base. In order to place in proper context the major and technical portion of this paper, it is desirable and necessary to discuss these four points:

A. INCREASING RESOLUTION (OR DEFINITION) IN THE AERIAL PHOTOGRAPH BY IMPROVEMENTS IN AIRBORNE RECONNAISSANCE SYSTEMS AND TECHNIQUES

The battle for definition has been a long and arduous one, and it is not yet won. Previously published papers by Col. George W. Goddard (5) and A.H.

Katz (6, 7) have described in considerable detail the problem of getting what may be loosely called "sharper pictures." Better lenses, better shutters, continuous strip cameras, image motion compensation applied to conventional cameras, anti-vibration mounts, better optical windows, pressure-temperature focussing of long focal length lenses—all of these have been and continue to be developed for one and for only one reason. They have been developed to capture more ground detail on the aerial negative. Of course poor techniques of exposure could invalidate and render useless most of these improvements. It is abundantly clear and well established that this phase of the research and development program has associated with it most of the drama, publicity and general interest.

B. INCREASING RESOLUTION IN THE LABORATORY BY IMPROVED DEVELOPMENT AND PRINTING TECHNIQUES AND THE USE OF POSITIVE TRANSPARENCIES

Resolution captured in the air at great expense, at great inconvenience to pilot and crew who must fly long missions under uncomfortable environmental conditions and, in time of war, with considerable danger, can be easily lost or greatly reduced in the Processing Laboratory. High resolution (i.e., 20, 30, or 40 lines/mm.,) is a tenuous and evanescent substance; in the plain language of aerial photographer with many years of experience "it's easily loused up." Resolution brought back in the latent image can be lost by poor processing techniques.

Assuming everything has gone well so far—a high quality lens, high shutter speed, proper image motion compensation, were used properly and good negative development followed in the laboratory—the high quality negative is now printed on contact paper; it is at this point that much information is lost. The best contact printer designed to make paper prints must perforce employ a broad light source, to permit dodging. The presence of this broad light source has been shown by the Photographic Laboratory to lower resolution in the process of transferring detail from negative to print. In yet unpublished experiments, the Photographic Laboratory has shown that a standard resolving power target (which goes down to 200 lines/mm.) when printed on a standard printer will show no more than 40 lines/mm. in the contact paper print; in fact this figure is obtained only under optimum conditions. This limitation is not imposed by the paper however, for when a reflector photoflood lamp was used as a slight source approximately 5′ away from a standard printing frame, 200 line/mm. were printed. It does not follow that the print of an aerial negative which just barely exhibits a resolution of 40 lines/mm. will also show 40 lines/mm. A further deterioration, to perhaps about 25 lines/mm., will take place.

The very choice of a reflection (or paper) print is bad, for the maximum range of density on paper is far less than that in the original negative. The answer to this problem has been at hand for a long time: use of point-source printed positive transparencies for photo-interpretation purposes. Positive transparency material, having a scale or range of tonal values far exceeding that of the paper print, requires no dodging and hence can be printed by the technique which preserves high resolution. Use of the term "point source" is perhaps bad in this connection, for any light source whose diameter is of the order of 4% of the distance from source to printing frame is effectively a point source for printing purposes.

It is realized by all that it is less convenient to look at a transparency than at a paper print. One must have a light source behind the transparency, and difficulties arise in stereoscopic viewing with the conventional pocket stereoscope. The writer and many other people feel that minor and easily minimized "inconveniences" of this type suffered by an analyst sitting at a desk in a comfortable environment do not compare with the inconvenience incurred in the securing of the reconnaissance negative. The use of positive transparencies, like the weather, has been discussed for years. At long last something systematic and definite is being done about it.

There is no question that in the future, sensitometric control, heretofore regarded as an esoteric or black art, will be a routine matter in the field laboratories. In a technical world of radar, B-36's, and the atomic bomb, the use of the H & D curve does not appear to be a major complication.

C. IMPROVING THE PHOTO-INTERPRETER'S OPTICAL AND MECHANICAL AIDS

In a speech to the November 1948 RAF, RCAF, USAF Reconnaissance Symposium held at Topeka, Kansas, the writer had the following to say, among other things, "this matter of photographic-interpretation strikes us as being a very much unbalanced situation, wherein we may take a million dollar airplane, a hundred thousand dollars worth of cameras, a half dozen rolls of film, one of which (for the K-40 camera) is going to cost about $400 . . . we take off on a very hazardous mission in the sense of military and social economics . . . when the photo-interpreter gets around to abstracting the information he uses a 10 cent magnifying glass. . . . "

The numerous reasons for this tremendous disparity between reconnaissance equipment and interpreter's equipment are simultaneously operational, historical, technical and illogical. A program is underway to develop better and high quality monocular and stereoscopic viewers and magnifiers, better light tables (for illumination of the transparencies), better measuring devices and better techniques in using these devices. Again, it is hoped that before too long proper balance will be restored.

D. DEVELOPING FAST MEASURING AND COMPUTING TECHNIQUES, SIMPLE IN USE AND WITH ADEQUATE PRECISION AND ACCURACY

The photo-interpreter will shortly be confronted with very large numbers of oblique photographs. See Col. Goddard's paper (5). He will be overwhelmed unless radically new methods are developed which will enable him to measure lengths, distances, heights, and areas in the photographs which he analyzes. Photogrammetric techniques are in general unnecessarily precise for the photo-interpreter, because:

a. The accuracy of the photo-interpretation process is too low
b. The photo-interpreter very seldom needs accuracy or precision greater than several per cent, and
c. The price paid for these ultra precise techniques is tedium in use plus a requirement for considerable technical ability.

The major technical portion of this paper is concerned with the last of these four lines of attack on the problem of getting the information.

PART III. THEORY AND SYSTEMS FOR MEASUREMENTS AND COMPUTATIONS IN VERTICAL AND OBLIQUE AERIAL PHOTOGRAPHS

A. THE PHOTO-INTERPRETER'S SLIDE RULE

As noted in an earlier section, the photo-interpreter's primary concern to date has been the vertical photograph. The only equation he had to solve was the simple equation relating ground object size to image size, focal length and altitude (Figure 1 and Eq. 3/$G=HI/f$). This solution has been accomplished in several ways, ranging in complexity from individual solution of the problem for each photograph to the tables prepared by the R.A.F. during the war. In these tables each possible value of the scale, or Representative Fraction (R. F.), was tabulated, with successive values differing by small increments. For each of these values of the scale, a complete tabulation was made of the pairs of values for image size and the corresponding ground object size. It can be readily imagined that this collection of tables was very extensive and bulky. Scale, of course, had to be known or calculated before using the tables. Because only discrete values of essentially continuous variables were tabulated, interpolation for both scale and image size was necessary.

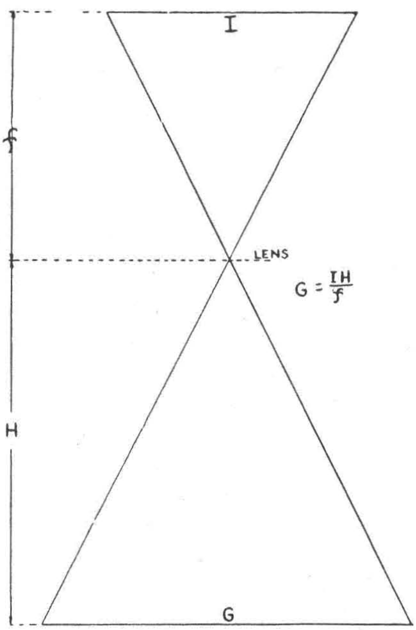

FIG. 1. Fundamental relationship between object size, image size, focal length and altitude, for vertical photographs.

All of these systems suffered from the same defect that would be apparent in comparing a multiplication table of numbers up to 1,000 (requiring a table with one million entries) with an ordinary 10″ slide rule having only C & D scales: they were unhandy solutions to a very simple problem. In April 1942, the writer constructed a nomograph for the solution of the basic equation (Equation 3), and shortly thereafter realizing that even a nomograph is somewhat clumsy to use, designed and constructed an experimental model of the slide rule shown in Figure 2.

Determination of the length of the rule and the length of the basic log cycle used on the rule were based on the philosophy that first, the photo-interpreter should have no concern with decimal points and that second, he should be able to set and read the slide-rule accurately and precisely enough so that no significant error in the calculations is introduced by the rule itself.

Photographs of the photo-interpreter's slide rule in several settings are shown in Figure 2. In the first photograph, Figure 2a, a 36″ camera, with 9″×18″ format, is flown at 30,000 ft. The slide is moved, setting 36″ on the focal length scale opposite 30,000 ft. on the altitude scale. Opposite the 12 on the focal length scale, the scale or representative fraction (R.F.) of the photograph is read—1:10,000. This particular setting of the slide rule enables the immediate reading of several other quantities of interest. Opposite the 1 on the focal length scale, the ground feet per inch of photograph is read. The reading is 830 feet per inch (the correct value is 833 ft. per inch). Without moving the slide, the complete set of pairs of values—ground size and image size—can be read; for example, a 0.3 millimeter image length corresponds to a 100 ft. ground object. The reverse side of the slide has an image size scale calibrated in fractions of a foot. Use of this unit seems to appeal to photo-interpreters more than do image size measurements in either centimeters or inches. To read the coverage of the 9″×18″ negative, the coverage indicators, opposite 9″and 18″ on the focal length

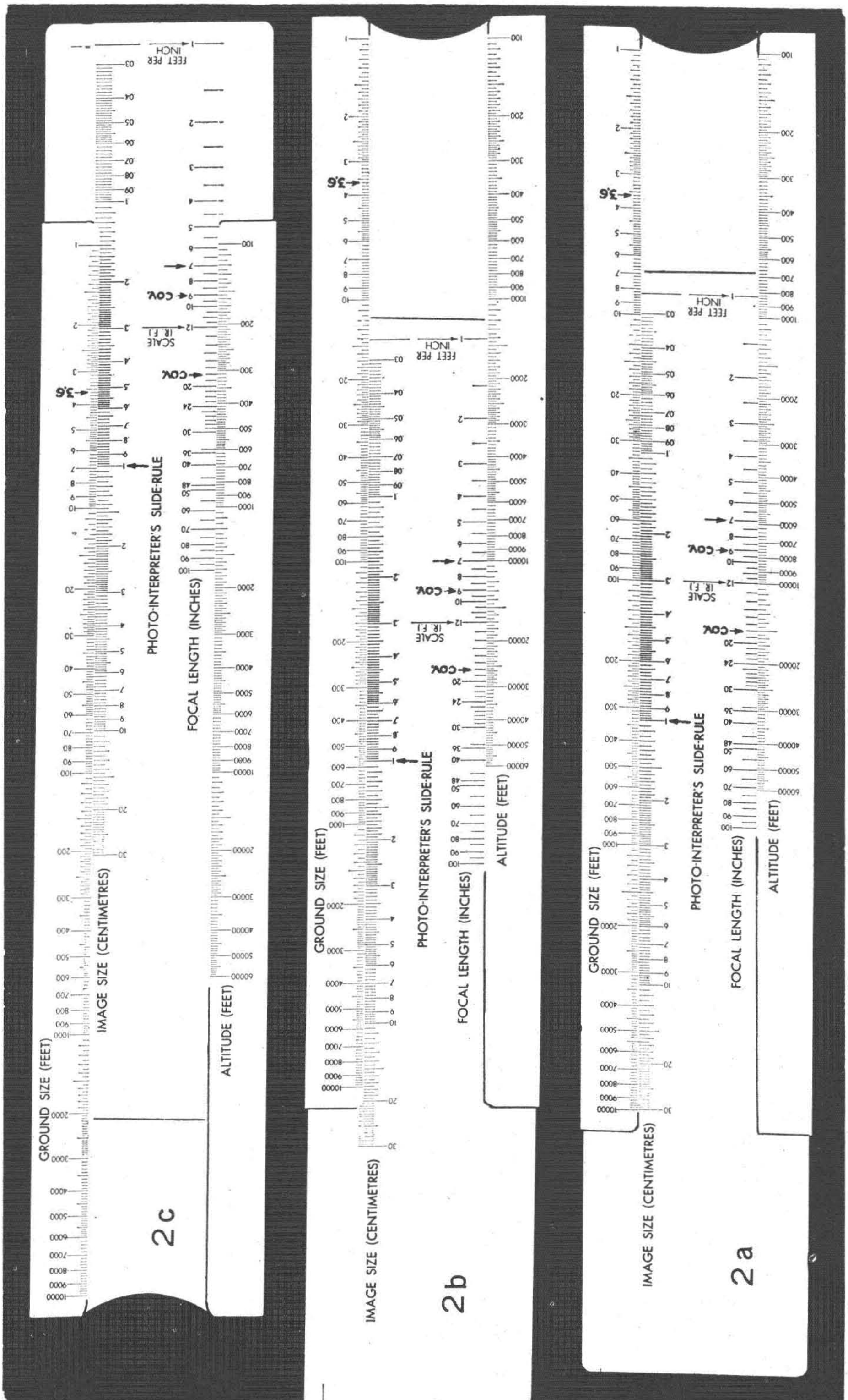

FIG. 2. The Photo-Interpreter's Slide Rule, showing its use in various problems.

scale, yield the immediate result that the 36″ focal length 9″×18″ camera at 30,000 feet covers 7,500 ft.×15,000 ft. on the ground.

It has been found recently that this same slide rule can be used for various other problems arising in aerial photography. By reading the units on the ground size scale in miles/hour and the units on the image size scale (centimeters) in inches per second, the film speed due to forward aircraft motion can be readily found. This is of ever increasing importance as strip cameras and moving film magazines increase in use. This change of units is accomplished by use of the arrow at 7 on the focal length which is set opposite the scale (R.F.) values. A dimensional analysis shows that the slide should really be moved up by the ratio 12/6.91 instead of 12/7, for the former figure is the more accurate value of the quantity 2.54 divided by 88/60; these constants will be recognized as the appropriate conversion factors. The convenience of using 7 instead of 6.9 outweighs the slight error introduced. For the particular example cited above—that of a 36″ lens flying at 30,000 feet—we previously read the scale to be 1 to 10,000. The second photograph of the rule, Figure 2b, shows the single step necessary to read image speed directly. The 7 on the focal length scale is placed opposite the scale (R.F.) value of 10,000 read previously. Opposite 300 miles/hr. (on ground size scale), image speed of .525″ per second is found. By setting this value of image speed opposite the arrow at 3.6 (on the ground size scale) as shown in the next photograph (Fig. 2c), cycling time for 60% overlap (with 9″ wide film in line of flight) is read opposite the arrow at one (on the image size scale). This cycling time is read as 6.9 seconds between photographs.

Other and more specialized uses of this rule have been made in connection with low altitude night photography calculations, resolving power calculations, and similar problems. All that is required is addition of one or two index marks to change units. The night photography calculations are made for the specially-modified tri-K-24 night camera set-up, wherein 100° of lateral coverage is obtained. The particular problem solved in two settings of the rule is as follows: For a given desired lateral ground coverage, find the flying height. At this height, find the flash-cartridge ejection interval, and the number of flash-cartridges required to cover a flight line of x miles.

