A FURTHER NOTE CONCERNING SHADOWS ON AERIAL PHOTOGRAPHS*

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MR. BELTMAN'S excellent and provocative article serves to clarify the geometric relationships between objects and their cast shadows, and to point out the effect of these relationships upon the pictorial image. Three additional points may be made briefly.

First, Mr. Beltman quotes a formula from my book and shows that it may only be applied when the direction of the shadow is measured at the principal point or nadir. He is entirely correct when the direction of the shadow is defined by a line drawn from the object to its shadow, as is the case when cloud shadows are measured. If, however, the shadow is cast by a linear vertical object, such as a flagpole, look-out tower, building, or tree, then the direction of the shadow can be determined anywhere on the photograph. Referring to his Figure 1, if the objects at C' and C'' were flagpoles rather than clouds, their linear shadows on flat ground would be parallel, and the direction of the shadow could be measured at either point. In this sense, of course, the shadow direction is determined solely from the shadow itself and not from the relationship between the object and its cast shadow.

Second, Mr. Beltman's interesting observations suggest the possibility of a valuable short-cut when the height of an object is to be found from the length of the shadow. On aerial photographs where point P (which we may call the *no-shadow* point) is registered, the tangent of the angle of elevation of the sun can be found directly. This tangent is found by dividing the elevation of the camera above the ground at the no-shadow point by the distance from the no-shadow point (P) to the nadir. In the special case of vertical photographs taken of level terrain, the tangent is found by dividing the elevation of the camera above the ground (H) by the distance from the no-shadow point to the principal point. From Mr. Beltman's Figure 2, the height of the aircraft is 4,600 meters. The distance from the no-shadow point to the principal point on the contact print is 2.7 cm. At the scale of 1:25,000, this amounts to a ground distance of 675 meters. The tangent of the sun's elevation, therefore, is 4,600/675 or 6.81. This tangent may be used directly in shadow-height determinations. By averaging tangents obtained in such a way from two or three successive exposures, the effect of normal tilt should be averaged out and a reasonably accurate shadow conversion factor should be obtained. If the angle of the sun's elevation is desired, it may be read from natural trigonometric tables. The angle for Figure 2 is 81 degrees and 39 minutes.

Third, the relationship between object displacement and shadow displacement may be used to determine the height of objects. The requirements are: (1) that the no-shadow point (P) can be located either on or off the photograph either by its characteristic light spot or by the intersection of lines drawn through object images and their respective shadow images; (2) that the top of the object may be seen; and (3) that the shadow cast by the top of the object may be seen.

Referring to Mr. Beltman's Figure 1, let the nadir (i.e. the point directly beneath O) be designated as N and the map position of the cloud (i.e. the point directly beneath C') be designated as R. Since the planes defined by triangles

ONP and *C'RS'* are parallel (the sun's rays being considered parallel), the two tetrahedrons constructed from each of these triangles and point *T'* are similar. Therefore, from the horizontal similar triangles:

$$\frac{S'T'}{PT'} = \frac{RT'}{NT'}.$$

While from the vertical similar triangles:

$$\frac{ON}{NT'} = \frac{C'R}{RT'}$$

Equating these two relationships, we obtain:

$$C'R = \frac{ON \cdot S'T'}{PT'}.$$

In other words, the height of an object is the product of the height of the camera above the ground and the distance from the object-image to the shadow-image divided by the distance from the object-image to the no-shadow point:

$$h = H \cdot \frac{\text{Dist. object-image to shadow-image}}{\text{Dist. object-image to no-shadow point}}.$$

Since the two distances measurable on the aerial photograph form a ratio, they may be measured on the photographs in any units of measure desired, as long as the same unit is used for both. The object-height will then be found in terms of the same unit as that used for the camera height.

Referring to Figure 2 again, the height of the cloud to the right of point *B* was found by measuring the distance from the cloud-image to the shadow-image on the contact print (2.10 cm.) and the distance from the cloud-image to the no-shadow point (8.45 cm.). The height of the cloud above the ground is, therefore:

$$h = 4,600 \cdot \frac{2.10}{8.45} = 1,143 \text{ meters.}$$

Measurements of two other points on the same cloud yielded 1,132 and 1,192 meters respectively.

This simple formula may be used wherever the object and its shadow are visible. Strictly, it yields the elevation between the top of the object and the point on the ground where its shadow appears. It has application in all phases of photo-interpretation, not only for the determination of cloud heights, but also for that of tree heights and of other lesser objects. Conversely, if the actual height of an object is known, the elevation of the aircraft can be determined from this relationship.