A NEW APPROACH TO FLIGHT PLANNING*

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INTRODUCTION

D^{URING} recent years significant developments in aerial mapping methods and instruments have taken place that point unmistakably toward one goal. That goal is ever-increasing precision in photogrammetric operations. Yet the fact remains that even the most precise instrumentation and procedures cannot overcome the disadvantages of poorly planned photographic coverage of the area to be mapped.

DEFICIENCIES IN PREVIOUS PLANNING METHODS

The methods of planning vertical aerial photography that were formerly in general use, have proved to be unsatisfactory because of several important deficiencies. Chief among these shortcomings are:

(a) Planning was done in terms of inches on the photograph, and inches on the flight map instead of at natural scale, thereby necessitating extra calculations that can be avoided by using a more scientific method of approach.

(b) The manner of expressing the spacing of photographs, in terms of per cent forward lap and per cent side lap, was misleading and unscientific, and resulted in inefficient photographic coverage.

(c) Data were not available, in convenient form, for the incorporation of the effects of tilt and relief in the flight design.

In this paper, methods of planning flights for vertical aerial photography are developed, in which the following principles are embodied:

(a) All measurements are at natural scale.

(b) The spacing of photographs, expressed in terms of the Base/Height and Width/Height ratios, is designed to provide efficient photographic coverage.

(c) The effects of tilt and relief are readily ascertained from simple graphs.

(d) The design procedure is standardized and simplified through the use of a Standard Flight Design Work Sheet.

In addition to facilitating the flight design, this Work Sheet affords a permanent record of the design. This paper is devoted essentially to the development of the principles used in the formulation of the Work Sheet. We will make a closer examination after examining the considerations involved in its preparation.

NATURE OF THE PROBLEM

When an aerial mapping project is under consideration, it is the function of general planning or policy-making personnel to determine:

(a) Completion dates of various phases.

(b) The boundaries of the project.

(c) The method to be used in map compilation.

(d) The compilation and publication scales, and the contour interval or intervals of the map.

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FIG. 1. Utilization of Negative Area and Corresponding Coverage on the Ground.

FIG. 2. Effect of Spacing of Photographs on Model Area.

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(e) The nature, extent, and location of existing control.

Given this basic information, the following factors in the flight design can be determined:

- (a) Focal length, format, and type of camera to be used.
- (b) Direction of flight lines.

(c) Flight heights.

- (d) Spacing of photographs along flight line.
- (e) Number and spacing of flight lines.

The discussion of the choice of camera, direction of flights, and flight heights, which are included in the *complete* version of this paper, will be omitted here, in order to maintain a reasonable brevity. Instead, the main problem will be discussed, namely—the spacing of photographs and flight lines. Before doing this, however, it is pertinent to examine certain geometrical facts as illustrated in Figure 1.

(Although the principles developed in this paper are illustrated only insofar as they apply to the commonly-used 5.2'' and 6.0'' cameras, they apply with equal force to other cameras.)

EXPLANATION OF FIGURE 1

Figure 1 shows the utilization of negative area, and corresponding coverage on the ground, for focal lengths of 5.2'' and 6.0'' with a 9'' square format, assuming the same exposure station for both cameras. Angle α is the apex angle of the largest cone of rays that can be used with dependable results throughout the entire sequence of steps in the photogrammetric procedure. It is necessarily

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assumed to be something less in value than the full angular coverage of the camera lens, which may be as much as 93° for some wide-angle cameras. For want of a better name, α will be referred to as the "Working value of the angle of coverage," and other quantities derived from it will be designated as "working values". As a result of Geological Survey experience, α has been given a value of 85° throughout this paper, but there is no inclination to debate the issue too strenuously with anyone advocating the use of say 83° or 87°.

It is seen that the 6'' camera utilizes a larger portion of the negative area than the 5.2" camera, but the 6.0" photograph is at a larger scale inasmuch as the scale is equal to the focal length divided by the flight height. On the other hand, the 5.2" camera records a larger ground area on the negative. It should be noted that the ground area encompassed in the field of the lens is a circle that depends only on the angle of coverage and is independent of the focal length. If the negative format were sufficiently large to record the entire circular coverage for either case, there would be no difference in the ground area recorded on the negative.

SPACING OF PHOTOGRAPHS

Prior to June, 1949, the Geological Survey followed the common practice of expressing the spacing of photographs in terms of percentage overlap. The spacing *along* the flight lines was indicated by the percent forward-lap, and the spacing *between* the flight lines was indicated by the per cent side-lap.