As with most instructions regarding the use of a slide rule, reading of the instructions is more difficult than actual performance of the calculations. This was found to be especially true with this slide rule. It has been the writer's experience that personnel unfamiliar with slide rules and calculations can perform calculations with this slide rule after but several minutes of instruction. Further, and most important, these calculations can be performed with speed and confidence, and with no worry about decimal points. An indication of the simple instructions which suffice to explain the use of the rule are the actual instructions which appear under the slide (these instructions apply only to the photo-interpreter's use of the rule):

Instructions for Photo-Interpreter's Slide Rule

1. Set focal length against altitude. Read scale (R.F.) and feet per inch opposite marked arrows.
2. With any setting of focal length against altitude, read ground object size against the image size (measured on the photo). Make sure that the units of measurement are the same as on the slide, which has two sides, making possible the use of either centimeters or feet for measurement of photo distance.
3. To convert from other units of measurement, use conversion scales on the back of the rule.
4. This rule is accurate for verticals only, since obliques have varying scale. Errors

in interpretation of measurements made on photographs may come from errors in altitude, the normal several per cent variation of actual focal length in lenses of the same marked F.L., and non-uniform distortion produced by focal-plane shutters.

5. Other methods of calculation with this rule will suggest themselves to the user.

This slide rule has been heretofore manufactured out of inexpensive vinyl plastic, and it is this version which is found in the new photo-interpreter's kits. There have been some minor difficulties with easy reading of the rule because of parallax between the outside scales and the slide. Plans are under way to produce this rule in a higher quality version (in the standard 10 inch size) which will have the scales for the various image size units all on one side. The slide and the outside scales will be flush-mounted as in a high grade commercial engineering slide rule. The altitude scale will be extended to about 250,000 to permit use of this slide rule in oblique photography computations (to be described in a subsequent section), and the various arrows and marks for image speed (for ground speed in knots and mph) and cycling time computations, which have been inked in as afterthoughts, will become part of the rule. The focal length scale will have special indices added for 96, 144, and 240 inch lenses.

The new slide rule, while especially useful to the photo-interpreter, will also be of considerable use to the aerial photographer and the staff photo officers. This new rule will have to be renamed; suggestions will be welcomed. Photo-interpreters have neither direct professional interest in nor concern with image speed and cycling time, but everyone else concerned with the taking of the photograph is so interested. All of the scales mentioned above will be located on one side of the slide rule. The reverse side of the rule will be a newly arranged standard 10 inch rule, having the normal, folded and inverted scales (C, D, CF, DF, DIF, DI), the square and cube scales (A & K), the sine and tangent scales (S & T), and the log scale (L). The logarithmic cycle on the interpreter's side of the rule will be 2.5 inches long, for four of these cycles are needed to cover the range 10^4 to 1. A ten inch logarithmic scale, such as the C scale on a standard 10 inch slide rule, can be read to approximately 0.1%; hence the 2.5 inch log cycle can be read to about 0.4%—one part in 250. It will become clear in the next section that such precision, if accompanied by the expected accuracy in manufacture, is at least ten times better than the accuracy of most photo-interpretation.

B. ERRORS IN PHOTO-INTERPRETATION MEASUREMENTS MADE FROM VERTICAL PHOTOGRAPHS

At the time the writer designed the photo-interpreter's rule described above, it became necessary to estimate the accuracy of measurements made from reconnaissance photographs. In order to properly design the slide rule, it was necessary to ensure that the error in manufacture and reading of the slide rule was smaller than the errors accruing from other causes in photo interpretation, and yet not so small as to give the user a false and unwarranted sense of precision of the photo-interpretation process.

It is unfortunate but none the less true that in the past few photo-interpreters knew much about aerial photography. This typical and evil by-product of specialization has led some photo-interpreters into an erroneous belief, eagerly communicated to the writer, that normal, routine measurements made from vertical reconnaissance photographs were accurate to at least one part in three hundred—approximately $\frac{1}{3}$%.

It is perhaps in order at this point to interject some thoughts regarding precision and accuracy and their application to aerial photography.

The concepts of precision and accuracy, and more important, the distinction between them, have been either ignored or misused in most published work on photogrammetry.

It is a serious and pernicious fallacy to believe that because computations are made with a modern electric calculator, "answers" can be written down embodying all the figures cast up by the machinations of the calculator. Because one is able to measure a length, of say 6 inches, on a photograph, to the nearest 0.01 inch, it does not follow that subsequent operations with this number are "good" to the same degree; however, it is exactly this heresy that has afflicted some phases of photo-interpretation. Briefly stated, precision refers to the reproducibility of the measurement operation, whereas accuracy refers to the essential truth of the measurement, i.e. its nearness to the true value. Thus conceivably (and in fact, quite often) one can have a measurement system which is capable of yielding excellent precision, and yet, because of a systematic error or bias in the measurement operation, be considerably removed from the true value. The writer's attention was first drawn to this problem in photogrammetry when he was associated with the scientific analysis of the technical photography of the Bikini Atomic bomb tests in 1946.

As a result of these and other more recent experiences, the author is preparing an article for PHOTOGRAMMETRIC ENGINEERING on this subject; it is hoped to thus open up the subject for thorough and definitive discussion.

The following discussion pertains to the case of the vertical photograph, i.e. photographs made with a camera mounted in a roughly vertical position. The comparable analysis of stereo-measurements merits, and will eventually receive full and separate discussion. Much of the subsequent analysis is as applicable to stereo-measurements as it is to measurements made from a single photograph.

The fundamental measurement problem in photo-interpretation is the determination of the size of ground objects. This can be accomplished by measuring the size of the image and computing the size of the object from data on either:

a. The image size, the focal length of the lens, and the altitude at which the photograph was taken, or

b. The actual (known) sizes of other objects in the photograph, and calculating the size of the object from the ratio of image sizes.

The main errors which are present in the two processes outlined above are the following:

a. Error in measurement of image size

b. Deviation in actual focal length of the lens from the marked or nominal focal length.

c. Error in determination of altitude.

d. Distortion produced by focal plane shutters.

e. Deviation from perpendicularity of the optic axis of the camera, arising from either

(1) Variation in angle of attack of aircraft,

(2) Tilt caused by pitch and roll, or

(3) Tilt chargeable to improper mounting.

The fundamental relationship between ground size G, image size I, focal length f, and altitude H is shown in Figure 1 whence

$$G = \frac{IH}{f} \cdot \qquad (3)$$

From Eq. (3) by logarithmic differentiation, is obtained

$$\frac{dG}{G} = \frac{dI}{I} + \frac{dH}{H} - \frac{df}{f}\,. \tag{4}$$

Eq. (4) expresses the fact that the relative error in determination of ground size equals the sum of the relative errors of the quantities on the right. Replacing the quantities on the right side of Eq. (4) by their maximum absolute values, we obtain the maximum relative error in G:

$$\left|\frac{dG}{G}\right|_{\max} = \left|\frac{dI}{I}\right|_{\max} + \left|\frac{dH}{H}\right|_{\max} + \left|\frac{df}{f}\right|_{\max}. \tag{5}$$

The lenses used in aerial photography may vary from their nominal or marked focal length by as much as 2 or 3%. Although recent practice has been to mark the exact axial focal length somewhere on the lens, these data are in general not available or difficult to present to the photo-interpreter, who must perforce use the nominal value of the focal length in his calculations.

For large image sizes, say several inches, dI/I, the relative error in image measurement, may be quite small and negligible. If we assume that image measurements can be made to 0.01 inch, then for an image of length, say 2 inches, $dI/I = .01/2 = 0.5\%$. For very small image sizes, dI will remain constant, thus increasing the value of dI/I to several per cent or more.

The determination of altitude by the radio altimeter has greatly increased the accuracy of this determination over that prevailing during most of World War II, when at great distances from friendly meteorological stations, altitude errors of 1000′ or more (at about 30,000′) were not uncommon. For the purposes of this study we can take dH/H as being considerably less than 1%.

All of these percentages are additive, yielding, from Eq. (5)

$$\left|\frac{dG}{G}\right|_{\max} = 4\% \text{ or } 5\%. \tag{6}$$

The foregoing analysis applies to the truly vertical photograph.

The assumption of perpendicularity of the optical axis cannot be justified. Varying fuel loads and other flight parameters change the angle of attack of the aircraft. Roll of the aircraft is an even more serious effect. The tilt so induced results in a variation of scale from one side of the photograph to the other. Hence, measurements made on one side of the photograph (of known objects) cannot be used to accurately determine unknown object sizes on the other side of the photograph. Relative errors arising from tilt are in general additive to the 4 or 5% error noted above.

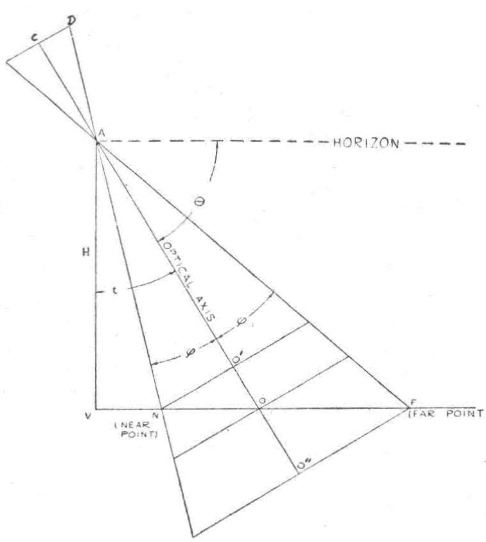

FIG. 3. The geometry of a "vertical photograph" with some tilt.

Consider Figure 3, from which will be derived an equation for the variation in scale across a photograph as a function of tilt angle t. In this figure, as in all subsequent discussions, θ is the depression angle of the optical axis, ϕ is the angle off axis (in this case, the half side angle of the camera) and t the angle of tilt off perpendicular (or assumed position). All angles will be measured positive in a clockwise sense, so that angles measured above the optical axis are negative, those below the optical axis being positive.

Assume now that two ground objects of length L are lying on the ground at N, the near point and F, the far point, and these objects are lying on lines perpendicular to the drawing at N and F. Throughout this paper we shall be using scale numbers, S_i. These numbers are defined so that when an image height, length, or area, is multiplied by the appropriate S_i, we obtain the corresponding ground height, length, or area. For the truly vertical camera the scale number is

$$S_v = \frac{H}{f}. \tag{7}$$

It should be clear from Figure 3 that the corresponding H for the object lying at N (in a line perpendicular to the paper) is AO'. This distance AO' is called the effective altitude for objects at N; similarly for objects at F, the effective altitude is $AO.''$ This concept of effective altitude plays an important part in continuous strip camera theory and operation, and in all image speed and synchronous film speed calculations. This statement about AO' may be easily proved. Imagine a small angle $d\psi$ = angle NAN' where N' lies a small ground distance dG along L (dG is thus perpendicular to the plane of the drawing at N). The small ground area dG will yield a small image length dI, and we have at once, from similar triangles ACD and ANO' that

$$\frac{dG}{dI} = \frac{AO'}{f}. \tag{8}$$

Comparison with Eq. 7 shows that AO' plays the same role as H in that Eq. (7), and is truly an effective altitude for objects at N and oriented as described above. We have

$$AO' = H \sec (t - \phi) \cos \phi \tag{9}$$

and

$$AO'' = H \sec (t + \phi) \cos \phi. \tag{10}$$

The scale numbers for objects lying along NOF is obviously different than for those lying perpendicular to NOF, for there is the added projection effect, occurring because these objects are not parallel to the focal plane of the camera.

The latter scale numbers we will call S_y throughout; the scale numbers for the objects lying perpendicular to the plane of Figure 3 will be called S_x. This choice follows the conventional x and y axes on an oblique photograph held so that the horizon is at the top of the photograph.

From Eqs. (9) and (10),

$$\left.\begin{aligned} \left(\frac{dG}{dI}\right)_{\text{at } N} &= S_x(\text{at } N) = \frac{H \sec (t - \phi) \cos \phi}{f} \\ \text{and} \quad \left(\frac{dG}{dI}\right)_{\text{at } F} &= S_x(\text{at } F) = \frac{H \sec (t + \phi) \cos \phi}{f} \end{aligned}\right\} \tag{11}$$

where ϕ in these equations is the numerical value of ϕ, disregarding sign. Further, let R_x = Ratio of scale on one side of the photograph to scale on other side. Then

$$R_x = \frac{S_x \text{ (at } F)}{S_x \text{ (at } N)} = \frac{\cos (t - \phi)}{\cos (t + \phi)}. \tag{12}$$

Expanding yields

$$R_x = \frac{\cos t \cos \phi + \sin t \sin \phi}{\cot t \cos \phi - \sin t \sin \phi}. \tag{13}$$

Assuming that tilt angle t will be less than 10°, we may set $\sin t = t$ and $\cos t = 1$. Eq. (13) reduces to

$$R_x = \frac{1 + t \cdot \tan \phi}{1 - t \cdot \tan \phi} = 1 + 2(t) \tan \phi \tag{14}$$

or

$$R_x = 1 + \frac{tw}{f} \tag{15}$$

where

w = width of film in inches
f = focal length in inches
t = tilt angle measured in radians

The magnitude of this scale ratio effect for several cameras is shown in the accompanying table wherein R is computed exactly from Eq. (12) and approximately from Eq. (15), thus furnishing an estimate of the value of the approximation; wherever a 9×18 camera is tabulated, the ratio effect is computed across the long dimension.

TABLE I. VALUE OF SCALE RATIO EFFECT FOR VARIOUS CAMERAS AND SMALL AMOUNTS OF TILT

Tip or Tilt		6″(9×9) 24″(9×18)	12″(9×9) 24″(9×18)	24″(9×9) 48″(9×18)
1°	Exact	1.026	1.013	1.007
	Approximation	1.026	1.013	1.007
5°	Exact	1.14	1.068	1.033
	Approximation	1.131	1.065	1.033
10°	Exact	1.300	1.142	1.068
	Approximation	1.261	1.130	1.065

Anticipating the results of one of the subsequent sections, it should be noted that the scale ratio effect for the S_y's is greater than that for the S_x's. The ratio $R_y = S_y$ (at F)/S_y (at N) is approximately twice R_x. It is now clear that the tilt error is of the order of several per cent for several degrees of tilt.

Another effect which contributes to the error in measurement of ground object sizes is caused by focal plane shutters. A discussion of this effect for focal plane shutters, travelling at right angles to the line of flight and in line of flight, may be found in "Camera Shutters," a previous paper by the author (8 p. 13). The case of most interest is that in which the shutter is travelling in line of flight, producing either a positive or negative linear distortion on images parallel to the flight line. If the shutter is moving toward the rear of the aircraft, the effect will be to foreshorten the image; if the shutter is moving in the same direction as the image (forward) the effect will be to stretch the image.

