particularly as regards the methods used. The photographic material, which was put to many uses, was secured by scheduled flights conducted in the course of about two months. Flying costs were considerably reduced, not to mention the fact that, in urgent cases, it was possible to start the mapping work at different points without having to wait on the flying operation. This made it possible to achieve a high degree of freedom in over-all planning.

Adjustment for the various plotting scales was accomplished by distributing the work between the Wild Model A5 Autograph and the Wild A6 Stereo Mapping Instrument, as well as by determining the control points by means of aerial triangulation on a scale of 1/5,000 and, in part, of 1/2,500. The allotment of the work to the A5 and A6 plotters used in the Institute of Photogrammetry at Jerusalem proved both practical and effective. With trained personnel, 8 to 10 pairs are triangulated daily (in two six-hour shifts) with one instrument, a figure which constitutes a normal output for territory of average difficulty. This is entirely sufficient for the purpose of obtaining plotting data for four Model A6 Stereo Mapping Instruments, even in those cases where scant planimetry and topography result in very rapid evaluation of detail.

In regard to plotting output, it must be considered that our data refer to the fourth month from the setting up of the equipment. Up to that time, none of the operators had had an opportunity of working with any precision plotting instrument. In spite of this, one square kilometer (0.4 square mile) of markedly hilly country was evaluated on an average in 25 hours, on a scale of 1/5,000 (including the work of preparation and fitting in). The more skillful operators, when working in easier territory, reached an output of up to 1 sq. km. in 8 to 10 hours. Even though some of the operators first had no technical preparation for this work, it should be pointed out that the morale of the personnel was on an average very high. Without such ambition and good will, it would hardly have been possible to obtain such results in so short a time.

AERIAL TRIANGULATION WITH THE STEREOPLANIGRAPH*

Robert S. Brandt, Army Map Service, Washington, D. C.

I SHALL attempt to outline briefly the objectives of aerial triangulation and our use of the Stereoplanigraph in pursuit of these objectives. Moreover, some of the methods of triangulation analysis and adjustment will be mentioned in conjunction with their use on one sample project.

Like other sciences today, the science of Photogrammetry is constantly changing. Aerial-triangulation is no exception to this general observation. In fact, it may be the leading source of change in the general application of Photogrammetry to the problem of making maps. Within the brief period allotted, it is difficult to recapitulate the various techniques employed in the past, and to establish the basis for their use in theory. Let it suffice to say, that our present triangulation methods have evolved from an accumulation of theoretical and empirical data over a period of almost twenty years and from a variety of sources both foreign and domestic, and it can be said that these methods will change insofar as additional knowledge of the process warrants.

* This paper was presented in the form of a fifteen minute talk at the annual spring meeting of the American Society of Photogrammetry on May 11, 1950.

NOTE: Comments on this paper are invited. To ensure consideration for publication in the December issue, receipt before October 15 is necessary.

OBJECTIVES OF AERIAL TRIANGULATION

In the year 1950, I believe the principal objective of aerial triangulation is to make possible the compilation of maps in areas where the density of photo-identified field control is low and where additional field work is either expensive or impossible. For example, at the Army Map Service we are employing aerial triangulation to establish a network of photogrammetric control for purposes of controlling individual stereo-pairs in map compilation, or similarly we are attempting to reduce the density of field control needed to practical limits. These are honorable goals because almost everyone would agree that maps are desirable, and that saving money is likewise desirable.

The crucial point, however, is whether or not the use of aerial triangulation will satisfy the accuracy and economy requirements for the type of mapping needed. At the Army Map Service, we are convinced that various degrees of judicious use of aerial triangulation will satisfy most requirements in terms of practical values.

USE OF THE STEREOPLANIGRAPH

In our triangulation work we use the Multiplex and the Stereoplanigraph. For most rigorous requirements, the Stereoplanigraph is used. Use of the Stereoplanigraph in pursuit of these aforementioned objectives provides a very precise means of establishing the necessary geometrical relationships.

Our interior orientation is precisely made using plotting cameras which are duplicates of the aerial camera. This serves to re-establish the relationship between the light rays which entered the aerial camera and their homologous counterparts which emerge from the plotting camera. With similar precision we are able to attain relative and absolute orientation. All measurements made with the instrument are recorded to the hundredths of millimeters. The accuracy of these measurements requires a rather involved analysis; I mention the hundredths of millimeters to give some idea of the precision available.

By means of the instrument then, we are attempting to establish the true relationship between the image points on our photographs and those points on the earth's surface referred to as field control. In single stereo models these relationships are normally easy to establish and hence offer no problem. When the disposition of field control points is such that more than one stereopair is needed to establish the proper relationships, we have what we call a stereo-bridge or we have aerial-triangulation.

From experience, we know that it is practically impossible to have a stereo-bridge with zero closures. Therefore, no attempt is made to attain the impossible. Our main objective using stereo methods, is to build up a systematic accumulation of errors in the machine triangulation. To achieve this end, systematic methods of orientation, measuring, and plotting are used. In doing this we are making essentially three assumptions.