This manner of expressing spacing proved to be unsatisfactory in several respects:

(a) It is not a sound mathematical expression. "Percentage overlap" is an arbitrary measurement which does not adequately represent the interrelation between the photographs.

(b) Its use necessitates circuitous calculations that can be shortened by direct mathematical approach.

(c) Its ambiguity makes it unenforceable in a contract.

In seeking a method of expressing spacing that would overcome these objections, the problem was analyzed with the purpose of determining the best values for B, the air base, and W, the width between strips, or strip width. Figure 2 indicates some of the possible model proportions resulting from various spacings of photographs. The problem is to determine what are the proportions best suited to the needs.

EVALUATION OF B (SPACING OF PHOTOGRAPHS ALONG FLIGHT LINE)

The best value for the air base, B, can be determined on the basis of each of several criteria:

(a) Maximum area covered in a stereoscopic neat model.

(b) Most accurate reading in the stereoscopic model.

(c) Most accurate setting of the stereoscopic model.

(d) Reduction of camera distortion effect to a minimum.

(e) Assurance of ample stereoscopic coverage.

The best effect, with respect to accurate reading, accurate setting, and reduction of distortion is obtained by making the air base as *long* as possible. But lengthening of the base is limited by loss of stereoscopic coverage.

The best effect with respect to maximum gross stereoscopic coverage is obtained by shortening the air base. But shortening of the air base beyond a certain amount is limited by the physical and mechanical limitations of plotting equipment. It is seen that the factors other than maximum area of neat model have a balancing effect on each other. Therefore, the best value for B is evaluated on the basis of maximum area covered in a stereoscopic neat model. (It should be kept in mind, however, that maximum area covered in a neat model is not the *only* consideration determining the most efficient use of the photography. In some circumstances, a longer or shorter air base is desirable.)

The problem is first analyzed with the assumption of no tilt and no relief, and with the photocenters in adjacent strips opposite each other. (Such a disposition of photo centers is the worst condition with respect to overlap.) The effects of tilt and relief are analyzed separately farther along in this presentation.



FIG. 3. Spacing of Photographs

EXPLANATION OF FIGURE 3

The upper part of Figure 3 is a view on a vertical plane containing the flight line. O_1 , O_2 , O_3 , and successive camera stations. f is the focal length, H the flight height, B the air base, R the radius of circular coverage on the ground, and r the radius of circular coverage.

The lower part of the figure is a plan view on the ground, showing two adjacent flight lines, 1 and 2, W, the width between strips, the camera stations O_1 , O_2 , etc., and the area on the ground included in the circular coverage (working value) of each exposure. The cut-off of circular coverage that is shown in Figure 1 is not shown in Figure 3, since it does not affect the analysis for customary formats. If shown, it would only complicate the figure. The heavily outlined rectangle indicates the neat model. E is the distance from the flight line to the limit of the neat model.

We now proceed to determine the value of B that gives the maximum

area of the neat model. Referring to Figure 3, the area of the neat model is seen to be equal to $B \times 2E$, which can be evaluated in terms of R and B.

By application of the calculus, the conditions for the maximum value of this area can be derived. The necessary differentiation is demonstrated as an appendix to the complete paper, but will not be given here.

The mathematical procedure gives us this result: the maximum area of the neat model is obtained when $B = R/\sqrt{2}$.

This indicates an angle of 45° between *B* and *R*, hence B = E is the condition for maximum area of the neat model. In other words, the half-model is a square.

Having determined that the area of the neat model has its maximum value when $B = R/\sqrt{2}$ it is now possible to determine whether the old method of using 60 per cent forward-lap gave the most efficient photographic coverage.

The calculation will not be demonstrated at this time, but it works out to this result: for a 9" square format, and $\alpha = 85^{\circ}$, the maximum area of neat model

is obtained when the percent forward-lap is 57 per cent for a 6" focal length and 63 per cent for a 5.2" focal length.

These values for per cent forward-lap indicate that the old method of using 60 per cent forward-lap, regardless of the focal length and format, did not provide the most efficient photographic coverage.

To obtain an expression for spacing that *does* provide the most efficient photographic coverage we evaluate B/H as follows:

B has been shown to be equal to $R/\sqrt{2}$ for the condition of maximum area. From the upper part of Figure 3, $H=R \cot \frac{1}{2}\alpha$. Using these values, B/H reduces to .707 tan $\frac{1}{2}\alpha$. For $\alpha = 85^{\circ}$, B/H works out to the constant value of .65. Thus for any value of α , we can determine a constant value of B/H that is *independent* of the focal length and gives the maximum stereoscopic coverage in the neat model.

