SUPPLEMENTARY HEIGHT-CONTROL*

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Introduction

THE aim of photogrammetry has been to reduce field work to a minimum. At the present stage of development of this science however, the proper number and distribution of points with known heights have been problems for purposes of contouring. In the case of perfectly vertical pairs of photos unaffected by distortions, the normal parallax method is found to provide quite satisfactory supplementary height-control. But owing to the presence of tilts in normal photography, the simple parallax formula viz., $H \cdot dp/b$ leads to very dubious results. The most general method viz., the inverse transformation in h from the equations

$$\xi_1 = \frac{\alpha_1 e + \alpha_2 n + \alpha_3 h + \alpha_4}{\theta_1 e + \theta_2 n + \theta_3 h + 1}, \qquad \xi_2 = \frac{\beta_1 e + \cdots}{\theta_1 e + \cdots}, \qquad \xi_3 = \frac{\gamma_1 e + \cdots}{\psi_1 e + \cdots}$$

is also laborious and requires six ground control points unless some assumptions or other are made.

In this paper two simple methods for providing supplementary height-control will be indicated. They require only four or five points per overlap with known heights.

The Four Point Method

In the most general case the equations of transformation are "Bi-Projective" viz.,

$$\xi_1 = \frac{\alpha_1 e + \alpha_2 n + \alpha_3 h + \alpha_4}{\theta_1 e + \theta_2 n + \theta_3 h + 1}, \qquad \xi_2 = \frac{\beta_1 e + \cdots}{\theta_1 e + \cdots}, \qquad \xi_3 = \frac{\gamma_1 e + \cdots}{\psi_1 e + \cdots}.$$

If the photo planes are supposed to be coplanar, then the equations become Projective of the form,

$$\rho\xi_1 = \alpha_1 e + \alpha_2 n + \alpha_3 h + \alpha_4, \quad \rho\xi_2 = \beta_1 e + \cdots, \quad \rho\xi_3 = \gamma_1 e + \cdots, \quad \rho = \theta_1 e + \cdots$$

From the inverse transformations e, n, h can be derived in this case from five control points. If however, only the heights of four points are supposed to be known, then an affine transformation may be assumed to subsist between the photo-coordinates and the heights of points viz.,

$$h = a_1\xi_1 + a_2\xi_2 + a_3\xi_3 + a_4.$$

EXAMPLE

Point No.	Photo-Coordinates			
	ξ_1	ξ_2	ξ_3	h
G 366	9.91	52.32	10.00	5,761
C 14	39.34	65.48	40.65	5,482
G 6	77.60	−14.73	84.38	4,422
S 18	36.27	−43.51	39.11	5,203
G 10	50.93	63.63	53.90	5,142
G 15	15.54	38.25	14.23	5,994
S 30	13.08	37.08	11.50	6,030
G 11	76.45	33.12	80.94	4,822

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Assuming the first four points to be control points and substituting their photo-coordinates in the affine transformation

$$h = a_1\xi_1 + a_2\xi_2 + a_3\xi_3 + a_4$$

and solving we obtain,

$$a_1 = 173.09188, \quad a_2 = 0.12974, \quad a_3 = -175.38147, \quad a_4 = 5{,}791.810.$$

Hence the heights of the other four points may be derived. Following is a table of values for comparison:—

Point No.	Computed height	True height	Error
G 366	5,760	5,761	−1
C 14	5,480	5,482	−2
G 6	4,423	4,422	1
S 18	5,205	5,203	2
G 10	5,163	5,142	21
G 15	5,991	5,994	−3
S3 30	6,044	6,030	14
G 11	4,834	4,822	12

The Five Point Method

If five known heights are supposed to be present in an overlap, then a slightly more general form of transformation is found to give good results. The equation is

$$h = a_1\xi_1 + a_2\xi_2 + a_3\xi_3 + a_4\xi_4 + a_5.$$

EXAMPLE

Point No.	Photo-Coordinates				
	ξ_1	ξ_2	ξ_3	ξ_4	h
A 12	.390	2.060	.397	2.025	5,761
C 10	1.549	2.578	1.597	2.549	5,482
S 5	3.055	− .580	3.322	− .573	4,422
B 251	1.428	−1.713	1.545	−1.700	5,203
C 25	1.765	1.090	1.885	1.082	5,112
C 26	2.005	2.505	2.126	2.472	5,142
A1 15	.612	1.506	.560	1.485	5,994
S 12	.515	1.460	.456	1.440	6,030
C 13	2.078	< .582	2.239	.580	4,936

Solving the equation with the help of the first five points we obtain

$$a_1 = 4{,}882.68055, \quad a_2 = 6{,}070.69022, \quad a_3 = -4{,}884.19443,$$

$$a_4 = -6{,}143.76099, \quad a_5 = 5{,}731.33213.$$

The comparative table of check points is as follows:—

Point No.	Computed height	True height	Error
C 26	5,157	5,142	15
A 15	6,003	5,994	9
S 12	6,039	6,030	9
C 13	4,911	4,936	−25