The equation for relative distortion of an image of length l is given below. The upper sign refers to the case of the shutter slit moving forward in the aircraft, the lower sign referring to the opposite case.

$$\frac{\Delta l}{l} = \frac{88}{60}\left\{\frac{Vf}{H\left[V_c \mp \left(\frac{f-d}{f}\right)\frac{88}{60}\frac{VF}{H}\right]}\right\}\left\{\frac{\left(\frac{d}{N}\right)+w}{l}+\left(\frac{f-d}{f}\right)\right\} \tag{16}$$

where

$\Delta l/l$ = relative distortion
V = ground speed, mph
f = focal length, inches
V_c = shutter curtain speed, inches/sec.
d = distance from shutter to film, inches
H = altitude, feet
$N = f/$Aperture of lens (e.g. 3.5)
W = shutter slit width
l = length of image

The derivation of this equation has never before appeared in print, and on the assumption that it may be of interest to the reader, is given in Appendix A, together with several examples of the use of the formula.

If image motion caused by forward aircraft motion is uncompensated, this distortion effect, for high-altitude long focal length photography, is in general of the order of 1% or more (depending on the image size). Image motion compensation, one of the most important advances in aerial photography in the last dozen years, will prevent this distortion. In the future, image motion compensation will be universally used, for oblique photography as well as vertical photography.

It is clear from all of the foregoing material that:

(a) In a truly vertical photograph dI/I may be 4%.
(b) In a slightly tilted photograph, assumed to be vertical, the scale may easily vary by several per cent from one side to the other.
(c) Focal plane shutters may easily contribute about one per cent distortion.

In the absence of non-availability of probability (or frequency) distributions for each of the quantities contributing to final error, it is impossible to estimate the probability of errors of a given size. It can be stated that under some easily realized circumstances, errors of 8% to 10% are not impossible. More important it can be stated that accuracies of 1% or less (such as the photo-interpreter's one part in 300) are extremely improbable.

C. SCALE IN OBLIQUE PHOTOGRAPHS

It cannot be over-emphasized that unless the photo-interpreter is able to measure those quantities of interest in oblique photographs, his utility as a collector of intelligence data will be sharply diminished. The distinction between qualitative and quantitative data, and their relative value, is much more important in intelligence work than in most other branches of scientific inquiry.

In connection with preceding studies of the effect of tilt on the scale in an (assumed) vertical photograph, it was necessary to derive an equation for S_x, the scale number for a line on the oblique photograph which is parallel to the horizon. S_x, is given in Eq. (11) with reference to Figure 3.

For purposes of this and succeeding sections, it will be desirable to refer S_x and S_y (the corresponding scale number for local measurements along the y axis of the photograph, where the y axis is the principal line) to the geometry of the oblique photograph as shown in Figure 4.

From Eq. 11 and Figures 3 and 4, it is clear that

$$S_x = \frac{H}{f} \frac{\cos \phi}{\sin (\theta + \phi)} \cdot \tag{17}$$

Consider now the problem of finding S_y. It will be remembered that the S number when multiplied by the image length yields the corresponding ground length. Hence, for the case of a ground object dy (Figure 4) whose image dI is located an angle ϕ off axis of a camera of focal length f, the optical axis of which is depressed an angle θ from the horizon, we have

$$S_y = \frac{dY}{dI} \cdot \tag{18}$$

The small increment $d\phi$ is greatly exaggerated in the figure; because we are deriving the value of S_y at the point D of Figure 4, $d\phi$ can be assumed to be a true infinitesimal. Under these conditions, the enlarged drawing of triangle DCE, gives the correct angles. It is assumed in this drawing that AE and AD are parallel, which follows from the previous discussion about $d\phi$. The procedure to be followed is to project dy on DB. The scale number for this projection of dY is clearly the same as S_x at D, for the effective altitude AB applies in both cases.

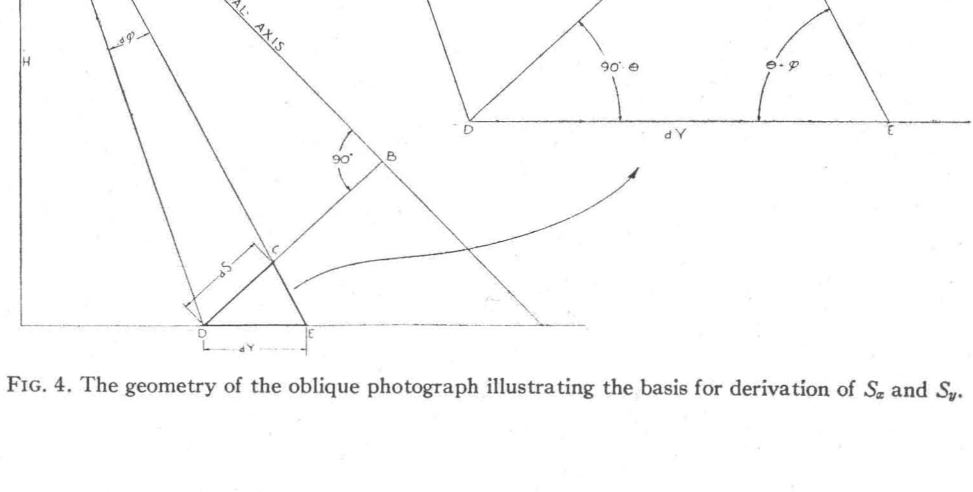

FIG. 4. The geometry of the oblique photograph illustrating the basis for derivation of S_x and S_y.

We have therefore

$$\frac{dS}{dI} = S_x \tag{19}$$

and, from the triangle CDE, by the law of sines,

$$\frac{dS}{\sin(\theta + \phi)} = \frac{dY}{\sin(90 - \phi)} = \frac{dY}{\cos\phi}. \tag{20}$$

From Eq. (20)

$$\frac{dY}{dS} = \frac{\cos\phi}{\sin(\theta + \phi)}. \tag{21}$$

Multiplication of both sides of Eq. (19) by dY/dS results in

$$\frac{dS}{dI}\frac{dY}{dS} = S_x \frac{\cos\phi}{\sin(\theta + \phi)}$$

which yields, utilizing Eq. (17)

$$S_y = \frac{dY}{dI} = \frac{H}{f}\left[\frac{\cos\phi}{\sin(\theta + \phi)}\right]^2. \tag{22}$$

The similarity of this equation for S_y to that for S_x should be noted. If S_v is the scale for a vertical camera, we may write the three formulae together

$$\left.\begin{aligned} S_v &= \frac{H}{f} \\ S_x &= \frac{H}{f}\frac{\cos\phi}{\sin(\theta + \phi)} \\ S_y &= \frac{H}{f}\left[\frac{\cos\phi}{\sin(\theta + \phi)}\right]^2 \end{aligned}\right\}. \tag{23}$$

Careful note should be made of the circumstances under which the equation for S_y was derived. S_y holds (exactly) only for distances measured along the principal line. Its utility and application to other areas on the photograph will be demonstrated later.

It is of interest to determine how S_x and S_y vary with slight changes in y. From Eq. (17) for S_x

$$\log_e S_x = \log_e \frac{H}{f} + \log_e \cos\phi - \log_e \sin(\theta + \phi). \tag{24}$$

Because y and ϕ are related, we may differentiate Eq. (24) with respect to ϕ only, holding H, f, and θ constant

$$\frac{dS_x}{S_x} = -\left[\tan\phi + \cot(\theta + \phi)\right]d\phi. \tag{25}$$

Now $y = f\tan\phi$, where y is measured along the principal line from an x axis which passes through the principal point.

Hence

$$dy = f \sec^2 \phi d\phi. \tag{26}$$

Substituting for $d\phi$ into Eq. (25) from Eq. (26) yields

$$\frac{dS_x}{S_x} = -\left[\tan \phi + \cot (\theta + \phi)\right] \frac{\cos^2 \phi}{f} dy. \tag{27}$$

A similar operation on S_y yields

$$\frac{dS_v}{S_y} = 2 \frac{dS_x}{S_x} \cdot \tag{28}$$

Thus the relative change in S_x and S_y for a slight increment in y is given by Eqs. (27) and (28). Two recent experimental camera installations, made for a special test, used K-38 cameras. The K-38 camera has a 9 inch × 18 inch format, and in these tests mounted a 36″ focal length lens. The two depression angles used were

$$\theta = 52° \quad \text{and} \quad \theta = 71°.$$

The three portions of the photograph for which the following table is calculated are on the axis, and at $\phi = \pm 13.5°$. The increment in y, dy is 0.5 inch.

TABLE II. PER CENT VARIATION IN S_x AND S_y FOR +.5 INCH INCREMENTS ON y AT SELECTED POINTS. 36″ (9×18) CAMERA

	$\theta=52°$			$\theta=71°$		
$\phi=$	+13.5°	0°	−13.5°	+13.5°	0°	−13.5°
dS_x/S_x	0.90%	1.08%	1.31%	.45%	.48%	0.53%
dS_y/S_y	1.80%	2.16%	2.62%	0.90%	0.96%	1.06%

Note that a positive increment in y corresponds to a negative increment in ϕ, for ϕ is always measured (in a positive sense) down from the horizon (as is θ). S_x and S_y are increasing functions with y, so that a $dy = +0.5$ inch implies a positive percentage change in S_x and S_y.

A problem of great importance is the investigation of the effect of roll of the aircraft on values of S_x and S_y, and the calculation of ground lengths, where the latter are calculated for an assumed angle of depression θ.

Let θ_n be the assumed or nominal depression angle and t be the angle of roll, with t being measured positive down from the horizon. Let us assume that we have a ground length L_x whose image lies on a horizontal line in the oblique photograph. The estimated value of L_x using erroneous data on true θ (i.e., assuming true $\theta = \theta_n$) is

$$\text{Estimated } L_x = (\text{Image size})(\text{Est. } S_x). \tag{29}$$

The image size is determined from the actual (but momentarily unknown) ground distance L_x and the true S_x (which depends on ϕ and the actual θ_a).

Hence

$$\text{Estimated } L_x = (\text{True } L_x)\left(\frac{\text{Est. } S_x}{\text{True } S_x}\right). \tag{30}$$

Hence

$$\frac{\text{Estimated } L_x}{\text{True } L_x} = \left(\frac{\text{Est. } S_x}{\text{True } S_x}\right). \tag{31}$$

Now

$$\text{Est. } S_x = \frac{H}{f} \frac{\cos \phi}{\sin (\theta_n + \phi)} \tag{32}$$

and

$$\text{True } S_x = \frac{H}{f} \frac{\cos \phi}{\sin (\theta_a + \phi)} \cdot \tag{33}$$

Substituting $\theta_a = \theta_n + t$ into Eqs. (32) and (33) the values of True S_x and Est. S_x into Eqs. (30) and (31) we obtain

$$\frac{\text{Est. } L_x}{\text{True } L_x} = \frac{\sin (\theta_n + \phi + t)}{\sin (\theta_n + \phi)} \cdot \tag{34}$$

Writing $s = \theta_n + \phi$, and reducing the right side of Eq. (34) yields

$$\frac{\text{Est. } L_x}{\text{True } L_x} = \cos t + \cot s \sin t. \tag{35}$$

For angles of roll up to say, $t = \pm 10°$ we may use the rather close approximations $\cos t = 1$ and $\sin t = t_1$ with t in radians. Eq. (35) reduces to

$$\frac{\text{Est. } L_x}{\text{True } L_x} = 1 + t \cot (\theta_n + \phi). \tag{36}$$

For $t = \pm 5° = \pm .08725$ radians, and for the top, axial, and bottom edges of the 36″ K-38 camera, we have the following table for the error in estimating ground lengths.

TABLE III

$$1 - \frac{\text{Est. } L_x}{\text{True } L_x}, \quad \text{for} \quad 36'' \text{ K-38}$$

	$\theta = 52°$			$\theta = 71°$		
$\phi =$	$+14°$	$0°$	$-14°$	$+14°$	$0°$	$-14°$
$t = \pm 5°$	$\pm 3.8\%$	$\pm 6.8\%$	$\pm 11.2\%$	0.1%	3.0%	5.7%

The error is proportional to t, so that a roll of 1° introduces errors $\frac{1}{5}$ of those in the table above.

An analogous demonstration could be made to show the effect of roll on S_y, and it would be found that the ratio

$$\frac{\text{Est. } L_y}{\text{True } L_y} = \left(\frac{\text{Est. } L_x}{\text{True } L_x}\right)^2 \tag{37}$$

making the percentage errors for y approximately twice the value of the corresponding errors listed in Table III. It is clear that unless one is prepared to tolerate large and substantial errors which might easily and often exceed 10%, tilt data are required, together with a method of using such data simply and quickly to make necessary corrections. Further discussion of this point will be found in the following sections.

D. VARIATION OF SCALE NUMBERS FROM TOP TO BOTTOM OF AN OBLIQUE PHOTOGRAPH

It is of some interest to know the total variation of scale (S_x and S_y) within an oblique photograph.

We have from previous equations:

$$\frac{S_x \text{ (at top)}}{S_x \text{ (at bottom)}} = \frac{\sin [\theta + |\phi|_{\max}]}{\sin [\theta - |\phi|_{\max}]}. \tag{38}$$

Further, if S_v is H/f, it is readily seen that we may write, for a particular θ and ϕ,

$$S_x = KS_v$$

$$S_y = K^2 S_v$$

whence

$$S_y = \frac{S_x^2}{S_v}$$

and

$$\frac{S_y \text{ (at top)}}{S_y \text{ (at bottom)}} = \left[\frac{S_x \text{ (at top)}}{S_x \text{ (at bottom)}}\right]^2. \tag{39}$$

The following table gives these ratios for the two experimental installations of the K-38, 36″ (9×18) camera, $\theta = 71°$ and 52°. An instructive and alternative

TABLE IV

	$\frac{S_x \text{ (at top)}}{S_x \text{ (at bottom)}}$	$\frac{S_y \text{ (at top)}}{S_y \text{ (at bottom)}}$
$\theta = 52°$	1.485	2.205
$\theta = 71°$	1.188	1.412

method of deriving the ratio for S_x follows from consideration of Figure 5. The actual lengths of the horizontal image lines cut by the two perspective lines are equal. Hence the respective S_x's are inversely proportional to the image lengths, yielding at once that

$$\frac{S_x \text{ (at top)}}{S_x \text{ (at bottom)}} = \frac{l + w}{l - w}$$

$$= \frac{f \tan \theta + w}{f \tan \theta - w}. \tag{40}$$

E. AIMING A CAMERA SO AS TO MAXIMIZE THE IMAGE AREA OF A GIVEN GROUND AREA

In a discussion some six or eight months ago, Mr. Eldon Sewell, Corps of Engineers resident representative at the Photographic Laboratory, raised the

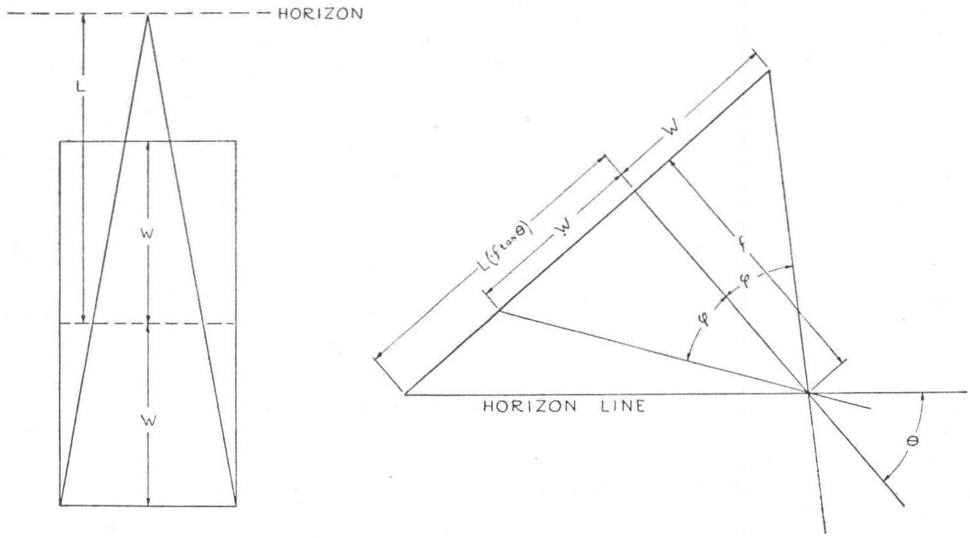

FIG. 5. A simple method for obtaining the ratio $\dfrac{S_x \text{ (at top)}}{S_x \text{ (at bottom)}}$

very interesting question of maximizing the image area of a ground area located at a given angle of depression with respect to an aircraft. Stating the question as it would be presented by the aerial photographer, it would read "where should I aim my camera to get the largest possible image of the ground object?" The concepts of S_x and S_y scale and their equations facilitate an easy and instructive solution to this problem.

