Einpassen von Luftaufnahmen," *Anzeiger d. öst. Akad. d. Wiss., math.-nat. Kl.*, 85 (1948), S. 72–74.

(32) ———, "Genauigkeitssteigerung der gegenseitigen Einpassen von Luftaufnahmen auf Grund noch nicht beachteter Bedingungsgleichungen zwischen den Orientierungsgrössen," *Öst. Zeit. f. Verm.*, 36 (1948), S. 25–45 und 56–61.

(33) ———, "Graphische Lösung der Hauptaufgabe beim Normalfall der Luftphotogrammetrie," *Anzeiger d. öst. Akad. d. Wiss. math.-nat. Kl.*, 86 (1949), S. 93–99.

(34) ———, "Gegenseitige Orientierung von Luftaufnahmen mittels liniengeometrischer Konstruktionen, *Anzeiger d. öst. Akad. d. Wiss., math.-nat. Kl.*, 86 (1949), S. 128–135.

(35) ———, "Über ein graphisches Verfahren zum gegenseitigen Einpassen von Luftaufnehmen," *Österr. Zeit. f. Verm.*, 37 (1949), S. 13–29.

(36) ———, "Über des Wegschaffen von Restparallaxen mittels graphischer Konstruktionen," *Schweiz, Zeitschr. f. Verm. u. K.*, 47 (1949), S. 256–262.

(37) J. Krames, "Zur Abhängigkeit zwischen den Orientierungsgrössen beim gegenseitigen Einpassen von Luftaufnahmen," *Anzeiger d. öst. Akad. d. Wiss., math.-nat. Kl.*, 87 (1950), S. 7–11.

(38) E. Kruppa. "Zur Ermittlung eines Okjektes aus zwei Perspektiven mit innerer Orientierung," *Zitzgsber. Akad. d. Wiss. Wien, math.-nat.*, 11a, 122 (1913), S. 1939–1948.

(39) G. Poivilliers, "Propriété perspective de certaines surfaces et son application aux levers photographiques aériens," *Intern. Archiv f. Photogrammetries*, VIII/2 (1937), S. 244–246.

(40) ———, "Formation de l'image plastique dans les appareils de restitution," *C. R. Ac. Sc. Paris*, 226 (1948), S. 1938–S. 1941.

LARGE AND INTERMEDIATE SCALE MAPPING OF EXTENSIVE AREAS WITH APPLICATION OF SPATIAL AERIAL TRIANGULATION. MAPPING EXAMPLE OF ISRAEL

T. Blachut dipl. Ing., Henry Wild Surveying Instruments Supply Co. Ltd., Heerbrugg, Switzerland

THERE is an ever growing need for the mapping of extensive areas (which may, in some instances, cover the whole territory of a country) at large or intermediate scales such as 1/2,500 and 1/5,000. As a matter of fact, this type of work is by no means limited to such highly developed areas as make up most of Europe, but also applies to regions that have yet to be fully developed.

Whatever the detailed specifications may be, such projects always entail major surveying assignments which must be completed as quickly as possible while meeting very exacting precision requirements. Large scale maps have many uses and must be kept up to date systematically, so that this precision of the techniques and the final accuracy of the results are essential requisites if the whole operation is not to be repeated after a short time.

Bearing the above in mind, there has been a tendency to supplement the photogrammetric work with a great deal of ground control when using such scales for the actual mapping, which would increase costs to a variable extent, depending on the nature of the terrain and on some local factors. Nevertheless with the many refinements embodied in modern photogrammetric cameras and plotting equipment, it becomes advisable briefly to review some of the techniques currently used in aerial surveying.

Attention will be drawn, in particular, to use of spatial aerial triangulation in the determination of geodetic control points, with the simplification and economies which this entails in large and intermediate scale mapping, especially in those regions which have to be surveyed for the first time.

In the case under review, two methods could be contemplated:

NOTE: Comments on this paper are invited. To ensure consideration for publication in the December issue, receipt before October 15 ss necessary.

spatial aerial triangulation,
the establishment of pass points by high-order leveling.

Radial triangulation would hardly ever have to be thought of. Since it only gives plane coordinates, it is not sufficiently precise.