In the expression B/H, H is the elevation of the camera station above the point on the ground that is observed in the camera view finder for determining the instants of exposure of two successive photographs. This definition indicates the manner of interpreting H in an area of variable elevation. If a constant altitude above sea level is maintained, the effect of keeping the B/H ratio constant is that B varies in proportion as H varies. The simplicity of the use of this ratio is well demonstrated in the setting of the two lines on the aerial camera view finder. As B/H = b/f or net gain/focal length, then if B/H is .56, net gain = .65 f. In this case, the photographer simply multiplies the focal length of his view finder by .65 and sets off that distance between the lines. He then maintains his B/H ratio automatically by picking up an image point crossing line 1 at the instant of an exposure, exposing again when that point crosses line 2, immediately picking up another image point on line 1, and so on.

EVALUATION OF W (SPACING OF FLIGHT LINES)

Referring again to Figure 3, it is seen that the width between strips, or strip width, W, is equal to 2 E.

It has already been demonstrated that, for maximum area of neat model, each half-model is a square, or, B = E. Therefore W = 2B, for maximum area of neat model. Furthermore, W > 2B, gapping occurs in the stereoscopic coverage. As B/H for maximum area of neat model has been shown to be .707 tan $\frac{1}{2}\alpha$, W/H must be twice as great, or W/H = 1.414 tan $\frac{1}{2}\alpha$.

For $\alpha = 85^{\circ}$, W/H works out to the constant limiting value of 1.30.

It can be shown from this analysis that, for a 9" square format and an assumed angle of coverage of 85° , the minimum sidelap for a focal length of 6" is 13 per cent and for a focal length of 5.2" is 25 per cent.

These values indicate that the old standard of using a fixed percentage of sidelap (say 30 per cent), regardless of focal length or format, did not provide the most efficient photographic coverage.

It is apparent from this analysis that, for most efficient flying with respect to area covered, the following conditions should exist for vertical photographs of flat terrain:

Condition I. $B/H = .707 \tan \frac{1}{2}\alpha$ (specifically, if $\alpha = 85^{\circ}$, B/H = .65)

Condition II. $W/H = 1.414 \tan \frac{1}{2}\alpha$ (specifically, if $\alpha = 85^{\circ}$, W/H = 1.30)

These are the equations for no gapping at the corners, under ideal conditions. However, in practice it is necessary to make provision for the following factors:

(a) Relief

(b) Allowable tilt

(c) Allowable crab

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FIG. 4. Flight Spacing with Relief Allowance

(d) Allowable deviation from the flight line

(e) Allowable deviation from the flight height

EFFECT OF RELIEF

The effect of relief on the allowable strip width, W, is demonstrated as follows:

In Figure 3, let h = maximum relief above assumed ground elevation, and assume this elevation to occur in the extreme corner of the neat model.

By mathematical analysis (not demonstrated at this time) an expression can be derived for width between strips that includes an allowance for the relief of terrain, h. (See Figure 4.)

In this expression, W_h is the maximum width between strips that can be



FIG. 5. Tilt Displacement

tolerated, without the risk of producing a gap in the stereoscopic model, when the relief amounts to h.

In Figure 4, curves are plotted for various values of H, each of which indicates the limiting values of strip width, for varying values of relief of terrain. One such curve is drawn for each of the commonly-used flight heights.

It should be noted that these curves contain two inherent factors of safety: 1. It is assumed that the centers of photos fall opposite each other in adjacent flights.

2. It is assumed that the peaks fall in the extreme corners of the models where the relief has the most damaging effect.

Because of these assumptions, there is justification for taking a certain amount of calculated risk in not adhering rigidly to these requirements, especially if considerable economies may reasonably be expected to result in PHOTOGRAMMETRIC ENGINEERING

most instances. In many cases, the possible gaps may be covered by short reflights. Where good flight maps are available, it is, of course, wise to arrange the flights to cross directly over the peaks, if possible.

EFFECT OF TILT

Tilt about an axis along the flight line has the effect of displacing the limit of photo-coverage. The effect in a single photo is demonstrated as follows:

In Figure 5 let t and t' represent the displacements of the coverage limit due to an angle of tilt θ . As t' increases the coverage, there is concern only with t, which decreases it. From the figure, and a mathematical procedure, the indicated equation is reached.

This equation is plotted in Figure 5, for f=6'' and 5.2", and $\theta=2''$. From this figure, values of the displacement due to tilt can be found, given the flight height and focal length. The allowable value for W, as calculated without tilt, must be decreased by the displacement determined from Fig. 5.