We have as before

$$S_x = \frac{H}{f}\left[\frac{\cos\phi}{\sin(\theta+\phi)}\right]$$

and

$$S_y = \frac{H}{f}\left[\frac{\cos\phi}{\sin(\theta+\phi)}\right]^2.$$

The larger S_x and S_y, the smaller the image, and

$$\text{Image area} \sim \frac{1}{S_xS_y}\cdot \tag{41}$$

But

$$\frac{1}{S_xS_y} = \frac{f^2}{H^2}\quad\frac{\sin^3(\theta+\phi)}{\cos^3\phi}\,. \tag{42}$$

Remembering that in this formula θ is the angle of depression of the optical axis from the horizon and that ϕ is the angular distance off axis to the ground object (measured positive below the optical axis and negative above the optical axis), it is clear, as before, that $(\theta+\phi)$ is the depression angle of the object with respect to the aircraft, and is independent of the aiming angle θ. Hence we may lump all factors except $\cos^3 \phi$ into a constant K and write

$$\text{Image Area} = K \sec^3 \phi. \tag{43}$$

This yields the very interesting result that the image area of a given ground object is a maximum when the camera is so aimed as to image the area as far off axis as possible—and, interestingly and surprisingly enough, the result is the same whether the camera is aimed low or high with respect to the object. Although these results were obtained on the assumption that the ground object is a flat area, they hold as well if the ground object is, say, a large building. In a subsequent section, on height measurements in oblique photographs, the analogous problem for heights yields the same solution—maximum off-axis imagery. The numerical value of this effect may be of interest:

TABLE V

	0°	10°	20°	30°	40°
$\sec^3$	1	1.05	1.21	1.54	2.23

For example, it is seen from the table that a ground area imaged 30° off axis will yield an image area 1.54 times that of an axial image of the same ground area. Under certain marginal military conditions, utilization of this effect may be quite advantageous.

F. CONDITIONS UNDER WHICH AN OBLIQUE PHOTOGRAPH YIELDS LARGER IMAGES THAN A VERTICAL PHOTOGRAPH OF OBJECTS DIRECTLY BELOW THE AIRCRAFT

Closely related to, and in fact a special case of, the problem of camera aiming discussed above, is an investigation of the conditions under which an oblique photograph will show greater scale (larger images) than a vertical photograph. This situation requires that

$$S_x < S_v. \tag{44}$$

This condition reduces to

$$\frac{\cos \phi}{\sin (\theta + \phi)} < 1. \tag{45}$$

From Eq. (45), we have

$$\cos \phi < \cos [90 - (\theta + \phi)] \tag{46}$$

whence

$$\phi > 90 - (\theta + \phi). \tag{47}$$

The geometric significance of Eq. (47) is clear; $(\theta+\phi)$ is the depression angle of the given object with respect to the aircraft, and therefore we have: when the angular distance off axis is greater than the angular distance from the vertical to that off axis point, the oblique scales S_x and S_y are greater than the vertical scale S_v. It should be noted that when Eq. (44) holds, $S_y < S_x$. In most oblique photography $S_y > S_x$. If we set $S_x = S_v$, then

$$\frac{\cos \phi}{\sin (\theta + \phi)} = 1 \tag{48}$$

and

$$S_x = S_y = S_v. \tag{49}$$

This is a statement of the isoline condition and yields the well-known result

$$\phi = \frac{90 - \theta}{2} \cdot \tag{50}$$

It is clear from the discussion above that at a given altitude with a given camera, the largest obtainable image of an object directly below the aircraft is not obtained with the camera mounted vertically, but with the camera swung off vertical so that the object is imaged at the edge of the field. In this case the angular distance off the vertical of the optical axis is the half side angle of the camera, $\phi_{\max}$. The ground coverage, from vertical to principal point is $H \cdot \tan \phi_{\max}$. Were the camera mounted vertically, the ground coverage from vertical to $\phi_{\max}$ is again $H \cdot \tan \phi_{\max}$, so the average scale over the lower half of a photograph made with an oblique camera mounted as described above (with a depression angle of $\theta = 90 - \phi_{\max}$) is the same as it would be for a vertical camera. In the case of a slide oblique camera, (with the optical axis perpendicular to the aircraft axis) the discussion above holds not only for objects at the nadir point, but also for the entire line of objects lying in the ground line formed by intersection of the earth's surface and the vertical plane which contains the nadir point and the longitudinal aircraft axis. In the case of a forward or rear oblique camera (the optical axis lying in the vertical plane which contains the longitudinal aircraft axis), the discussion holds not only for objects at the nadir point but also for objects lying in the ground line formed by intersection of the earth's surface and the vertical plane which contains the nadir point and is perpendicular to the aircraft longitudinal axis.

G. MEASUREMENT OF GROUND DISTANCES FROM OBLIQUE PHOTOGRAPHS

An important phase of photo-interpretation measurements is the determination of lengths and distances on the ground—lengths of buildings, aircraft, runways, etc. and distances between given ground points. See the high altitude photo of Phoenix—Figure 6. The general situation is shown in Figure 7. The problem is to find the actual ground distance between P_1 and P_2. The horizontal lines at y_1 and y_2 are equidistant on the ground. If we can calculate the corresponding ground distances G_1 ($=x_1$ on photograph) and G_2($=x_2$ on photograph), the true ground rectangular components of P_1P_2 can be found, and thus also the true ground distance P_1P_2. We have

$$G_1 = S_x' X_1 \tag{51}$$

$$G_2 = S_x'' X_2 \tag{52}$$

$$G_3 = Y_2 - Y_1 = H \cot (\theta + \phi_2) - H \cot (\theta + \phi_1). \tag{53}$$

Therefore the true ground distance D between P_1 and P_2 is given by

$$\begin{aligned} D &= [(G_1 - G_2)^2 + G_3^2]^{1/2} \\ &= \{(S_x'' X_2 - S_x' X_1)^2 + H^2[\cot (\theta + \phi_2) - \cot (\theta + \phi_1)]^2\}^{1/2}. \end{aligned} \tag{54}$$

This is exact, and of little direct appeal to the photo-interpreter. If he must compute the length of a line with a y component much larger than say, four inches, he will have to use an exact formula. Of course, if the line is parallel to x

FIG. 6. A high altitude oblique photograph. Data: 48″ f/6.3 lens, 35,000′ altitude. Depression angle about 12°. Phoenix, in the center area of the photograph, is about 35 miles from the nadir point.

axis, a single S_x holds true over the entire length. Fortunately, his main interests will be found in areas (and distances) small enough to permit the use of approximations to Eq. (54). The equation for D can be rewritten as

$$D = \left\{ [\bar{S}_x(X_2 - X_1)]^2 + [\bar{S}_y(Y_2 - y_1)]^2 \right\}^{1/2} \tag{55}$$

where the $\bar{S}$'s are appropriate average scales in x and y. Clearly $\bar{S}_y$ will be an S_y that corresponds to some value of y lying between y_1 and y_2 and $\bar{S}_x$ will be the $\bar{S}_x$ for some horizontal line through a y lying between y_1 and y_2. The easiest and *a priori* most reasonable y for calculation of S_x and S_y is $y=(y_1+y_2)/2$, the point halfway between y_1 and y_2. A theoretical investigation of the accuracy of this

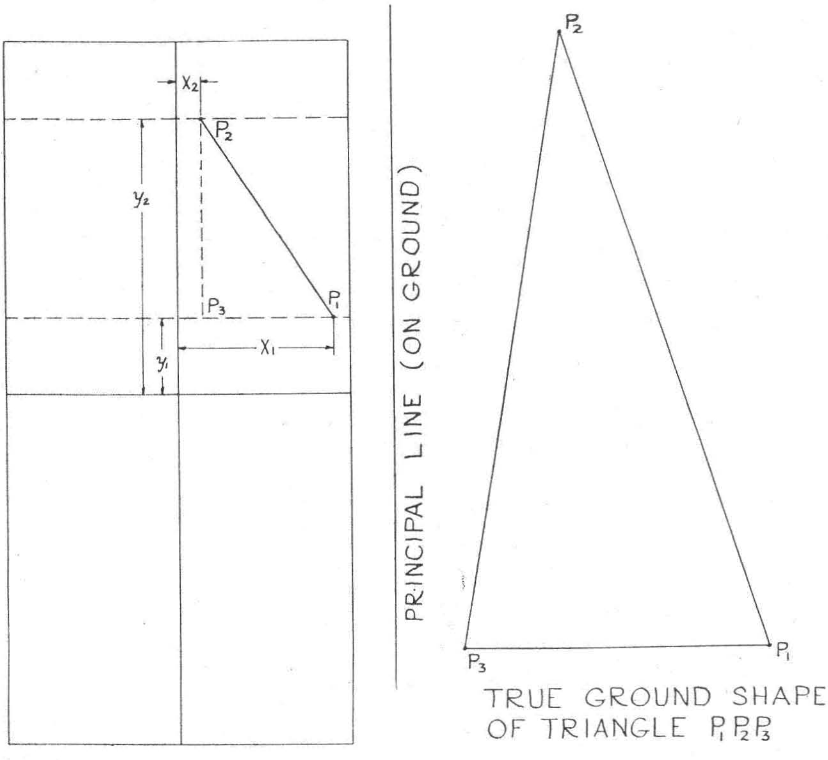

FIG. 7. The geometry of measuring P_1P_2 in an oblique photograph. Note that triangle $P_1P_2P_3$ in the oblique photograph (left) is not a right triangle; its true shape is something like that of the large triangle $P_1P_2P_3$ (right).

procedure could and should be made, being essentially straightforward. It has not been done, however, and will have to be deferred to a subsequent paper which will pick up this and other unfinished business.

In a subsequent section, on the measurement of area, an example is given, in which the ground equivalent of a 2 inch length along the y axis is measured by this technique. These 2 inches are at the upper edge of the oblique 36″ 9×18 camera, aimed with $\theta=52°$. In that case the error in estimation was only .7%, and it should be noted that dS_y/dy is a maximum in this region. S_x has been shown to be half as sensitive as S_y to variations in ϕ.

It seems likely, therefore that lines with y components up to perhaps four or five inches can be accurately computed from the streamlined version of Eq. (54), namely Eq. (55). As noted previously, this does not constitute proof. Detailed tests and actual examples (utilizing tilt correction methods to be discussed in a later section) will be presented in another paper.

If the photo-interpreter (or the reader) is willing to accept Eq. (55) for the moment, he must still do some computing. Two aids will be available to get the values of S_x and S_y. The Graphical Computer for Oblique Aerial Photographs can be used in the general case. This gadget is described in a later section. For fixed installations, the Oblique Computing Overlay (also described in a later section) can be used to get S_x and S_y. It is not too much to expect the photo-interpreter to multiply the x and y components by their respective S's, but we can certainly simplify and eliminate the squaring and extraction of square root called for by Eq. (55). Two methods are available. The value of $(A^2+B^2)^{1/2}$ can be determined (knowing only A and B) on a slide rule like the K & E Log Log Deci-trig, with one setting of the slide and two movements of the hairline. One starts with A and B, and writes down the square root of the sum of squares. The key to success with this method depends on the photo-interpreter's having a suitable slide rule. He doesn't have it yet, but will in the future.

The second method is to use the approximation $(A^2+B^2)^{1/2}=0.96A+0.40B$, $(A \geqq B)$ which is good within 4%, and simple to use.

The writer first saw this approximation in a handbook many years ago, but never since then has he seen a derivation. Realizing the impropriety of presenting a formula without a derivation, a derivation was prepared, and is found in Appendix B.

H. MEASUREMENT OF GROUND AREAS FROM OBLIQUE PHOTOGRAPHS

This problem, like every other measurement problem in photo-interpretation, is important. Without measurements, the photo-interpreter is committed to looking at photographs and saying "Ah, yes, I see a city (factory, lake, airport, etc.) on this photograph. But I haven't the vaguest idea how big it is or where it is."

Before proceeding with calculations, a few comments are in order. Figure 8 shows a portion of a perspective grid superimposed on an oblique 9×18 negative. The converging lines are parallel equidistant ground lines. The three parallel horizontal lines are also parallel equidistant ground lines, but the distance between them is not the same (in this drawing) as the distance between the converging lines. Consider a single row of the trapezoids cut out by two horizontal lines. It is clear that their print or photograph areas are all equal, for the distance between horizontal lines is constant, and these trapezoids have the same average base length, for they are actually parallel equal rectangles on the ground.

Similarly, were there a row of equal-print-area rectangles located between the same two horizontal lines, it could be shown easily that their respective ground areas are equal. Without much difficulty, it can be established that any set of congruent and similarly oriented areas, regular or irregular, lying between the same two horizontal lines (with the top and bottom edges of areas touching the lines) correspond to equal ground areas.