In high-order leveling, the minor control points used for actual mapping are taken from sets of pictures which are themselves connected with ground control points (established by field surveying). This method, which is reliable up to a certain point, calls for a double amount of photographic material, a condition which it is not always possible to fulfill.

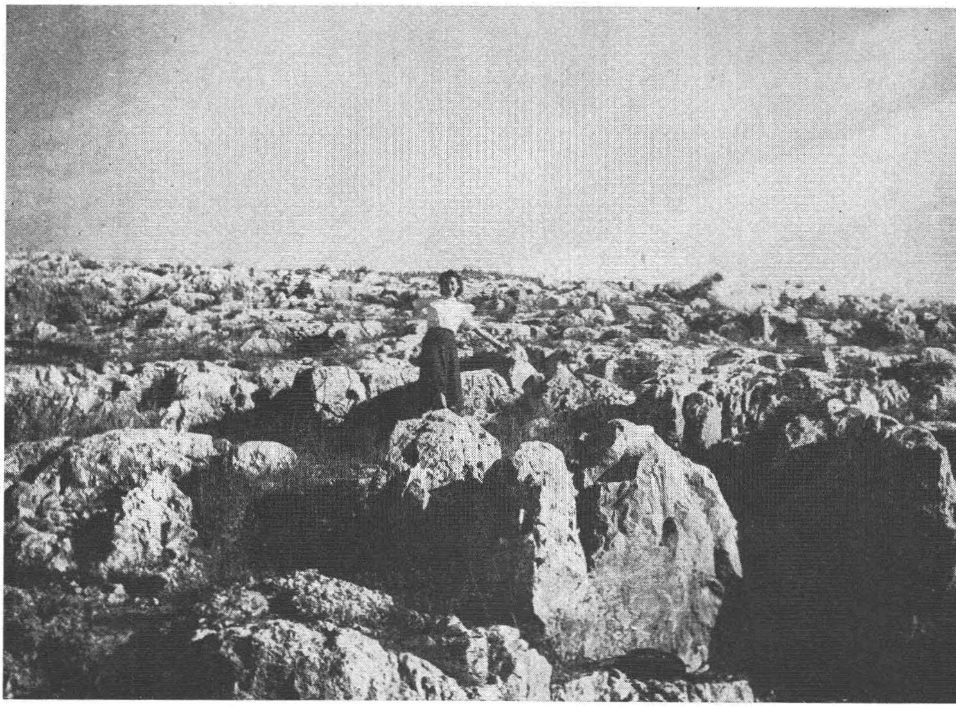

FIG. 1. Typical Examples of Ground Formation in a Part of Israel.

Such was the situation when mapping operations were started in the Fall of 1949 by the newly established Jerusalem Institute of Photogrammetry in Israel. The rapid development of Israel had created various problems which made it urgently necessary to map various large areas at scales of 1/5,000, 1/2,500 and, in parts of some of these, 1/1,000.

For various reasons, the only aerial survey that it was possible to carry out at once had to be from a uniform height of 6,900 ft. (2,100 m.), giving a representative fraction of 1/10,000 for a focal length of 210 mm. This survey covered about 2,500 sq. mi. and was conducted with a Wild fully automatic RC 5 camera equipped with an Aviotar lens. The excellent quality of the pictures made it possible to use them for all three scales, with different contour intervals, determined by the scale and topographic texture (5 m., 2.5 m. and 1 m. or roughly 16, 8 and 3 ft.).

The precision requirements were influenced by local conditions and technical considerations. Much of the country consists of hills which rise anywhere from about 600 to 1,000 feet above the surrounding country, and are dotted with rocks and boulders (Figure 1). Most of these hills are bare, except in the areas that are free from rocky formations, where they often are covered by a growth of thistles, ranging in height from one and a half to three feet. In other words the terrain is almost ideally suited for photogrammetric surveying and mapping, although the texture of the topography will occasionally make it advisable to resort to form lines, rather than actual contour lines. When a scale of 1/5,000 is used, in particular, with a contour interval of about 16 feet (5 m.), it would be altogether useless to endeavor to plot all the contour lines corresponding to each sizeable rock or edge with rigorous accuracy, because 16 feet on the terrain become 0.04 inch (1 mm.) on the map. Indeed, many boulders would then have to be represented by contour lines which would be reduced to a mere dot!