The tilt analysis includes the effect from one photo only, as it is not likely that photos in adjacent flights will be tilted in opposite directions. Here, again, an element of risk is involved, for sometimes the harmful effect of tilt *is* cumulative.



FIG. 6. Crab

If the tilt is about an axis normal to the flight line, displacement of coverage is automatically compensated for in holding B/H constant, provided the tilt is in the same direction in consecutive exposures. Ordinarily, there is not a sharp change in the direction of tilt between successive exposures; therefore, no further tilt allowance is made.

EFFECT OF CRAB

In connection with the analysis of spacing of photographs, it should be noted that the analysis holds true only if crab is kept within reasonable limits.

Figure 6 shows one quadrant of a 9" square format, and 5.2" and 6.0" circular coverage (working value).

If the 5.2" photograph is crabbed (i.e., rotated about 0) until the edge of the format passes beyond its critical point, N_1 , a gap will occur in the stereoscopic coverage. Likewise, if the

6" photograph is crabbed until the edge of the format passes beyond its critical point, N_2 , a gap will occur. The critical angles of crab, K_1 for the 5.2" camera, and K_2 for the 6" camera, can be evaluated, with the following results:

 $K_1 = 26^{\circ} K_2 = 10^{\circ}$

It is seen from this analysis that the limit of crab for the 5.2'' camera is 26° whereas it is 10° for the 6'' camera. However, 26° of crab is not desirable as it imposes a severe handicap in the orientation of stereo-plotting instruments. Geological Survey specifications allow not more than 10° of crab. As either

camera gives sufficient coverage with 10° of crab, no additional allowance for crab is made in the flight design.

EFFECT OF DEVIATION FROM FLIGHT LINE AND FLIGHT HEIGHT

Specifications for aerial photography necessarily permit a certain amount of leeway in allowable tilt, crab, deviation from the flight line, and deviation from the flight height. However, provision for these tolerances could be carried to an unwise extreme in flight design, if it were assumed that the photographic flight will be at the limit of the tolerances in all respects, and that the harmful effects of the allowable deviations will be entirely cumulative. Experience has shown that the occurrence of such an accumulation is very rare.

Geological Survey specifications permit a leeway in the position of flight lines equal to 10 per cent of the flight height. However, in practice, the great majority of photographic flights adhere fairly closely to the planned lines. The

deviation from flight line is considered as a risk that will occasionally cause gaps in the stereoscopic coverage. It would not be a sound economic procedure to design the flights in such a manner as to cover all allowable deviations from flight lines. A sounder solution, economically, is to provide for short reflights in the event that gaps do occur. Such situations are likely to arise, for example, when the best available flight maps are of very poor quality so that the pilot has no reliable means of holding the theoretical line.

The allowable deviation from flight height is 2 per cent below and 5 per cent above the specified flight height. Contractors uniformly take advantage of the latter tolerance and fly slightly higher than the specified height, in order to decrease the possibility of gapping. This is a further safety factor in the design; and no additional allowance should be made for



the possibility that the flight height will be too low.

SPACING SCALE FOR PHOTOGRAPHS

The spacing scale is used for the purpose of determining the B/H and W/H ratios of photographs taken with cameras having a focal length of 5.2" or 6.0" It makes possible very rapid approximate checking of assembled flight strips, based on measurements between picture edges. For accurate determinations of the Base/Height ratio, successive principal points are spotted on the photographs, and the Base-/Height ratio is read directly by measuring between the points. The spacing scale is not quite so convenient for accurate measurement of the Width/Height ratio, as this requires the drawing of intermediate lines common to adjacent strips, and the summation of two measurements for each determination.

SAMPLE FLIGHT DESIGNS

As sample flight designs, two cases are presented, illustrating the application of the principles already outlined, to actual flight design. Case I illustrates a routine design of no particular difficulty. It follows a procedure outlined on the standard Flight Design Work Sheet. As a part of the Flight Design Work Sheet, two nomographs are presented to facilitate the rapid determination of values for W_h and t. These nomographs are equivalent to Figures 4 and 5 in the information that they make available. The nomographs are somewhat more convenient to use; in the case of determining W_h , the nomograph has the added advantage that it can be used for any flight height without the necessity of interpolating between curves. When reference is made to values obtained from Figure 4 or Figure 5, it should be understood that the values may be obtained from either the curves or the corresponding nomographs. Figure 7 illustrates the flight plan for Case I made up from information obtained from the Flight Design Work Sheet.