Consider now the square of Figure 9, lying between horizontal lines at y_1 and y_2. The exact ground area may be found easily by breaking this square into two triangles, A_1 and A_2. The ground area A is given by

$$A = 1/2(S_x'dx)(\overline{S}_ydy) + 1/2(S_x''dx)(\overline{S}_ydy) \tag{56}$$

where $\overline{S}_y$ is an appropriate average. ("Appropriate" means that $\overline{S}_y\, dy$ yields the true ground distance, Y, corresponding to (y_2-y_1) and S_x' is S_x at y_1 and S_x'' is S_x at y_2. We may rewrite Eq. (56) as follows:

$$A = 1/2(S_x'' + S_x')(\overline{S}_y)dxdy. \quad (57)$$

Clearly

$$S_y'dy < Y < S_y''dy \quad (58)$$

where the single prime refers to y_1 and the double prime to y_2

Since

$$Y = \overline{S}_y dy, \quad \text{we have}$$

$$S_y' < \overline{S}_y < S_y''. \quad (59)$$

Also, writing

$$\overline{S}_x = \frac{S_x' + S_x''}{2}, \quad (60)$$

$$S_x' < \overline{S}_x < S_x''. \quad (61)$$

If dx and dy are very small, the differences between S_x' and S_x'', and S_y' and S_y'' will be of higher order, so that the ground area A of the very small rectangle will be given by

$$A = S_x S_y dxdy.$$

The ground area A corresponding to the large irregular area lying between y_1 and y_2 Figure 9, is then given by

$$A = \iint S_x S_y dxdy \quad (62)$$

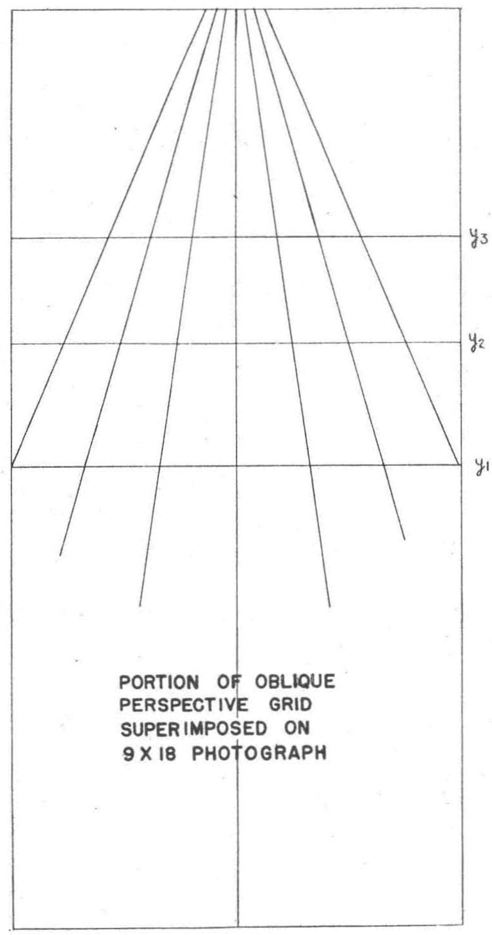

FIG. 8.

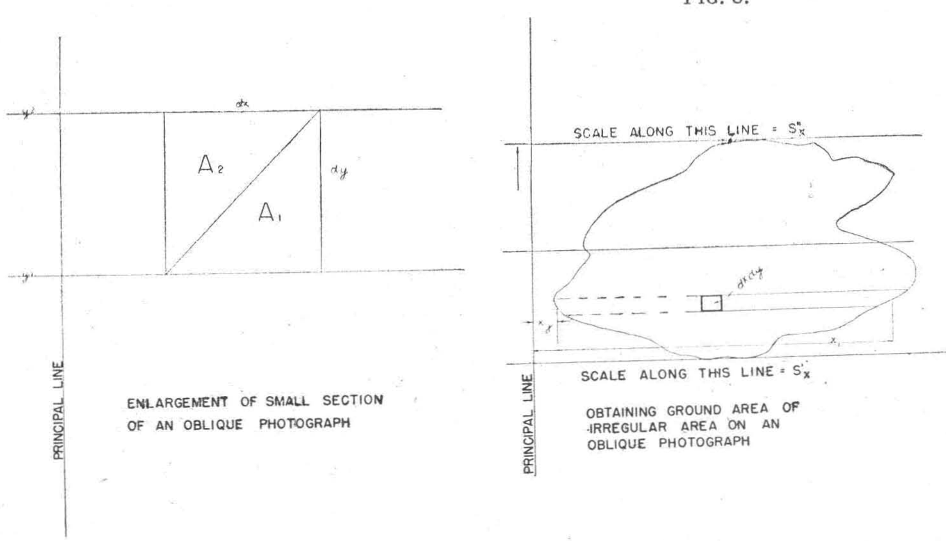

FIG. 9. The basis for derivation of the scale number, S_A, for area measurement from oblique photographs.

where S_x and S_y are of course functions of y. Eq. (62), although exact, is difficult and impractical to use. Because S_x is constant along any horizontal line, we may rewrite Eq. (62) as

$$A = \int_{y_1}^{y_2} S_x S_y (X_i - X_j) dy. \tag{63}$$

If the distance $(y_2 - y_1)$ is not excessive, say several inches, S_x and S_y are essentially linear functions of y in this interval, (which they are, because dS_x/dy and dS_y/dy are fairly small) we can, without serious affront to the mathematician, replace S_x and S_y by their average values $\overline{S}_x$ and $\overline{S}_y$. For the reasons given above, we may compute these values of $\overline{S}_x$ and $\overline{S}_y$ at a y value between y_1 and y_2. This choice of y is best made with reference to the shape of the area at hand. To avoid the complications arising from detailed consideration of many different area types, we will use this simple rule of thumb that $\overline{S}_x$ and $\overline{S}_y$ should be determined for the y that corresponds to the centroid or center of gravity of the irregular area. A good guess should be close enough. Rewriting Eq. (63) yields

$$\text{Ground } A = \overline{S}_x \overline{S}_y \int_{y_1}^{y_2} (X_i - X_j) dy. \tag{64}$$

The integral on the right side of Eq. (64) is the actual area of the irregular area. We then have

$$\text{Ground Area } A = \overline{S}_x \overline{S}_y \cdot (\text{actual print area}). \tag{65}$$

We may determine the limits of error by making upper and lower estimates of the true ground area A. If the actual area on the photograph is A', we have clearly

$$A' S_x' S_y' < A < A' S_x'' S_y''. \tag{66}$$

The ratio of the extremes in this inequality will furnish a measure of the maximum possible error. This ratio is

$$R = \frac{S_x'' S_y''}{S_x' S_y'}. \tag{67}$$

A saving feature of area measurement in obliques is that areas of interest—cities, airfields, industrial sites, and the like—are essentially circular or rectangular. In an oblique photograph, the projection effect insures that the y diameter of the area will almost always be less than x diameter of the area. S_x is constant along horizontal lines, making for accuracy in the x direction.

An example of the use of this method follows, and a description of a simple computing system to get $\overline{S}_x \overline{S}_y$ will be found in a subsequent section.

The photo-interpreter will be satisfied with accuracies of say 5% to 7%. This method should satisfy him.

I. EXAMPLE OF AREA MEASUREMENT

As an illustrative check on the area-measurement technique, consider a 2 inch square area on the print lying with its top edge at the top edge of a photograph made with the 36″ K-38 camera at a depression angle of $\theta = 52°$. It will be observed that (a) the 2 inch square area is larger than most areas which will be measured by the photo-interpreter and (b) this area is located in the most unfavorable portion of the oblique photograph, for S_y has its maximum rate of change at the top of the photograph. These conditions make this a severe test of the area measurement system.

The two drawings of Figure 10 clarify the location and geometry of this area. From previous considerations, it was seen that all print areas which lie between the same two horizontal line on the print (which are assumed to be parallel to the horizon) and are both congruent and similarly oriented, correspond to equal ground areas. Hence location of the 2 inch square could be anywhere along the top edge.

The true (actual) ground area A_T in square feet is given by

$$A_T = 1/2\left[\left(\frac{2}{12}\right)S_{X_1} + \left(\frac{2}{12}\right)S_{X_2}\right]\cdot[H\cot(\theta+\phi_2) - H\cot(\theta+\phi_1)] \tag{68}$$

where

$$H = \text{altitude in feet}$$

$$\overline{S}_x = \frac{S_{X_1} + S_{X_2}}{2}$$

$$\theta = \text{depression angle}$$

$$\phi_1 = \text{angle off axis to } y_1 \text{ (measured along the principal line)}$$

$$\phi_2 = \text{angle off axis to } y_2 \text{ (measured along the principal line).}$$

Substitution of the appropriate values of θ, ϕ_1, and ϕ_2—(52°, −11°, −14.05°) respectively yields

$$A_T = \frac{H}{6}\overline{S}_x(0.13188). \tag{69}$$

The approximate ground area, computed by the technique developed above is given by

$$A_E = (\text{Image area})\overline{S}_x\overline{S}_y \tag{70}$$

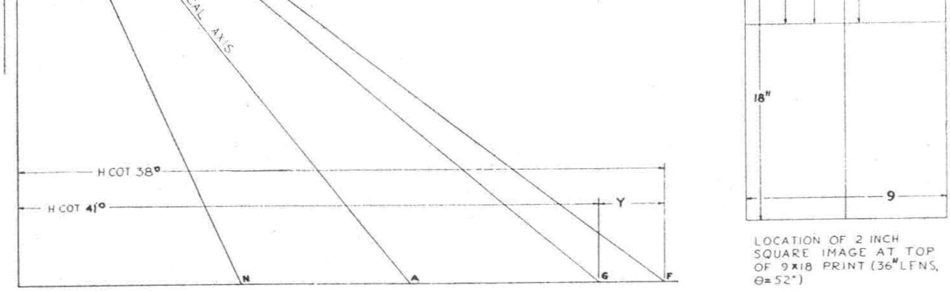

FIG. 10. Example of area measurement by the methods developed in this paper.

where

$$\overline{S}_x = \frac{Sx_1 + Sx_2}{2}$$

f = focal length in feet (3′)

ϕ_3 = angle off axis to y_3 (measured along the principal line).

The value of ϕ_3, at y_3, is given in Figure 10 as $-12.52°$ (note that for angular differences of the order with which we are herein concerned, ϕ_3 is given to 2 decimal places by the arithmetic average of ϕ_1 and ϕ_2). Substitution of the values of ϕ_3 and f into the formula for A_E (Eq. 70) reduces this equation to

$$A_E = \left(\frac{1}{6}\right)^2 \overline{S}_x \frac{H}{3} \frac{\cos^2(-12.52°)}{\sin^2(39.48°)}$$

$$= \frac{H}{6}\overline{S}_x(0.13097). \tag{71}$$

From Eqs. (69) and (71) we obtain

$$\frac{A_T}{A_E} = \frac{0.13188}{0.13097} = 1.00695. \tag{72}$$

Hence the ground area A_E measured by this system developed above is within 0.7% of the true ground A_T computed by formal methods. It is true that an example is not a proof, but it will have to be admitted the approximation is unusually close..

Part of this unusual closeness is chargeable to the easy and obvious choice, in this example, of the proper point to compute $\overline{S}_x$ and $\overline{S}_y$; the "feel" developed after handling these formulae for only a short time will be enough to convince the reader of the efficacy of the method.

In general the technique would have been to estimate the ground area by use of a single $\overline{S}_x$ and a single $\overline{S}_y$. Let us see how good the $\overline{S}_x$ would have been computed at the same point as the $\overline{S}_y$ (just found to be within .7%). In this case it is easily shown (the calculations are routine, and omitted) that

$$\overline{S}_x \text{ (calculated at } y_3) = 0.51177H$$

and that

$$\overline{S}_x \text{ (the average of } S_{x_1} \text{ and } S_{x_2}) = 0.51247H.$$

The ratio is 1.0014, showing the power and accuracy of this method of finding $\overline{S}_x$.

J. HEIGHT MEASUREMENTS IN OBLIQUE PHOTOGRAPHS

As previously discussed, the photo-interpreter is concerned in no small degree with making measurements on photographs, and from these measurements calculating the corresponding dimension of the ground object. In general, the interest of the photo-interpreter is in man-made objects, as distinct from the object of interest to photogrammetrists. In this section, we shall derive a simple formula for finding the height of (man-made) objects from a single oblique photograph. This includes objects of interpretation interest such as bridges, antennae, ships, smoke-stacks, hangars and the like which in normal high-altitude photography will yield small images. The conditions under which use of this formula is reducible to a simple and quick procedure are exactly the conditions under which most oblique photography will be produced in future operations: the camera will be mounted with a known and fixed depression angle in a side oblique position as one of a bank of long focal length cameras affording wide lateral coverage. The amount of tilt induced by the normal aircraft roll will be measured and available. Under these circumstances all calculations can be made from a single photograph; stereo measurements are unnecessary.

Consider again the geometry of the side oblique photograph (Figure 11), wherein the optical axis is perpendicular to the aircraft longitudinal axis. The ground object of height h is located as shown, at an angle ϕ below the optical axis which is at an angle of depression θ. The angle ϕ is always measured in a positive sense clockwise from the optical axis; thus angles above the optical axis are negative and the situation in Figure 11 is entirely general. Further, the subsequent theory holds exactly over the entire line given by a particular combination of ϕ and θ, for it is a common observation that the images of any set of vertical objects on a line at a constant distance from the flight line (in the case of the side oblique photograph) are themselves essentially parallel; the plane perspective grid doesn't apply to vertical assemblages. Of course, if the heights of the ground objects are a substantial fraction of the flying height, as in the case of the low altitude oblique photographs of Chicago (Figures 12, 13) the tops of the vertical objects are photographed at a larger scale than the bottoms of the objects, and the objects seem to diverge. Even this effect will not affect the calculations, the effect necessitating a small and negligible cosine correction. The following discussion, while made for the side oblique photograph, holds equally well for the forward or rear oblique photograph.

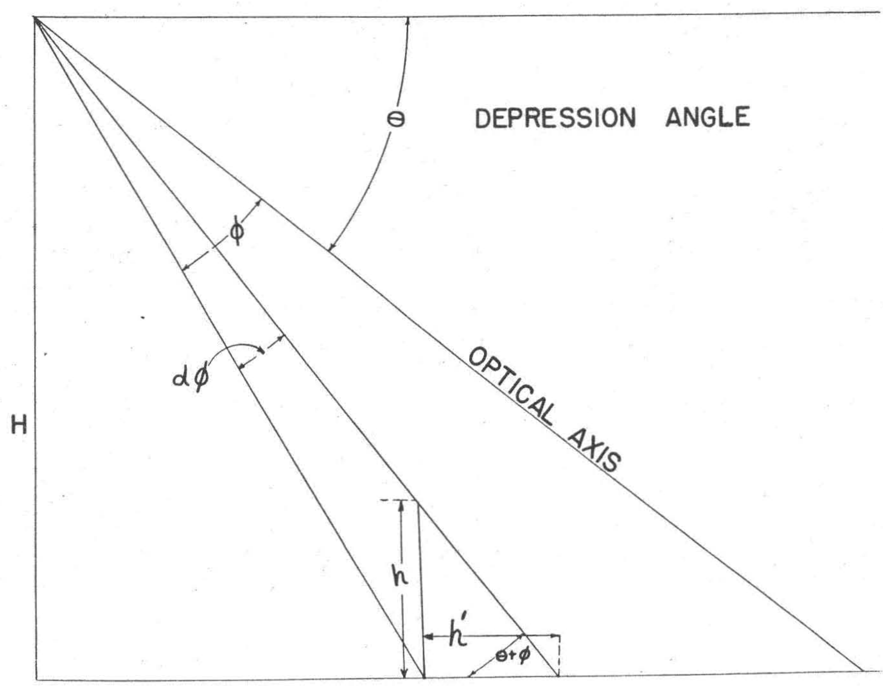

FIG. 11. Geometry of the oblique photograph, with especial reference to the measurement of height h.

FIG. 12. Low altitude oblique photograph of Chicago. Camera used was K-27 (12″ f/6.3 lens on 9″×18″ format). Note the perspective effects, especially prominent in wide angle photographs.