The planimetric content of these regions is rather meager as compared to the culture to be observed in the flat coastal districts, some of which are very thickly settled.

All the maps were to be used primarily for the agricultural development of the country and as guides for the installation of settlers. Nevertheless, the accuracy of the results was to be sufficient to allow further development and the filling in of details. Such requirements call for a high degree of precision in the planimetry. In fact, this precision had to be adequate for a final tracing accuracy of 0.3 mm. There was some question as to the precision of vertical control, for the reason that the existing fourth order triangulation points to which the plotting was supposed to be related, were believed to be up to ± 3.3 ft. in error (± 1 m.). A survey conducted in the field to check on these points showed that such large errors were exceptional and, allowing for the limitations imposed on the precision of the work by the topographic texture, it was hoped that it would be possible to keep the vertical error to a maximum of ± 4 ft. (1.2 m.). This appeared quite adequate for the purposes of the operation.

In flatter country, with scales of 1/2,500 and 1/1,000, vertical control had to be more exacting, leveling was resorted to when and as needed, and the mean error in the positions of the contour lines varied between $\pm 1\frac{1}{2}$ and $\pm 2\frac{1}{2}$ ft. (± 0.5 and ± 0.8 m.) depending on the cover and configuration of the ground and the methods used to determine the control points. Low growing vegetation, in particular, limits the precision of vertical plotting, and thus the accuracy of the final results.

It would seem inconsistent, at first sight, to allow for different tolerances according to the scale used, while all the plotting (regardless of the actual final scale) had to be done from pictures having the same representative fraction. It should be remembered, however, that this was inevitable since all the pictures available at the time had been taken from the same height. Furthermore, there were instances in which the scale that would finally be used for the mapping was not known in advance.

Another point to be borne in mind, in order to make this seeming inconsistency less evident, is the fact that the plotting at 1/5,000 scale was done with the A6 autograph, while 1/2,500 and 1/1,000 plotting was carried out with the A5. Finally, it had been decided at the very beginning of the work to relate all 1/5,000 plottings to control points obtained by spatial aerial triangulation.

In view of the fact that the season was well advanced and the rainy period imminent, the most difficult task which confronted the Institute was the determination of these pass points, particularly in view of the fact that the personnel was none too conversant with the methods of aerial photogrammetry. This meant that the rainy season might come before sufficient data had been obtained for starting field control operations in which case some of the A6 equipment would have had to remain idle. For the 1/2,500 maps, it was decided to wait until the result of the 1/5,000 aerial triangulation results were known, from which it would be possible to determine whether and within what limits one could extend this technique to this larger scale.

It was first proposed to bridge strips of five pairs by spatial aerial triangulation, using an A5 Autograph which would provide the required accuracy. However, it became apparent at once that the length of the strips could safely be extended to eight pairs, compensation for errors being computed from three ground control points suitably distributed about the middle of the strip.

In order to keep the number of the points to be determined in the field to a minimum, triangulation was carried out in part as shown in Figure 2.

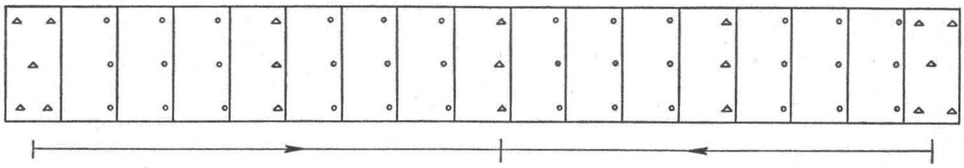

FIG. 2

Triangulation, of course, may be conducted in either direction. This method makes it possible to reduce by $\frac{2}{3}$ the number of control points to be determined in the field.

The minor or aerial control points were connected to ground control by resection or transection using stones, small bushes and the like as readily identifiable points. Nevertheless, there were some slight difficulties in the early stages of the work. The minor control points, in the Autograph, were identified on the photographs with suitable marks, and properly related to field sketches. This type of work requires some practice. As a consequence, the operators first had to use pass points that were not defined with adequate clarity, so that their precision and the speed were adversely affected. On the other hand, the advantages of the photogrammetric techniques became apparent because, even with the few points used to connect spatial aerial triangulation with ground control, it became possible readily to discovery those major control points that were not accurately determined.