(1) That accidental error or random operator error will be small in magnitude.
(2) That the accumulated errors from aerial camera to plotting table will have geometrical relationships of a systematic nature that consistently repeat, and
(3) That by careful analysis of the errors in a bridge it is possible to correct them in a systematic manner.

Our methods for establishing the vertical and horizontal bridge are exactly the same, and usually the horizontal and vertical instrument work is done in a single run. The length of these bridges in terms of number of models depends usually upon the disposition of control, and may vary considerably also from the various flight heights. For each stereo bridge a starting model is set to a horizontal scale and to a vertical datum. Each succeeding model in the strip is likewise a very close approximation of the preceding model scale. At the end of a strip—the end being a point where more field control is available—the errors are recorded and the strip is ready for adjustment.

METHODS OF ANALYSIS AND ADJUSTMENT

Our methods of adjustment are similar for both horizontal and vertical errors; we use a form of graphical interpolation to determine the unknowns between our known values or between our control points (Exhibit I). This is a very practical method in that only a few computations need be made and then the *X*, *Y*, and *Z* ordinates of the photogrammetric control can be read directly from a correction line drawn on graph paper. These correction lines or curves that we are using are usually curves of empirical radii. The basic approach is to use the simplest type of equation from which a reasonably good fit can be obtained.

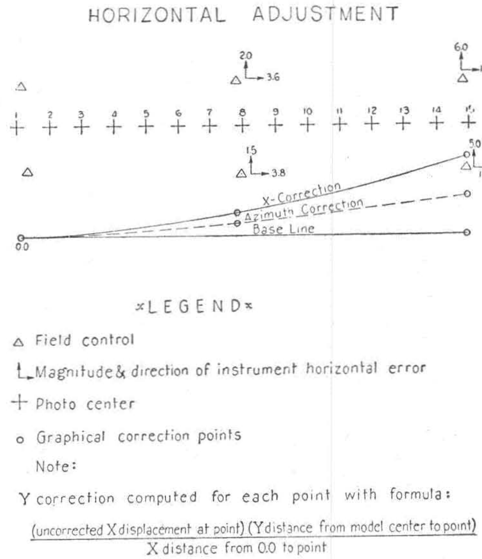

EXHIBIT 1. Shows the quadratic horizontal adjustment by means of graphical curves.

To get a better understanding of the methods, let us separate the *X*, *Y*, and *Z* corrections and consider the *X* adjustment first. Since we have an *X* closing error, we do *not* assume that the scale of the first model was wrong. Rather, if the *X* error is plus, we assume that the scale of each model in the strip is consistently slightly larger than its predecessor. Conversely, if our *X* error is minus, we assume that each model in the strip is consistently slightly smaller scale than its predecessor. This assumption leads to our conclusion that the total *X* closing error is directly proportional to the square of the number of different scales in the bridge. Consequently, our *X* error correction is not linear but is quadratic.

The *Y* closing error adjustment is a combination of two computations: one for scale or size of the model, and the other for azimuth. The *Y* scale correction is linear, and is directly proportional to the differential change of the model centers in any given model as determined from the *X* correction. The *Y* azimuth error is almost always a curve because we do not assume that there was any swing in the first model. This curved azimuth is further substantiated by the fact that there is nearly always swing in the last model. Hence, the azimuth is also corrected on a quadratic curve. It may be a little more enlightening to point out also that due to the curved azimuth, two *X* corrections are necessary, one for the top of a strip and one for the bottom so that the models move radially along with their *X* and *Y* shift.

The vertical or *Z* errors are similarly corrected on curves. The shapes of these curves are determined by bridging across more than one line of field control, and then interpolating the unknowns graphically. We are continually investigating other methods for adjusting errors in vertical datum, but none of the experimental techniques other than the *Bz* correction have been applied to regular production jobs. Among these so-called experimental techniques are the altimeter adjusted *Bz* correction, the straight line earth curvature correction, and lastly a correction combining earth curvature with another curve.

REVIEW OF CAMP GRAYLING, MICHIGAN PROJECT

Briefly, for this sample project, the area mapped consists of 16, 7½ minute quadrangles. The compilation planning was for multiplex and the desired contour interval 10 feet. Based upon the methods to be used, the flight height was 6,200 feet. Field control requirements were planned utilizing the stereoplanigraph for bridging between bands of horizontal and vertical control. The extension of horizontal and vertical control was performed simultaneously in the Stereoplanigraph at the scale of 1:5,000. The lengths of the horizontal bridges of the distance between lines of horizontal control averaged 9½ miles or 13 stereo pairs. The lengths of the vertical bridges averaged three (3) miles or four stereo models.