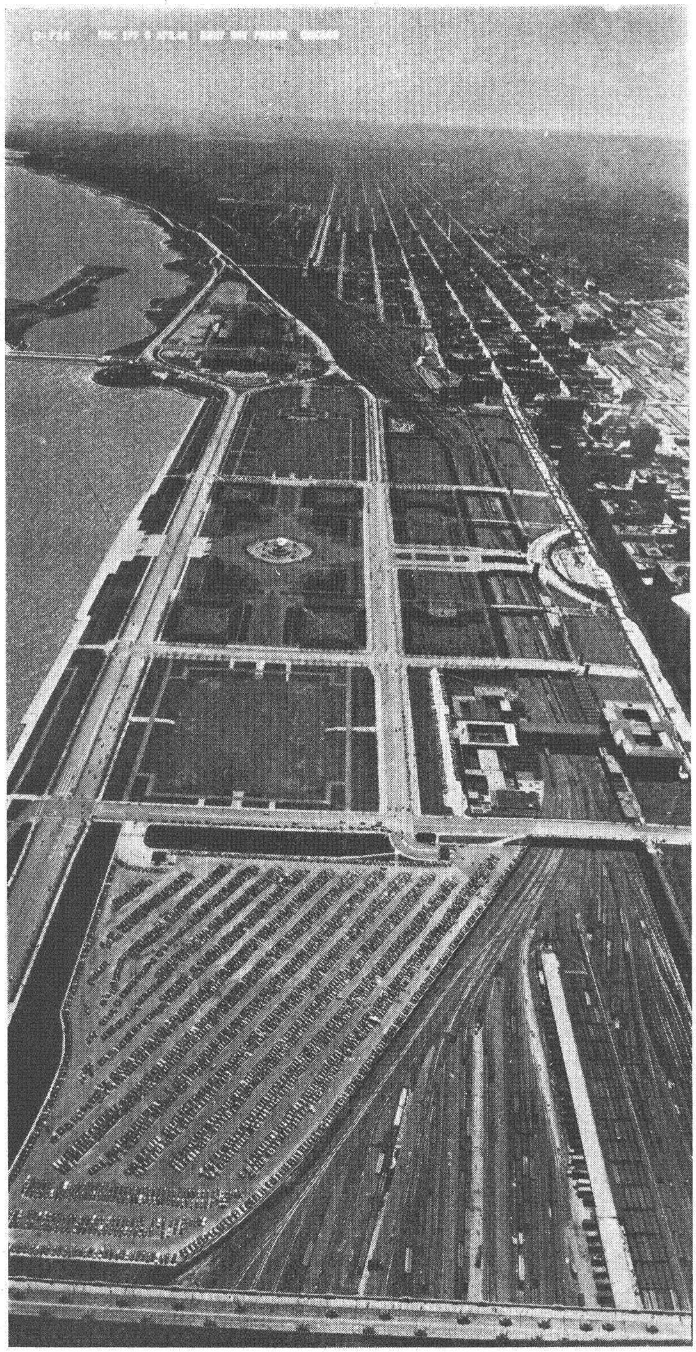

FIG. 13. Unusual photograph of Chicago's lake-front. Camera same as for Fig. 12. Note how closely the streets form a perspective grid on the photograph.

In Figure 11, the angle $d\phi$ is greatly exaggerated, as are h, the height of the ground object and h', the projection of h on the ground. This is easily seen when it is realized that the ratio H/h will in general be of the order of 300 or more. Under these circumstance $d\phi$ will be a small fraction of a degree, and it is order to write

$$h = h' \tan (\theta + \phi). \tag{73}$$

The ground distance h', is perpendicular to the flight line; the corresponding image length I must be multiplied by S_y (Eq. 23) to obtain h'. Hence making the appropriate substitutions in Eq. (73) we have

$$h = \frac{HI}{f} \frac{\cos^2 \phi \tan (\theta + \phi)}{\sin^2 (\theta + \phi)}.$$

This formula is easily rewritten in the form

$$h = \frac{2HI}{f} \frac{\cos^2 \phi}{\sin 2(\theta + \phi)} \tag{74}$$

where

h = Height of ground object in feet
H = Flying height in feet
f = Focal length in feet
I = Image size of ground object, in feet
θ = Angle of depression of camera axis
ϕ = Angle off optical axis, as defined above

Defining S_h as that factor which when multiplied by the image height I yields ground object height h, we have:

$$S_h = \frac{2H}{f} \frac{\cos^2 \phi}{\sin 2(\theta + \phi)}. \tag{75}$$

To estimate the effects of aircraft roll on the measurement of height by this system, we will investigate the error introduced by assuming that no roll exists (i.e. that the actual value of θ is the same as the installation angle or nominal depression angle θ_n). Tilt will be measured in the same sense as θ and ϕ: a positive tilt angle means further depression of the optical axis, and a negative tilt angle results in decreasing the depression angle. We therefore have

$$\text{Est. } h = (\text{Image Size})(\text{Est. } S_h) \tag{76}$$

where

Est. h = Estimated (calculated) ground object height

Est. S_h = S_h determined from ϕ and nominal depression angle θ_n.

The image size is determined from the actual (but momentarily unknown) or true ground object height and the actual or true S_h (which depends on ϕ and the actual depression angle θ_a).
Hence

$$\text{Est. } h = (\text{True } h)\left(\frac{\text{Est. } S_h}{\text{True } S_h}\right). \tag{77}$$

The estimated S_h, in ignorance of the tilt, is

$$\text{Est. } S_h = \frac{2H}{f} \frac{\cos^2 \phi}{\sin 2(\theta_n + \phi)}. \tag{78}$$

The actual (true) S_h is

$$\text{True } S_h = \frac{2H}{f} \frac{\cos^2 \phi}{\sin 2(\theta_a + \phi)}. \tag{79}$$

Substituting the values of Est. S_h and True S_h into Eq. (77) yields the ratio of Est. h to True h:

$$\frac{\text{Est. } h}{\text{True } h} = \frac{\sin 2(\theta_a + \phi)}{\sin 2(\theta_n + \phi)}. \tag{80}$$

The actual depression angle is given by

$$\theta_a = \theta_n + t \tag{81}$$

where t is the tilt angle
Writing

$$s = 2(\theta_n + t) \tag{82}$$

and substituting in equation (80) yields

$$\frac{\text{Est. } h}{\text{True } h} = \frac{\sin (s + 2t)}{\sin s}$$

which reduces to

$$\frac{\text{Est. } h}{\text{True } h} = \cos 2t + (\cot s)(\sin 2t). \tag{83}$$

Assuming that the tilt angle is no greater than 5°, (which is very probable) permits use of the approximations $\cos 2t = 1$ and $\sin 2t = 2t$ (measured in radians). For tilt measured in degrees, Eq. (83) can be modified as follows:

$$\frac{\text{Est. } h}{\text{True } h} = 1 + 0.0349t \cot (\theta_n + \phi). \tag{84}$$

The two tables below give the value of this ratio for the K-38-36″ camera, using the example values of $\theta = 52°$ and $\theta = 71°$. Distances on the print are measured positive toward the horizon along the principal line, starting at the principal point.

Study of Table VII shows that measurements in the lower half of the photographs made at $\theta_n = 71°$ are subject to inordinately large errors. The reason for this somewhat unexpected situation is that in this part of the photograph very little of the height of an object is photographed. Clearly, in the vertical (as in the case of +5° tilt and objects at the lower edge of the photograph) there is no image to measure. Were we sure that the tilt would vary between ±5°, it would be perfectly in order to use the height formula (uncorrected for tilt) for objects in the upper half of the photograph. True, large errors—as much as 20%—could occur. There is however no alternative in the absence of tilt data.

In the case of the camera mounted with $\theta_n = 52°$, the data of Table VI reveal

TABLE VI

$$\frac{\text{Estimated } h}{\text{True } h} = \cos 2t + (\sin 2t) \cot (\theta_n + \phi)$$

36″ lens, $\theta_n = 52°$

Tilt Angle t	Distance Off Axis, Inches						
	−9	−4.5	−2.25	0	+2.25	+4.5	+9
+5°	.827	.892	.918	.9415	.964	.986	1.028
+3°	.885	.938	.954	.968	.982	.995	1.021
+1°	.968	.981	.986	.991	.995	1.000	1.008
0°	1	1	1	1	1	1	1
−1°	1.031	1.018	1.013	1.008	1.004	.999	.991
−3°	1.089	1.051	1.035	1.021	1.007	.994	.968
−5°	1.142	1.078	1.052	1.028	1.006	.984	.941

TABLE VII

$$\frac{\text{Estimated } h}{\text{True } h} = \cos 2t + (\sin 2t) \cot (\theta_n + \phi)$$

36″ lens, $\theta_n = 71°$

Tilt Angle t	Distance Off Axis, Inches						
	−9	−4.5	−2.25	0	+2.25	+4.5	+9
+5°	0	.594	.695	.762	.810	.848	.907
+3°	.401	.760	.820	.860	.890	.907	.948
+1°	.802	.922	.942	.955	.965	.973	.985
0°	1	1	1	1	1	1	1
−1°	1.198	1.058	1.058	1.045	1.035	1.027	1.015
−3°	1.590	1.230	1.170	1.130	1.100	1.077	1.042
−5°	1.968	1.374	1.273	1.206	1.1576	1.120	1.061

that the worst error, and an improbable one, is about 17%. The errors should in general be much less than this, for objects of interest will be distributed uniformly over the print, and 5° is assumed to be an extremum for tilt. This analysis, however, is not designed to show what can be obtained by disregarding tilt. Such disregard can be accepted only if the accompanying errors are not excessive. Such judgment depends on the particular problem at hand. Tilt data are easily obtained from the stabilized mount which will be in the same aircraft as the multi-camera installation. A subsequent section will describe how such tilt data might be used to correct the computations, without introducing too many complications for the interpreter. As a suggested method, the latter could, if he had tilt data, take an expanded and reciprocal form of Tables VI and VII (computed for the particular cameras and installation angles producing his photography) and correct his calculated h's by the appropriate multiplying factor.

Of course Eq. (74), although valid enough, will have little appeal to the photo-interpreter, or to any one else engaged in hurried and routine calculation. The problem of reducing this height computation, as well as those for area and lengths to a simple procedure will be fully discussed in a subsequent section.

It should be further noted that the problem of obtaining the largest image of a ground height h has the same solution as the problem of maximizing the image of a ground area, i.e. imaging the object as far off axis as possible. It should also be obvious to the reader that another assumption herein made, and in fact throughout the paper, is that the ground objects of interest are in the horizontal plane containing the nadir point. More simply stated, it is assumed that the H of Figure 11 is applicable. Two comments regarding this assumption are in order. First, for those areas of intelligence interest, great relief throughout the photograph is unlikely. In the few and unlikely cases which may arise, a correction to H may be applied. Second, photography from high altitudes, say 40,000′, minimizes the effect of absolute altitude differences between the nadir point and the objects of interest.

K. THE GRAPHICAL CALCULATOR FOR OBLIQUE AERIAL PHOTOGRAPHS

Equations, formulas, and tables are helpful, important and basic tools, but it is difficult to develop a good sense of "feel" about such things. The author designed the Graphical Calculator for oblique aerial photographs (shown in Figures 14 and 15) to meet the requirement of presenting a graphic and dynamic analysis (of the oblique photograph) which at the same time would be useful in determining coverage, nadir point distances, effective altitudes, S_x, S_y, etc. Its use and theory are practically self-explanatory, and with this calculator (as with the P.I. slide rule) anyone can calculate with understanding and confidence, after but a few minutes of explanation.

It is intended to supply a base board on which will be drawn the coordinate axes (altitude in feet and ground distance, in both statute and nautical miles) and a set of transparent sectors for each of the various cameras in use. These

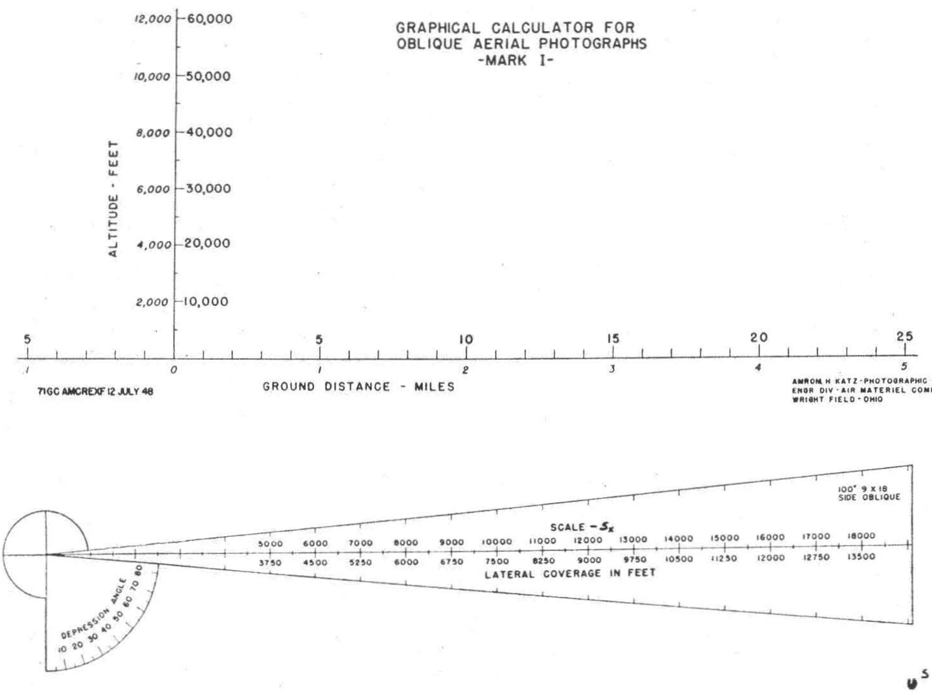

FIG. 14. The Graphical Calculator for Oblique Aerial Photographs. The sector shown below the base axes is for use with the K-30 camera (100″ focal length 9″×18″ format).

sectors will be hooked into a mechanical runner which will permit locking of either or both of the camera altitude and depression angle. Consider the example in Figure 15. The 100″ (9×18) K-30 camera, is flying at 40,000′, with depression angle $\theta=25°$. The short side of the camera format is horizontal, making for maximum oblique coverage. From Figure 15 we immediately read the following information:

(a) The near point of the photograph (foreground) is approximately 10.3 miles from the nadir point.
(b) The principal point is about 16.7 miles from the nadir point.
(c) The far point (background) is about 21.7 miles from the nadir point.
(d) The scale no.—S_x—at near, axial, and far points is about 9,800, 11,800, and 14,750 respectively.
(e) The lateral coverage at near, axial, and far points is about 7,400 feet, 8,850 feet, and 11,000 feet respectively.

Addition of a supplementary line pivoted at the same point as the sector, and linear calibration of the opposite end of the sector, would enable the operator to obtain nadir point distance, scale, and coverage for any point in the photograph, and not just for the points corresponding to the outside boundary and center lines of the sector. This modification, and finer subdivision of the axes, are being planned for the production proto-type of this calculator, along with certain changes in the scales.

The instrument will not have to be very large to provide more precision and accuracy then is required or significant. An 18″ long sector will allow readings on scale, for example, to be made to better than 5%. This gadget will be of especial value to staff planners, although it will also be very useful to the photo-interpreter.