AUTOGRAPH TECHNIQUES

At the beginning, all spatial triangulation work with the Autograph was carried out by the auther, because this is a technique which calls for a certain amount of skill, and a good deal of certainty as to the relative orientation of the consecutive models. In hilly country, the triangulation of one pair requires about an hour of work. This means that one operator was able to triangulate 4 to 6 pairs a day, which is equivalent to only 50% utilization of the A5 Autograph as against customary two shift operation. However, after three months of training, *it became possible to go over to two shift operation*, while retaining the same final accuracy and homogeneity of results as were achieved with one operator. As stated before, this work requires some theoretical knowledge and skill in the handling of the instrument. Conscientiousness, however, is the most important element, especially where there has to be a division of responsibility.

As a matter of fact, this probably also accounts for the fact that the operators must all have a reasonably broad general educational background.

The standard procedure is as follows: six aerial triangulation points are chosen from each pair of pictures. Every effort is made to maintain as thorough

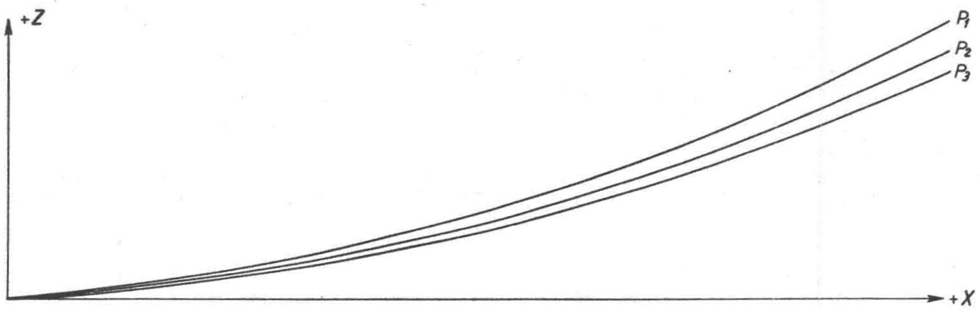

FIG. 3

a measure of vertical control as possible; this is not easy when dealing with steep slopes. It must be kept in mind that in rocky territory, the appearance of the same point will be different in adjacent pictures, a fact which may make identification difficult.

Triangulation was then carried out in accordance with the customary method used with consecutive pairs, scale transfer being effected by determining the exact elevation of the central or triangulation points.

It is advisable to choose triangulation points that are substantially equidistant from the X axis. This simplifies compensation and increases over-all accuracy.

As far as vertical compensation is concerned, it will usually be sufficient for the accuracy required to compute three parabolas from the compensation formula for the vertical errors in the triangulation point, plot these curves on graph paper and use them for graphic corrections. It will be convenient to use the same set of coordinates for all three parabolas. A suitable value of the Z scale should be selected in order to be able to read vertical corrections of 4″ (10 cm.) easily and accurately. The separation between parabolas at the points indicated shows the extent to which the difference in the Y coordinates of the aerial triangulation points affects the magnitude of the correction. With aerial triangulation points scattered at random throughout the strip, it will be advisable to resort also to graphic interpolation.

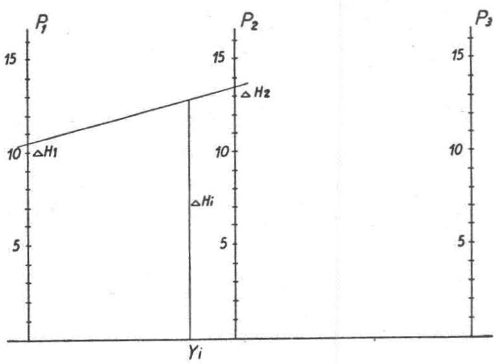

FIG. 4

In Figure 4, the straight lines P_1, P_2 and P_3 are the projections of the 3 parabolas on the YZ plane. One will first read ΔH_1, ΔH_2, or ΔH_3 from Figure 3, and then a final ΔH correction can be determined from the Y coordinate of the point on Figure 4.

