attributable to the operators." American experience does not confirm this statement since errors of stereoscopically measured spot heights usually are less than the errors of points whose elevations are determined by interpolation between contours. Even in the clear areas of the model non-uniformity of ground slope between contours weakens the interpolated values. Also, the contours vary greatly in strength throughout the model because of the variations in the appearance of the terrain which may range from light to dark in tone and from clear to heavily wooded in vegetation characteristics. Recognition of these conditions has led to the statement that an appreciable disparity exists between the *C* factor indicated by the mean square error of spot heights read in the stereoplotter as opposed to that indicated by the error of interpolated elevations based on stereoscopically drawn contours.

If a *C* factor comparison is to be made between stereoplotting instruments it should be based on the mean square error of spot height readings. Such a procedure will establish practical conditions which most nearly approach the ideal conditions of the grid model.

ROBERT E. ALTENHOFEN

METHODS OF FIELD CAMERA CALIBRATION*

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IV. THE STAR EXPOSURE METHOD OF CAMERA CALIBRATION

A. GENERAL

The purpose of this section is to present a method of calibrating cameras from zenithal exposures of stars. The method consists of making exposures of the zenith and calculating the camera constant (f, Δx, Δy) and residual lens distortion, with the plate and celestial coordinates of the stars imaged as given data. The concept of a negative exposure of the zenith for calibrating purposes is illustrated in Figure 16. Reasonably accurate camera calibration data can be calculated because of the multiplicity of star images, and the relatively high accuracy of the star's celestial coordinates.

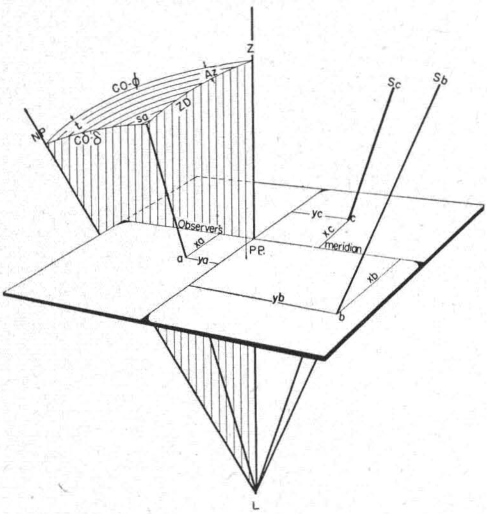

FIG. 16. Idealized camera orientation.

In the star exposure method, there are certain operations, such as identification of the stars imaged, reduction of the stars from the mean epoch of 1950 to their apparent places at the instant of exposure, and com-

* This is the completion of Mr. Merritt's paper. Parts I to III are in the September 1951 issue (Volume XVII, No. 4, pp. 610–635). The author is now a member of the staff of Photogrammetry, Inc., Silver Spring, Md.—*Publications Committee.*

† This was the author's title and occupation when this paper was prepared.

putation of coordinate corrections to the images for refraction, that do not occur in any other method.

One of the problems arising with the star exposure method is the photographic registering of the fiducial marks. Admission of sufficient light to register the fiducial marks photographically, without harmful deterioration of the star images, requires the exercise of care in the choice of aperture, shutter speed, and intensity of the light source. A mechanical device and a number of expedient means for accomplishing this are described.

Inasmuch as the celestial coordinates of identifiable stars have been predetermined, angle observations by the observer are conveniently eliminated. The more tedious general space calibration equations are necessitated, however, by the difficulty, and hence impracticability, of imposing rigorous orientation on the camera and the random distribution of the star images.

The lack of symmetry in the star images also gives rise to the necessity of discarding all images that do not exhibit a collective symmetry with respect to the fiducial axes, in the computation of the principal distance and coordinates of the plate perpendicular. On the other hand, all well defined images are used in the computation of lens distortion. Employment of star images not balanced about the approximate plate perpendicular results in large errors in the location of the plate perpendicular and consequently asymmetrical distortion curves.

The maximum resolution is necessary in exposures made expressly for the determination of a camera's interior geometry and linear lens distortion. Yet the small amount of light reaching the earth from stars necessitates the use of fast super-sensitive emulsions which results in an unavoidable reduction in the definition of the star images.

The remainder of this section covers the calibration procedure in detail, description of the calibration of a camera by the star exposure method, some discussion, and the numerical data supporting the calibration of a camera by the star exposure method.

B. CALIBRATION PROCEDURE

1. *Observation Procedure*

(a) Equipment.

- (1) Fiducial illumination assembly (optional).
  - (a) 12 dry-cell batteries.
  - (b) 4 frosted 6.2 volt bulbs.
- (2) Ground glass plate (optional).
- (3) Watch.
- (4) Barometer.
- (5) Thermometer.
- (6) Comparator.
- (7) Star charts.
- (8) American Ephemeris.
- (9) Boss Star Catalogs.
- (10) Calculating machine.
- (11) Natural trigonometric function tables (eight places).
- (12) Tablets, pencils, and computation forms.

(b) Choice of photographic materials.

It is commonly known that the resolution of emulsions decreases as its speed increases. Thus an emulsion is required that has less than the optimum resolution characteristics, in order that the pin-point star images will register as positive visible trails.