In Figures 14 and 15 the basic scales are the bold-face altitude and ground distance scales; the altitude axis (to 12,000′) and the ground distance axis (to 5 miles) are for use with low altitude operations. Clearly both the altitude and

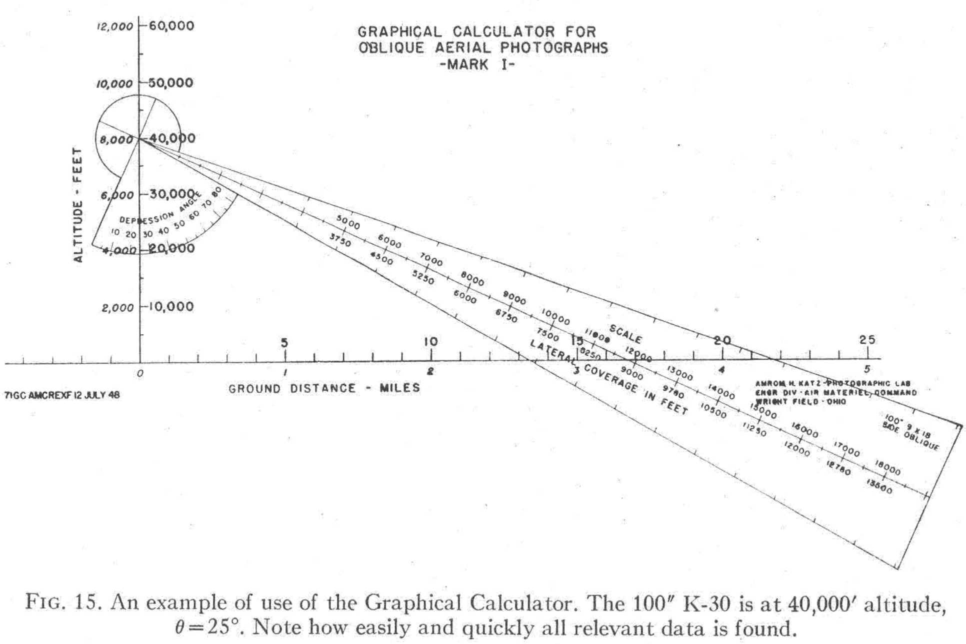

FIG. 15. An example of use of the Graphical Calculator. The 100″ K-30 is at 40,000′ altitude, $\theta=25°$. Note how easily and quickly all relevant data is found.

ground distance scales can be multiplied or divided by 10, (if the same operation is performed for both axes) giving expanded ranges useful for some problems.

L. THE OBLIQUE COMPUTING OVERLAY

(A system for easy and rapid computation of heights, lengths, and areas in oblique photographs made at a fixed depression angle.)

Enough has been said above to indicate that in the immediate future photo-interpreters will be handling, using, and interpreting at least four times as many oblique photographs as vertical photographs, for in a five-camera multi-camera installation there are four oblique cameras and one vertical camera. In seven-camera installations the ratio is six oblique cameras to one vertical camera.

This ratio—four or more to one—is not, however, an accurate measure of the ratio of interpretation time on oblique photographs to interpretation time on vertical photographs. Assuming a uniform probability for the location of areas of interest in the angular sector covered by multi-camera installations, the ratio of interpretation effort and time for obliques to that for verticals will be the ratio of lateral ground distances covered in the two cases. Consider the case of 5-K-38 cameras, with 36″ lenses, mounted so as to cover say, 120° across the line of flight. The total lateral angular coverage of 120° yields a ground coverage of 3.46 times the flying height H, whereas the single vertical camera covers .5H across the line of flight. The ratio becomes

$$\frac{3.46H - .5H}{.5H} = 5.92.$$

Hence, in this case the ratio of ground area on the oblique photographs to the ground area on the vertical camera is about 6.1.

It is imperative, therefore, that the photo-interpreter be given computing aids which incorporate and take advantage of all of the information known in advance, and when necessary, tilt data as well. Measurements made for other than mapping purposes, which include all of the measurements to be made from large scale multi-camera photography, usually can be in error by several per cent, and many times, by 10% without impairing their usefulness.

With these considerations as a basis, it is possible to design a simple and easily used aid, which will in advance incorporate all of the data and geometry of an oblique photograph made at a stated depression angle except the altitude factor, the actual image measurements, and tilt data. This computing gadget, which for want of a better name, is being called the Oblique Computing Overlay, is a transparent sheet, on which the 9×18 inch format is outlined. Lines one inch apart and parallel to the 9 inch side (perpendicular to the principal line) are drawn across this area. On one side are located five columns of numbers—for measurement of x, y, height, area, and distance from nadir point. These numbers are S_x/H, S_y/H, S_h/H, S_A/H^2 and cot $(\theta+\phi)$ respectively. Multiplying the first three of these by the flying height H yields S_x, S_y, and S_h. The area factor must be multiplied by H^2 to yield S_A, and H cot $(\theta+\phi)$ is the distance from nadir point. The appropriate S is then multiplied by the actual image length, height or area to obtain the corresponding ground measurement.

Figure 16 is a sketch of the 9×18 inch overlay showing the numbering system (above and below the horizontal line through the center of the 9×18 inch Photograph) and the principal line. In use, the overlay is centered using the fiducial marks, not with respect to the edges of the print, which are occasionally masked. Tables VIII and IX give the column data for each inch of the overlay,

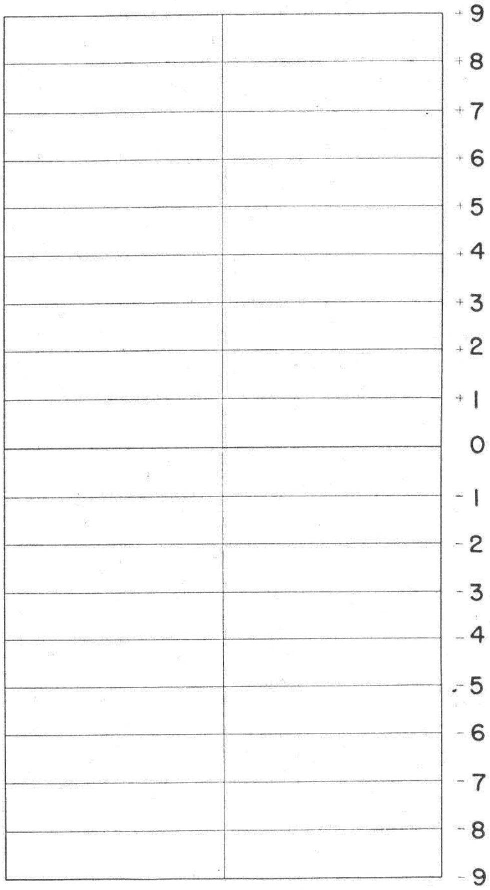

FIG. 16. The basic grid for the Oblique Computing Overlay.

and are reproduced in table form only for the sake of clarity of presentation. As noted above, these five columns are immediately adjacent to the overlay, so that the location of any object of interest in the photograph is referenced to the nearest horizontal line and the appropriate factor (of the five factors listed) is read directly.

Careful study of the set of numbers in Tables VIII and IX should convince the reader that one inch intervals are close enough. For anyone wishing to work with more accuracy, or in those fortunately infrequent cases where greater accuracy is necessary, the linear interpolation between the inch line values is permissible, easy and accurate.

The several examples of S_x, S_y, S_A, and S_h given in the separate sections above illustrate the power, utility, speed, and accuracy of these methods, hence no further examples are needed in this section.

It is abundantly clear from the discussion of the preceding sections that the absence of tilt information necessitates wide confidence limits on the computation of areas, lengths, heights and nadir distances. It is also clear that with tilt information which is obtainable by comparison of the true nadir point (photographed by a camera in a stabilized mount) and the apparent nadir point photographed by the vertical camera (in the rigid multicamera installation), the calculations can be made to a several per cent degree of accuracy. What is required is a simple method of exploiting the tilt data.

The author believes that the following method, developed in a joint discussion with Mr. Eldon Sewell, will fulfill the purpose and enable use of a single grid for calculations. The grid of Figure 16 will have to be extended say, $\pm 5°$ in ϕ. This distance is an additional distance d_1 where

$$d_1 = \pm\ 36(\tan 19° - \tan 14°) = \pm\ 3.42''. \tag{85}$$

To keep the even inch divisions, 4″ would be added on each end, and appropriate constants computed. The use of tilt data would enable finding of the exact location of θ_n on the photograph, and the zero line would be moved by translation and rotation, to its true position on the photograph. In this way both components of tilt—chargeable to roll and pitch—would be compensated for, and the new position of the grid would be accurate. There are certain assumptions underlying this contemplated use of the grid. Chief among them is the assumption that the number of inches off axis and ϕ are linearly related. For the 36″ lens, (and of course for lenses of longer focal length on the same format) this is true enough. Actually we have

$$y = f \tan \phi \tag{86}$$

and for the distance from $\phi = 14°$ to $\phi = 19°$ we computed above that

$$d_1 = y\,(\text{at } 19°) - y\,(\text{at } 14°) = 3.42 \text{ inches.} \tag{85}$$

TABLE VIII. FOR K-38, 36″ LENS $\theta = 52°$

Numbers to be used in conjunction with the Oblique Computing Overlay. Multiply appropriate columnar entry by the flying height (or in case of areas, by H^2) to get the proper value of S_x, S_y, S_A, S_h, and the distance from nadir point.

Distance from Principal point, Inches	$\frac{S_x}{H}$	$\frac{S_y}{H}$	$\frac{S_A}{H^2}$	$\frac{S_h}{H}$	Nadir point distance / H
+9	.526	.829	.436	.647	1.282
+8	.512	.786	.402	.647	1.215
+7	.499	.746	.372	.649	1.150
+6	.486	.710	.345	.651	1.090
+5	.474	.675	.320	.654	1.032
+4	.463	.644	.298	.659	0.977
+3	.452	.614	.278	.664	0.925
+2	.442	.587	.259	.671	0.875
+1	.432	.561	.242	.678	0.827
0	.423	.537	.227	.687	0.781
−1	.414	.514	.213	.697	0.738
−2	.405	.493	.200	.709	0.695
−3	.397	.473	.188	.722	0.655
−4	.389	.455	.177	.737	0.617
−5	.382	.437	.167	.754	0.580
−6	.374	.420	.157	.773	0.544
−7	.367	.405	.149	.794	0.510
−8	.360	.390	.140	.818	0.476
−9	.354	.376	.133	.845	0.445

TABLE IX. FOR K-38, 36″ LENS $\theta = 71°$

Numbers to be used in conjunction with the Oblique Computing Overlay. Multiply appropriate columnar entry by the flying height (or in case of areas, by H^2) to get the proper value of S_x, S_y, S_A, S_h, and the distance from nadir point.

Distance from Principal Point, Inches	$\frac{S_x}{H}$	$\frac{S_y}{H}$	$\frac{S_A}{H^2}$	$\frac{S_h}{H}$	Nadir point distance / H
+9	.386	.446	.172	.686	.650
+8	.382	.437	.167	.713	.614
+7	.378	.428	.162	.742	.577
+6	.374	.420	.157	.774	.542
+5	.370	.411	.152	.811	.507
+4	.367	.403	.148	.851	.473
+3	.363	.395	.143	.897	.440
+2	.359	.388	.139	.950	.408
+1	.356	.380	.135	1.012	.376
0	.353	.373	.131	1.083	.344
−1	.349	.366	.128	1.166	.314
−2	.346	.359	.124	1.267	.283
−3	.343	.352	.121	1.389	.254
−4	.340	.346	.117	1.539	.225
−5	.336	.340	.114	1.731	.196
−6	.333	.333	.111	1.986	.168
−7	.330	.328	.108	2.331	.141
−8	.327	.322	.105	2.838	.114
−9	.325	.316	.103	3.644	.087

We may compare this with the y distance corresponding to the central 5°. In this case the distance d_2 is

$$\left.\begin{aligned} d_2 &= 2.36 \tan 2.5° = 3.14 \text{ inches} \\ \text{and} \qquad d_1 - d_2 &= 0.28 \text{ inch.} \end{aligned}\right\} \tag{86}$$

Detailed description of the development, technique of use of the tilt-correcting Oblique Computing Overlay, and thorough error studies will have to be deferred to a subsequent paper. At this point it looks to be practical, reasonable, and feasible.

The sole objective of the author has been to develop an easily used system of computation and measurement such that the photo-interpreters who use it need know a minimum of mathematics. Quite clearly the photo-interpreter who has the appropriate Oblique Computing Overlay need know nothing about the derivation of the formulae. Unfortunately, he must still use multiplication, for it has not yet been possible to eliminate this process and simultaneously keep a simple system devoid of big, expensive, precision gadgetry.

PART IV

CONCLUSIONS AND COMMENT

The reader may rightly wonder at this point that experimental data and reports of texts of this system are missing from this report; the choice confronting the author was between (a) holding off publication of theoretical considerations till they could be supplemented by thorough experimental tests, and (b) publication of the experimental test results following publication, dissemination, and criticism of the basic theory.

That this choice was resolved in favor of the latter alternative is obvious. Two basic considerations dictated this choice. First, and less subtle, is the author's conviction that it is unnecessary to take vertical photographs with a 12 inch lens at 10,000 feet to prove that under such conditions the scale number is 10,000 (R.F. = 1:10,000). Second, experimental tests of this system, when conducted, will not be designed to verify examples like that cited immediately above. Such experimental tests will be essentially statistical tests, designed to establish the probability distributions of the quantities entering the calculations. For example, assume that the probability distribution of focal lengths of production 36″ lenses around the nominal 36″ focal length, the probability distribution of angles of roll $d\theta$, estimates of the precision of tilt determination, and similar distributions for altitude readings, image measurements, and the other variables were available.

Using the methods of modern statistics, one could then calculate the variance (square of the standard deviation) for each of these variables, and then determine the master distribution (for the quantity under measurement—say, distance, area, or height) around the true value. Calculation and use of the term "probable error" is obsolete by at least a dozen years, but modern statistical methodology has available the much more useful concept of "confidence limits." Confidence limits, for any desired degree of confidence, could be calculated and used for estimates of the precision and accuracy of the particular method, and presence or absence of systematic bias would be determined. Successful execution of such a test program, depending as it must on the usual bedevilments and vagaries of flight testing, will take a comparatively long time.

It is hoped that this presentation of the several aspects of the photo-interpretation problem (and the systematic exposition of a measurement and computing system) will provoke much comment and discussion. Were this paper circulated exclusively within the military services, it is clear that such discussions would be severely limited, and thus be eventually detrimental to the interests of the photo-interpreter.

Of relevance and interest in connection with this point are the viewpoints on security of the Joint Committee on Atomic Energy of the 81st Congress. In their majority report on the Investigation into the United States Atomic Energy Commission, published in October, 1949, this Committee discussed clearly, convincingly, and at length the nature and philosophy of "security by achievement" and "security by concealment."

The writer feels that only by presenting problems to the broadest type of technical group, such as the readers of PHOTOGRAMMETRIC ENGINEERING, can solutions be found which, by meeting the tests and rigors of open discussion and criticism, are tempered and sharpened into useful tools.