Compensation of X and Y coordinates was still simpler. This was due to the fact that the errors in the planimetry were mostly very small, not exceeding a few meters at the ends of the strips. Furthermore, the curvature of the X and Y parabolas was so small that it would have been superfluous to determine it accurately. Wherever such curvature was present, the parabola was drawn approximately with a flexible ruler which was made to pass through the three values of the errors found for the connecting points. For the Y compensation, a single compensation curve usually was found adequate.

ANALYSIS OF THE RESULTS

Aerial triangulation has often been likened to traversing. This comparison is justified as long as only one strip is concerned. On the other hand, one must observe, when the whole area to be surveyed is covered by aerial triangulation that the coordinates of the lateral points of the strips are determined twice, from two independent triangulations. This greatly increases the dependability and accuracy of the results. The only points determined only once are those that are axially located. Due to their position, they usually are subject to smaller errors than the laterally located points. In the case of those points that are subject to double determination, the difference between their mean value and single determinations will be averaged. This was done for a triangulation group chosen at random, giving:

For the X coordinates ± 0.35 cm. (14″)
For the Y coordinates ± 0.30 cm. (12″)
For the Z coordinates ± 0.40 cm. (16″)

In this connection, it should be noted as already mentioned, that the larger deviations frequently are caused by the differences which will appear in the identification of the same points in successive mosaics and in the adjoining strips. *Therefore, the most exacting care should be devoted to the choice of these points during aerial triangulation.*

The maximal Y and X differences (mean value minus individual value) range up to 1 m. (40″) or 0.2 mm. at a scale of 1/5,000, a value which still lies within the limits of tracing accuracy. The greatest Z deviation amounted to 1.3 m. (51″).

As regards the point showing maximal deviations, it should be observed that these may be left out in the evaluation of the corresponding pairs; this is particularly true in the case of large X or Y deviation. In the planimetry there are six instead of two points from which to determine positions, and major errors in isolated points are hardly ever carried into the whole mapping operation.

A somewhat more delicate problem is posed by the maximal elevation differences. In general, four control points suffice for fitting in the model in elevation, and if one of the six aerial triangulation points available in each model shows a substantial deviation from the norm in elevation, this point may be omitted in plotting. Such points usually being located in the corners of the mosaics calls for extrapolation, requiring greater care on the part of the operator.

The final verification and amendment of the results of the aerial triangulation thus will be carried out during the plotting. The reaction of A6 operators toward the data obtained by aerial triangulation was very interesting: most of them preferred these points to control points determined in the field, for the following reasons.

1. Identification of aerial triangulation points usually is easier and more positive than that of control points obtained by field surveying.
2. The symmetrical distribution of the aerial triangulation points simplifies and accelerates absolute orientation.
3. The accuracy of absolute orientation was more consistent than that of the field data, which were not completely dependable. This applies in areas where the elevation of control points is not known exactly, a situation which is apt to be encountered frequently in actual practice. Furthermore, the fact that *the same original photographs* are used in both plotting instruments favorably affects precision as well as the smoothness of the work. There are many

fitting-in rules, to be followed with the A6, providing for a maximal elevation deviation of 0.2 to 0.3 m. (8 to 10″) for the six control points. (Photograph scales: 1/10,000; plotting scale 1/5,000.) In regard to the planimetry, complete agreement with the data also was found. (See Figure 5.)

MAPPING ON 1/2,500 SCALE

The above facts concerning the results obtained with aerial triangulation demonstrate to what extent aerial triangulation may safely be used at a 1/2,500 scale. The very high precision obtained in the planimetry unconditionally