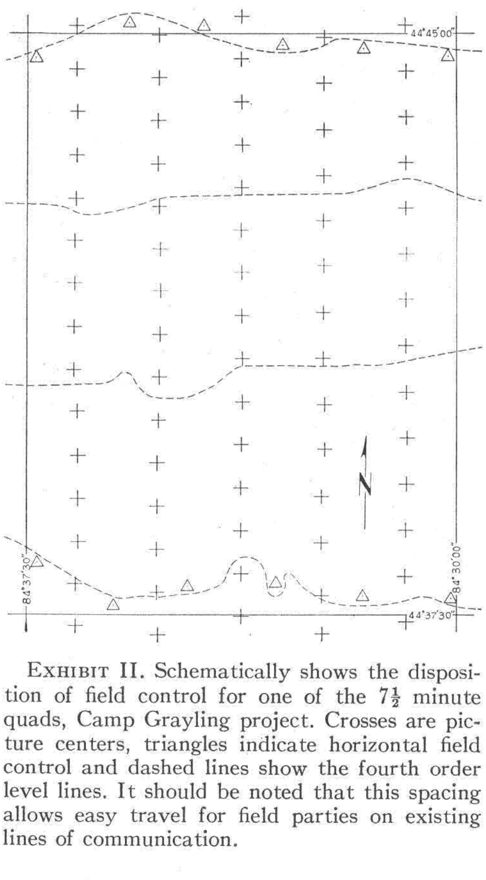

EXHIBIT II. Schematically shows the disposition of field control for one of the 7½ minute quads, Camp Grayling project. Crosses are picture centers, triangles indicate horizontal field control and dashed lines show the fourth order level lines. It should be noted that this spacing allows easy travel for field parties on existing lines of communication.

All but one of the horizontal field control points were used as a basis for establishing photogrammetric control. After applying corrections, no difference exceeding 20 feet was found between horizontal photogrammetric control points common to adjoining strips. After compilation, it is expected that all the surrounding map features will fall within the allowable tolerance of 42 feet.

The vertical analysis and adjustment was made for each strip at distances of about 9.5 miles. The spacing of vertical lines of field control was about three (3) miles so that each bridge was supported in the middle. With this type of adjustment, the photogrammetric vertical control common to adjoining strips showed no differences greater than ± 2 feet.

SUMMARY

Although we do not know what the final check of this project will reveal regarding absolute accuracy, we feel confident that it will check well within the domestic standards for map accuracy (Exhibit II). Summarizing, it can be said that the horizontal field control was used at 7½ minute quad spacing, and that the vertical field control density was reduced about 25% from what would have been needed with no aerial triangulation. From the experience gained on this project, the indications are that horizontal field control spacing may be pushed apart to 15 minute quad distances, and that vertical density needed will approach 50% of what would be needed, not using aerial triangulation.

From a qualitative standpoint one may consider the amount of effort expended to do a particular job. If this is an important factor then, one can see the advantages of placing lines of field control along roads or easily accessible routes. In the Camp Grayling Project the control disposition was planned in this manner. The advantages of being able to do this for the planner and for the geodetic field parties are self evident. These practical considerations being for all intensive purposes the main considerations in efficient map production, I believe that one can assert, that a modest degree of progress has been made toward more efficient production of maps.

AFFINE TRANSFORMATIONS APPLIED TO THE MULTIPLEX AERO PROJECTOR

*William A. Allen**

IN HIS last paper[1] the author derived certain closed expressions specifying the effects in the multiplex model due to errors of relative orientation. These results can be easily generalized to include the effects of errors in interior orientation, and that is the purpose of this paper.

ERRORS OF INTERIOR ORIENTATION INTERPRETED AS AFFINE TRANSFORMATIONS

Dr. Wang has called attention to the role of affine transformations in photogrammetry.[2] As might be inferred from his paper, every error of interior orientation can be investigated by the use of these transformations. As applied to the multiplex, these errors consist of three types:

1. Improper centering of a diapositive.
2. An error in the principal distance of a projector
3. A diapositive lying in a plane not perpendicular to the optical axis of a projector.

These three types of errors can also result from maladjustments in the aerial camera and/or the reduction printer. Moreover, as Dr. Wang suggests, errors of type 2 are sometimes tolerated for the advantage of utilizing long focal-length photographs.

We consider a projector whose perspective center is at the point (a^i), $i=1, 2, 3$. We select the system so that the x^3-axis is parallel to the optical axis of the projector. The vector equation of a ray from this projector can be represented by the relation

$$\boldsymbol{r}_2 = \mu(a^i - x_1{}^i)\boldsymbol{e}_i, \tag{1}$$

where the repeated indices imply summation. The vectors, $\mathbf{e}^i$, represent a unitary orthogonal basis; the point, $(x_1{}^i)$, is an arbitrary image point in the model; μ is an arbitrary scalar. In this case, the effect of an error of type 1 can be

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[1] "Analysis of the Multiplex Model," William A. Allen, PHOTOGRAMMETRIC ENGINEERING, June, 1946.

[2] "Affine Transformation in Stereophotogrammetry," Dr. Chih-Cho Wang, PHOTOGRAMMETRIC ENGINEERING, June 1947.