Case II is a more complicated design because of the large amount of relief. Whenever circumstances necessitate the study of individual flight strips, line by line, as occurs in this case, the standard Work Sheets must be supplemented by additional calculations as shown in the continuation of the Design for Case II, which follows.

CONTINUATION OF THE FLIGHT DESIGN, CASE II (see Flight Design Work Sheet)

To be assured of no gaps with a uniform flight height above sea level, 9 flights would have to be flown at an elevation of 18,700+1,600=20,300'. But an alternate solution is to fly higher over the high area, lower over the low area, and fly only 8 lines.

The tabulation below indicates the flight heights chosen, and the resultant range of projection distances in the Multiplex. It will be noted that a shorter than optimum projection distance was favored somewhat, but in each case the range straddles the optimum distance of 360 mm.

The projection distance is calculated from the formula:

$$H = \frac{dS}{304.8}$$
 or, $d = \frac{304.8 \ H}{S}$

Line	Range of Relief	Assumed Ground Elev.	Flight Height above Sea Level	Range of Projection Distance
1	300-3,000	1,300	20,000	379-327
2	350-2,000	1,300	20,000	378-346
3	400-2,000	1,300	20,000	377-346
4	400-2,400	1,300	20,000	377-338
5	500-3,400	1,300	20,000	375-319
6	600-3,400	1,300	20,000	373-319
7	650-4,300	1,800	20,500	382-312
8	750-4,300	1,800	20,500	380-312

where H = Flight Height above Sea Level, minus Relief.

For flights 1-6, the maximum elevation is 3,400' above sea level, or 2,100' above assumed ground. For H=18,700 and h=2,100', the Width Chart indicates an allowable value for W_h of 18,300. Allowing 1,000' for tilt displacement as indicated on the tilt chart, allowable W=17,300'. Therefore, the spacing of 16,630' is acceptable and includes an ample factor of safety.







Flight Design Work Sheet-Case I.

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Flight Design Work Sheet-Case II.

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For flights 7-8, the maximum elevation is 4,300', or 2,500' above assumed ground. The Width Chart indicates an allowable value for W_h of 17,000' when H=18,700 and h=2,500'. Allowing 1,000' for tilt displacement, allowable W=16,000'. This is less than the calculated value of 16,630 for W. However, in this case, it is advisable to risk a gap rather than fly higher, for the following reasons:

- 1. The peak elevation is at only one point; gapping will not occur unless the peak is at the extreme corner of the model, and the photo centers are exactly opposite each other.
- 2. Over the peak, there is as much chance of the tilt displacement increasing the overlap as there is of reducing it.

From the foregoing tabulation, the range of H is calculated as follows:

Maximum H = 20,000 - 300 = 19,700Minimum H = 20,500 - 4,300 = 16,200

Since W is constant at 16,630, the range of W/H is as follows, assuming accurate flying.

Minimum W/H = 16,630/19,700 = .84

Maximum W/H = 16,630/16,200 = 1.03

Allowing ten per cent of the flight height for deviation from flight line, the allowable value of W/H ranges from .74 to 1.13. This range is automatically maintained when the photographic flight follows the flight line as drawn on the flight map, within allowable limits.

Figure 8 illustrates the resulting flight plan for Case II.

CONCLUSION

If the methods used in this approach seem unduly complicated, it should be remembered that the end product, the Flight Design Work Sheet, is comparatively simple. It is, after all, necessary to understand the complications before they can be reduced to acceptable simplicity.

LUNCHEON ADDRESS*

Arthur C. Lundahl, Naval Photo Interpretation Center, Washington, D. C.

LADIES and Gentlemen, Distinguished Guests and Friends of the American Society of Photogrammetry:

In the few minutes that I have to speak with you today, I should like to dwell on two subjects, very briefly. I should like to talk in general, first, about some of the things that are happening to the American Society of Photogrammetry—some of the trends which are developing and that I have observed in my year's service as the Chairman of the Publications Committee, and then I should like to relate in a remote fashion these events to some of the things which we have been doing at the U. S. Naval Photographic Interpretation Center where I am employed.

The American Society of Photogrammetry is becoming big business. You do not realize this until you attend some of the long sessions of the Publications Committee with the Editor. Each of the issues is growing larger and larger. This year marks the biggest year that we have ever had in pages published. Over 682 pages. I have no doubt that we will publish much more next year; the trend is definitely upward.

* Delivered extemporaneously at the Luncheon Session, Sixteenth Annual Meeting of the Society. Washington, D. C., January 14, 1950.