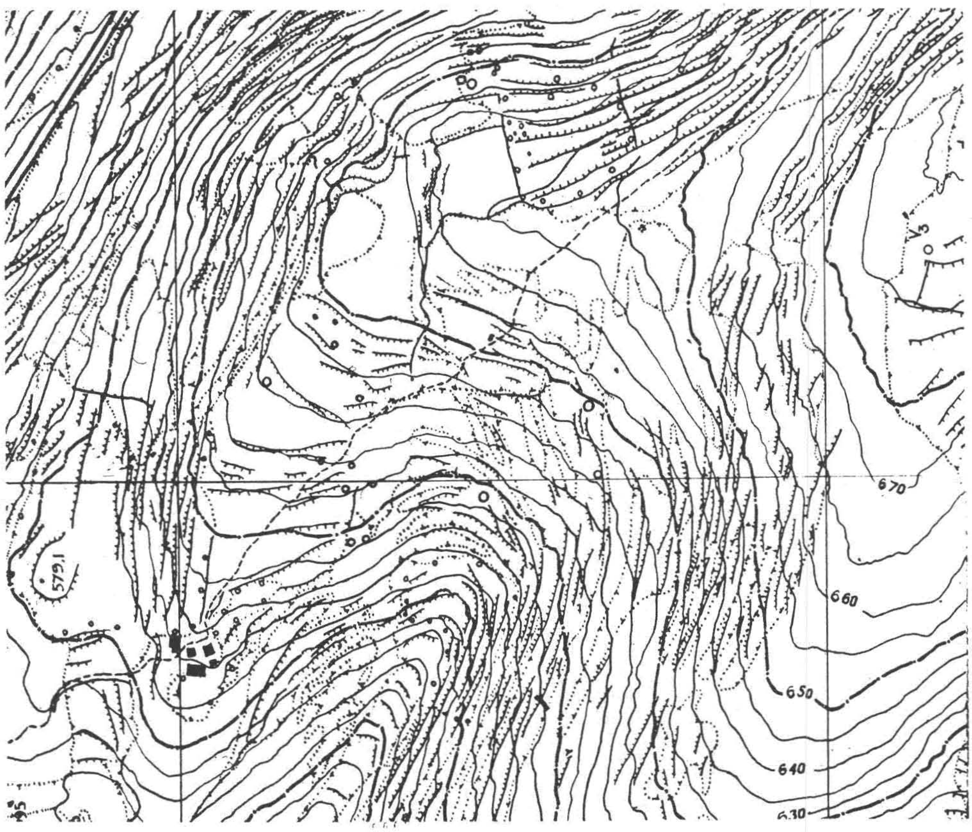

FIG. 5. Example of a Plotting on a Stereo-Plotter A6.

admits of first determining the X- and Y-components of the control points by triangulation, a procedure which eliminates the largest and costliest part of control point determination. The accuracy of elevation measurement in aerial triangulation also is adequate for the planimetry at 1/2,500 in hilly territory. In flatter country, and in cases where a higher accuracy in elevation is desired, the elevation of the aerial triangulation points may be determined by terrestrial methods, trigonometrical determination and aerial triangulation being then jointly employed to advantage as follows:

From the photographic copies, the control points are selected in the field, and the vertical angles of mutual lines of sight from these points measured.

The strips are then triangulated in the Autograph and the X- and Y-coordinates of the control points determined (see Figure 6). The elevation differences between control point trains are then computed.

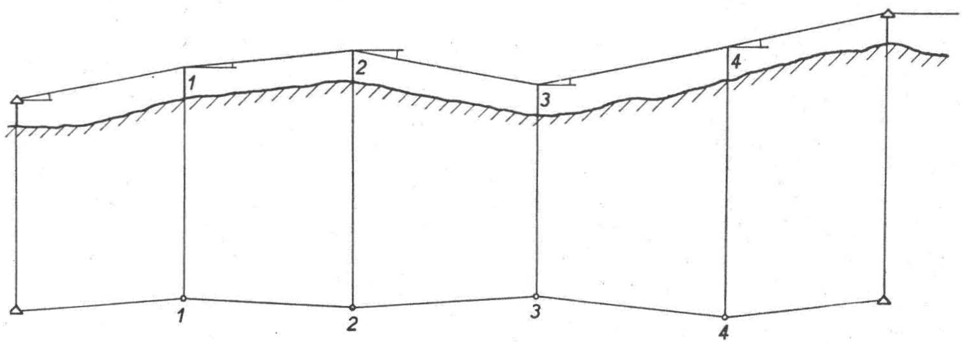

FIG. 6

Due to the limited accuracy of the planimetry, some additional errors in elevation, of a magnitude conditioned by the steepness of the line of sight, will become apparent during this process. The mean value of these errors can readily be computed from the data listed.