Acknowledgement

The nature of the author's daily office routine, organized around a quantum work unit of about twenty minutes, is such as to preclude doing research or calculations of the type described in this paper during working hours. Because practically all of this work was done at home, a grateful acknowledgment is due to my wife, Louise, who made possible the completion of this project by her continued interest and high-order efficient administration of household and family. Sincere thanks are also due to Mr. Eldon Sewell, of the Corps of Engineers, and Mr. Walter Levison and Lt. Col. Richard W. Philbrick of the Photographic Laboratory. Their constant interest, encouragement, and especially their helpful and illuminating discussions, aided the writer considerably; however, whatever heresies and inaccuracies persist are solely the author's.

APPENDIX A

DISTORTION PRODUCED BY A FOCAL PLANE SHUTTER TRAVELING PARALLEL TO LINE OF FLIGHT

The derivation of Eq. (16), for the relative distortion produced by a focal plane shutter which moves parallel to the flight line will now be given. The basic assumptions are that no image motion compensation is employed (in this case, there would be no distortion) and that the camera is essentially vertical. Under those circumstances the forward image speed in inches/sec., is obtained by Eq. (3) by differentiation, yielding

$$v_i = \frac{88}{60}\left(\frac{Vf}{H}\right) \tag{87}$$

where

V = Ground speed in m.p.h.
f = Focal length in inches
H = Altitude above terrain, in feet.

Figure 17 shows schematically the relationship between the lens, shutter, focal plane, and two images of length l, one lying close to the axis, the other far off axis.

From Figure 17, it will be seen that the distance the shutter must move to cover an image of length l is independent of the position of the image.

Also, it will be seen that if image l were infinitesimal in size, and if the shutter were infinitesimal in width, the shutter would still have to move the distance c to complete the exposure of the image point. Because the shutter slit is of width w and the image of length l, the shutter must move the additional distances c and Δ, shown in Figure 17. However, while the shutter slit is scanning l, the image is moving. In the following use of the double sign $\pm$ or $\mp$, the upper sign will always refer to the case in which the slit is moving in the same direction as the image, and the lower sign will always refer to the case in which the slit moves opposite to the image.

To cover the image l, the slit must move

$$E' = c + \frac{(f-d)}{f} + w. \tag{88}$$

During this time the image has moved $\pm\Delta l$, so the shutter must travel an additional

$$\pm\,\Delta l\,\frac{(f-d)}{f}\,.$$

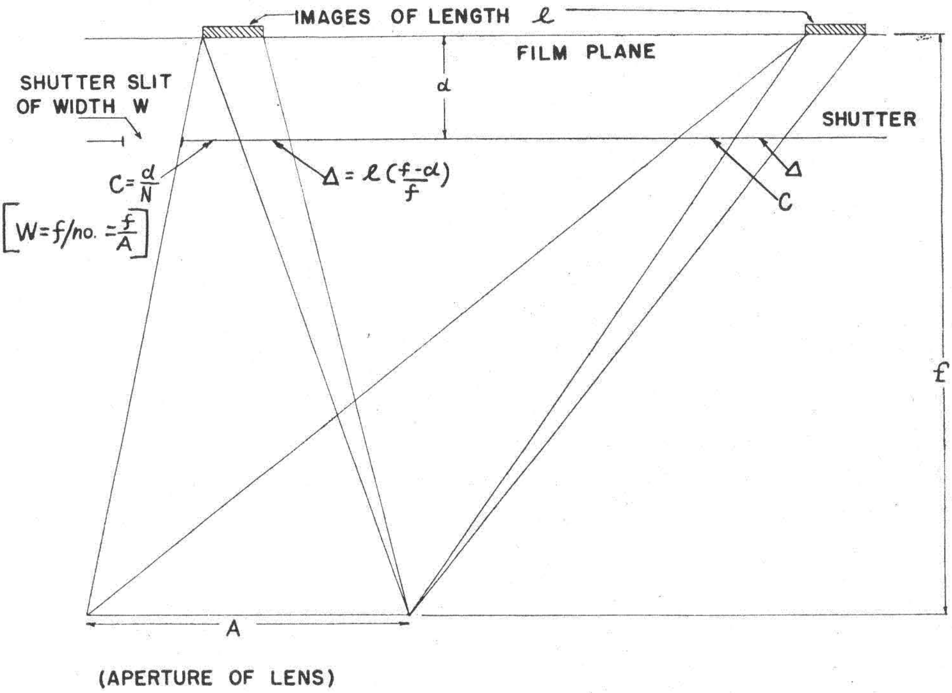

FIG. 17. Geometry of the focal-plane shutter relationship to a moving image.

Hence the curtain travels

$$E = c + w + (l \pm \Delta l)\,\frac{(f-d)}{f} \tag{89}$$

in a time t in which the image moves Δl. If v_c is the velocity of the curtain and v_i the velocity of the image, then

and

$$\left.\begin{aligned} E &= v_c \cdot t \\ &= v_i \cdot t. \end{aligned}\right\} \tag{90}$$

From Eqs. (90)

$$\Delta l = \left(\frac{v_i}{v_c}\right) E. \tag{91}$$

Upon substitution of α for $(f-d)/f$ and the value of E from (89) into (91) there results

$$\Delta l - \left(\frac{v_i}{v_c}\right)(\pm\alpha\Delta l) = \frac{v_i}{v_c}(c + w + \alpha l) \tag{92}$$

which, upon further simplification and division by l, yields relative distortion directly

$$\text{Relative Distortion} = \frac{\Delta l}{l} = \left[\frac{v_i}{v_c \mp \alpha v_i}\right]\left[\frac{c+w}{l} + \alpha\right]. \tag{93}$$

Upon making the proper substitutions for α and c the formula (93) becomes much more imposing, viz:

$$\frac{\Delta l}{l} = \left\{\frac{88}{60}\,\frac{Vf}{H\left[v_c \mp \left(\frac{f-d}{f}\right)\frac{88}{60}\,\frac{Vf}{H}\right]}\right\}\left\{\frac{(d/N)+w}{l} + \left(\frac{f-d}{f}\right)\right\}. \tag{16}$$

The absolute distortion, Δl, is of course obtainable from (16) by multiplying by l, the image length. Inspection of the formula for relative distortion shows that small images are distorted more, percentage-wise, than large images. The reason for this unexpected effect is that the linear blurring caused by image motion during the actual (motion-stopping) exposure time is independent of image size, and hence contributes a larger proportion of the total distortion in small images than it (the linear blurring) contributes to the total distortion in larger images. A single example will show the nature of this effect:

The focal plane shutter of the K-24 camera has a curtain velocity of 75 inches/sec. at some point in the focal plane. With the $\frac{1}{8}''$ width the shutter located 0.75 inch from the focal plane, with aircraft speed of 300 mph and altitude of 1,000′, the table below applies. (True image size refers to the theoretical, unblurred, undistorted image size.) Computations were made with a slide rule, and may be in slight error. (The shutter slit travels forward, yielding positive distortion):

TABLE X

Ground Object Length, Feet	True Image Size, Inches	Relative Distortion, %	Absolute Distortion, Inches
50	0.35	8.6	0.003
100	0.70	6.1	0.004
250	1.75	4.7	0.009
500	3.50	4.2	0.015

APPENDIX B

DERIVATION OF THE APPROXIMATION $(A^2+B^2)^{1/2}=0.96A+0.4B$

Consider $(A^2+B^2)^{/2}$, $A>B$.

This may be rewritten, setting $x=B/A$, as

$$(A^2+B^2)^{1/2} = A\left[1+\left(\frac{B}{A}\right)^2\right]^{1/2} = A(1+x^2)^{1/2}. \tag{94}$$

The plot of

$$y = (1+x^2)^{1/2} \tag{95}$$

is given in Figure 18, for $0 \leqq x \leqq 1$.

This curve is distorted by a factor of two in the vertical scale, to accentuate the curve shape. Clearly the dashed straight line connecting the end points of the curve is a poor linear approximation of the curve; a better linear approximation would cut across the curve at two points in such a way as to minimize departures from the curve. To find the point where the straight line connecting the end points departs most from the curve, construct the equation for the difference, Y, between the straight line

$$y_1 = 1 + 0.4142x \tag{96}$$

and the curve

$$y_2 = (1+x^2)^{1/2} \tag{97}$$

$$= y_1 - y_2 = 1 + 0.4142x - (1+x^2)^{1/2}. \tag{98}$$

Upon differentiation, we have

$$\frac{dY}{dx} = 0.4142 - x(1+x^2)^{1/2}. \tag{99}$$

At the maximum value of Y, $dy/dx=0$, and

$$x(1+x^2)^{-1/2} = 0.4142. \tag{100}$$

x can be found readily by algebraic processes; however, observation of the fact that if $x=\tan\theta$ the left side of the equation is $\sin\theta$, enables direct solution from a table by finding tan ($\sin^{-1}$ 0.4142): $x=.4550$, and y_2 at this point, as shown in the figure, is 1.0987. The criterion for picking the best approximating line is that it can depart from the curve by an amount proportional to the height of the curve. Let the line pass through the particular points indicated on the figure. The value of k is found by equating the slope m_1, of the line through $(0, 1-k)$ and $(0., 4550\ 1.0987+1.0987k)$ to the slope m_2 through $(0.4550, 1.0987+1.0987k)$ and $(1, 1.4142-1.4142k)$ and solving, whence

$$k = 0.03924. \tag{101}$$

The equation of this approximating line is therefore

$$y = 0.9608 + 0.3980x. \tag{102}$$

The assumption is herein made that the maximum departure of straight line from the curve is still

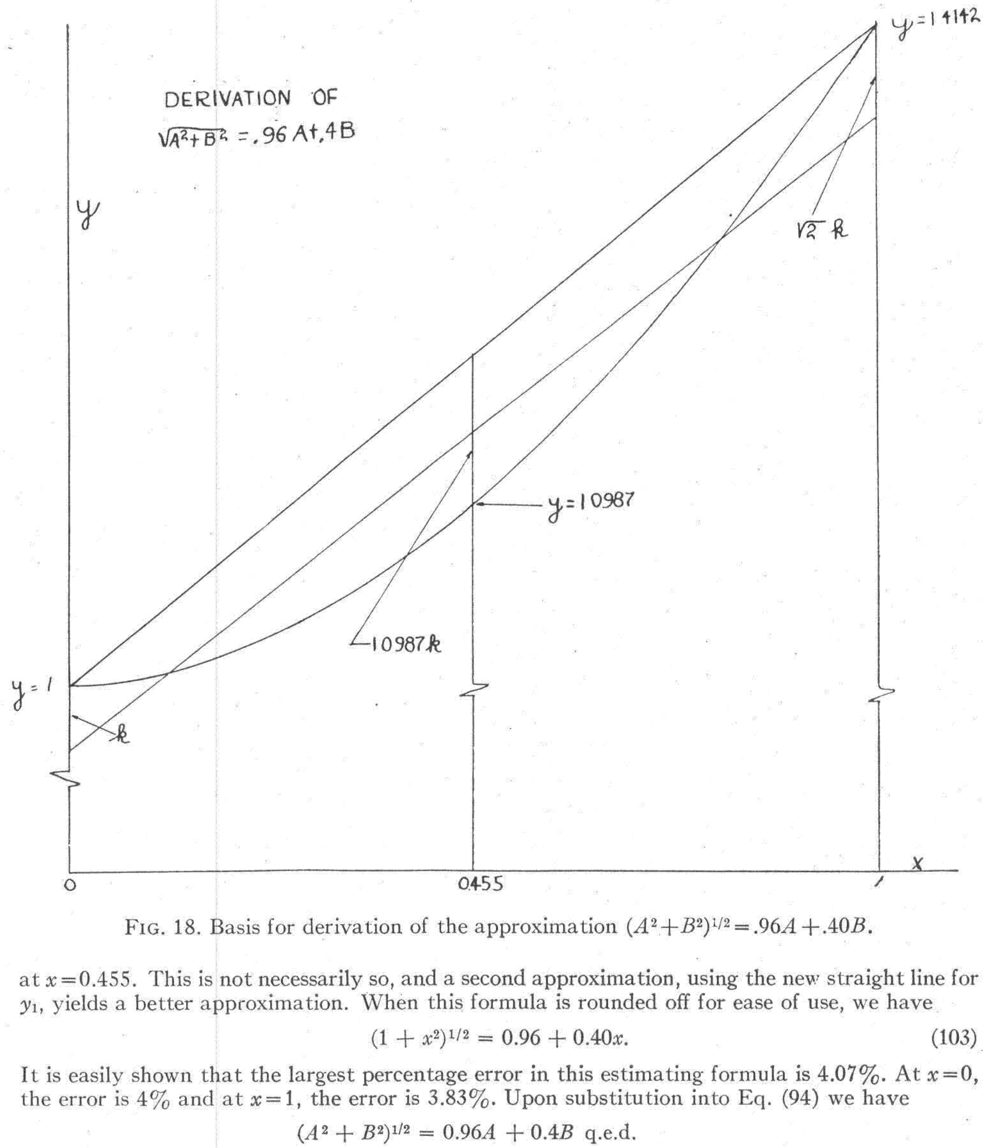

FIG. 18. Basis for derivation of the approximation $(A^2+B^2)^{1/2} = .96A + .40B$.

at $x=0.455$. This is not necessarily so, and a second approximation, using the new straight line for y_1, yields a better approximation. When this formula is rounded off for ease of use, we have

$$(1 + x^2)^{1/2} = 0.96 + 0.40x. \tag{103}$$

It is easily shown that the largest percentage error in this estimating formula is 4.07%. At $x=0$, the error is 4% and at $x=1$, the error is 3.83%. Upon substitution into Eq. (94) we have

$$(A^2 + B^2)^{1/2} = 0.96A + 0.4B \text{ q.e.d.}$$

REFERENCES

1. Coleman, C. G., and Lundahl, A. C., (Editors) "Symposium—Military Photographic Interpretation," PHOTOGRAMMETRIC ENGINEERING, vol. XIV, no. 4, pp. 453–521, Dec. 1948.
2. Kent, Sherman, *Strategic Intelligence*, Princeton University Press, 1949.
3. Wiener, Norbert, *Cybernetics*, John Wiley and Sons, 1948.
4. Jones, Thad, "Interpretation of Military Installations from Aerial Photographs," PHOTOGRAMMETRIC ENGINEERING, vol. XIV, no. 4, p. 501, Dec. 1948.
5. Goddard, Col. George W., "New Developments for Aerial Reconnaissance," PHOTOGRAMMETRIC ENGINEERING, vol. XV, no. 1, pp. 51–72, Mar. 1949.
6. KATZ, Amrom H., "Aerial Photographic Equipment and Applications to Reconnaissance," *Journal of the Optical Society of America*, vol. 38, no. 7, pp. 604–610, July 1948.
7. Katz, Amrom H., "Air Force Photography," PHOTOGRAMMETRIC ENGINEERING, vol. XIV, no. 4, pp. 584–590, Dec. 1948.
8. Katz, Amrom H., "Camera Shutters," *Journal of the Optical Society of America*, vol. 39, no. 1, pp. 1–21, January, 1949.