The mean error in the planimetry of a point, in our example, would be:

$$m_s = \sqrt{0.30^2 + 0.35^2} = \pm\, 0.46 \text{ m. } (18'')$$

Hence, the mean error in distance between two points (at any sightings and target distances) would be:

$$m_d = \sqrt{0.46^2 + 0.46^2} = \pm\, 0.65\ (25'')$$

This would give the following errors in elevation (depending upon the steepness of the line of sight) and pertinent differences in elevation H in the field, on the assumption that the distance between control points be about 700 m. (2,300 ft.) as in our example:

Distance 700 m.

	5g	10g	15g	20g
dh cm.	5.7 (2.2″)	11.5 (4½″)	16.9 (6½″)	23.6 (9.3″)
ΔH m.	61 (200 ft.)	124 (410 ft.)	188 (420 ft.)	255 (835 ft.)

As will be seen from above, the uncertainty of control point determination in the planimetry only slightly affects the elevation. On the other hand, the above error in elevation is independent of the target distance and is an accidental one, stemming from the errors in the planimetry.

MAPPING ON A LARGE SCALE (1/1,000)

The mean error in the planimetry of the aerial triangulation points, 0.4 m., exceeds the graphic accuracy at a scale of 1/1,000. Plotting at this scale therefore had to be based on control points directly surveyed in the field. In these cases, however, where maximal accuracy is not required, the reduction in the number of control points allowed by high level surveys may be counted on to give good results even at this scale. In our case, evaluation at the 1/1,000 scale was based on control points obtained by field surveying.

The large-scale mapping jobs carried out in different parts of Israel by stereophotogrammetric precision methods provide an interesting example, particularly as regards the methods used. The photographic material, which was put to many uses, was secured by scheduled flights conducted in the course of about two months. Flying costs were considerably reduced, not to mention the fact that, in urgent cases, it was possible to start the mapping work at different points without having to wait on the flying operation. This made it possible to achieve a high degree of freedom in over-all planning.

Adjustment for the various plotting scales was accomplished by distributing the work between the Wild Model A5 Autograph and the Wild A6 Stereo Mapping Instrument, as well as by determining the control points by means of aerial triangulation on a scale of 1/5,000 and, in part, of 1/2,500. The allotment of the work to the A5 and A6 plotters used in the Institute of Photogrammetry at Jerusalem proved both practical and effective. With trained personnel, 8 to 10 pairs are triangulated daily (in two six-hour shifts) with one instrument, a figure which constitutes a normal output for territory of average difficulty. This is entirely sufficient for the purpose of obtaining plotting data for four Model A6 Stereo Mapping Instruments, even in those cases where scant planimetry and topography result in very rapid evaluation of detail.

In regard to plotting output, it must be considered that our data refer to the fourth month from the setting up of the equipment. Up to that time, none of the operators had had an opportunity of working with any precision plotting instrument. In spite of this, one square kilometer (0.4 square mile) of markedly hilly country was evaluated on an average in 25 hours, on a scale of 1/5,000 (including the work of preparation and fitting in). The more skillful operators, when working in easier territory, reached an output of up to 1 sq. km. in 8 to 10 hours. Even though some of the operators first had no technical preparation for this work, it should be pointed out that the morale of the personnel was on an average very high. Without such ambition and good will, it would hardly have been possible to obtain such results in so short a time.

AERIAL TRIANGULATION WITH THE STEREOPLANIGRAPH*

Robert S. Brandt, Army Map Service, Washington, D. C.

I SHALL attempt to outline briefly the objectives of aerial triangulation and our use of the Stereoplanigraph in pursuit of these objectives. Moreover, some of the methods of triangulation analysis and adjustment will be mentioned in conjunction with their use on one sample project.

Like other sciences today, the science of Photogrammetry is constantly changing. Aerial-triangulation is no exception to this general observation. In fact, it may be the leading source of change in the general application of Photogrammetry to the problem of making maps. Within the brief period allotted, it is difficult to recapitulate the various techniques employed in the past, and to establish the basis for their use in theory. Let it suffice to say, that our present triangulation methods have evolved from an accumulation of theoretical and empirical data over a period of almost twenty years and from a variety of sources both foreign and domestic, and it can be said that these methods will change insofar as additional knowledge of the process warrants.

* This paper was presented in the form of a fifteen minute talk at the annual spring meeting of the American Society of Photogrammetry on May 11, 1950.

NOTE: Comments on this paper are invited. To ensure consideration for publication in the December issue, receipt before October 15 is necessary.