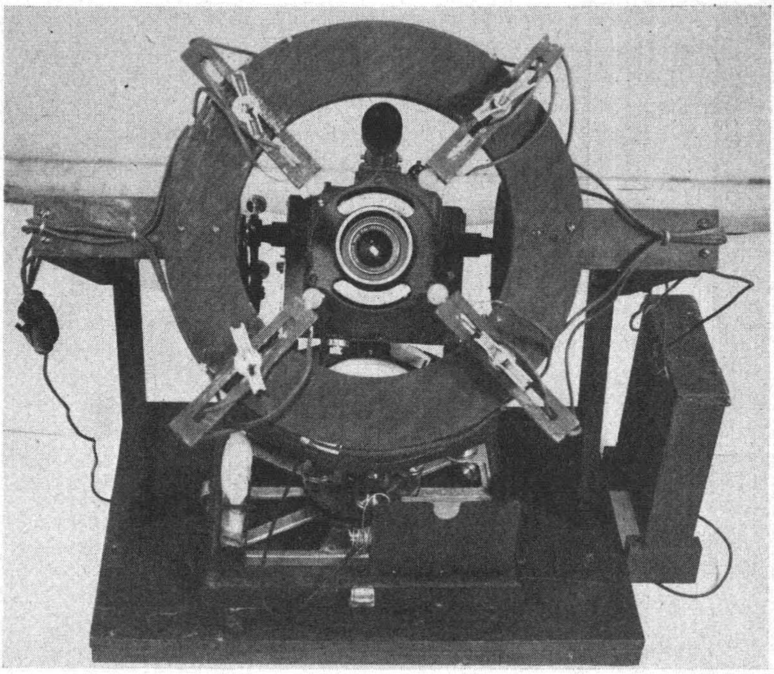

FIG. 17. Fiducial illumination assembly.

The slowest emulsion that will image the stars as positive trails is selected. Glass-based emulsions are preferred to acetate-based emulsions. At present the movements of the image caused by emulsion creep are unavoidable. Needless to say, these types of movements exist in any photographic method of camera calibration.

The following plates have given satisfactory results: Kodak Super XX panchromatic plates; Kodak Tri X panchromatic type B plates; Kodak Super ortho-press plates.

(c) Photographic register of the fiducial marks.

Light reaching the exposure plate from stars is insufficient to photographically register the fiducial marks. Light other than starlight must be employed to register the fiducial marks.

The problem is either to permit sufficient light other than starlight to reach the exposure plate to register the fiducial marks without overexposing the star images, or to localize the entering light so that only the immediate area of the fiducial marks is exposed during the fiducial register. The latter method of photographic fiducial register is done with a fiducial illumination assembly.

The fiducial illumination assembly shown in Figure 17 consists of a flat wooden circular frame on which are mounted four frosted bulbs, which may be moved in two directions in the plane of the ring, for trial and error illumination of the fiducial marks. The camera is mounted on a suitable base, lens up, with a focusing plate clamped against the fiducial marks. The fiducial illumination assembly is oriented over the camera, so that the camera optical axis is approximately perpendicular to the plane of the ring and collinear with the axis of the ring. With bulbs lighted, the camera is shifted horizontally and vertically below the assembly until only the fiducial marks and immediate area are illuminated. After the fiducial marks are illuminated, the camera and fiducial assembly are locked in place, and the focusing plate replaced with a loaded plate holder. With the fiducial bulbs illuminated, the dark slide is removed and a fiducial exposure made, followed immediately by the star time exposure.

The technique of registering the fiducial marks described was suggested by Dr. *F. E. Washer* of the U. S. National Bureau of Standards.

The star exposure method of camera calibration fulfills the definition of a field method in that the observation can be completed in a few minutes without auxiliary equipment. In conformity with this, three expedient techniques that have given satisfactory results are suggested for registering the fiducial marks photographically.

(1) Zenithal star exposures made in moonlight. Light reflected by the moon when half or more of its disc is illuminated is sufficient to register the fiducial marks, without deterioration of the star images in a 3-minute star time exposure.

(2) A pre-exposure made of a plain piece of white cardboard somewhat larger than the image plane format, and held a distance equal to the approximate focal length in front of the camera, yields a uniform illumination sufficiently subdued to register the fiducial marks without destruction of the star image. The plate is illuminated by a light from a dry-cell hand lamp reflected from the cardboard. The light source is adjusted until the card is uniformly illuminated without bright spots or shadows.

(3) A pre-exposure made of a dry-cell hand lamp subdued by transmission through a yellow filter covered with two layers of onion-skin typing paper yields uniform illumination sufficiently subdued to register the fiducial marks without overexposing the emulsion.

Experience showed that pre-fiducial exposures where exposing the fiducial marks is separate from the star exposure gives more satisfactory results than post-fiducial exposures. This is due to the fact that the pre-fiducial exposure pre-sensitizes the emulsion, causing the emulsion to respond more readily to the small amount of starlight reaching the plate.

The shutter speed and aperture for fiducial exposures vary with the lens speed, focal length, and film speed. The proper combination can be determined precisely by a few test exposures.

(d) The star exposure.

A time exposure is made with the camera optical axis approximately defining a plumb line and the camera *y* fiducial axis approximately in the observer's geographic meridian.

The length of the star exposure will vary according to the focal length of the camera; the shorter the focal length, the longer the exposure. The objective is to obtain a positive star trail so that the star image is not confused with the photographic register of foreign particles on the surface of the emulsion or with clamps of silver. Thus the length of the exposure serves only to insure positive identification of the stars. The time varies from 1 to 5 minutes for conventional focal lengths.

The date, the exposure number, the time of the exposure to the nearest minute, the approximate geographic position, the *f* number, the length of the exposure, the type of plate, and the temperature are recorded. Neither time nor geographic position is absolutely necessary. These factors are used to more expeditiously locate the area of the celestial sphere imaged on the negative.

The exposed plates are developed in accordance with the instructions accompanying each box of plates.

(e) Selection of star images.

Stars are selected for optimum definition and for conformity to the geometric properties required to give a reliable location of the plate perpendicular.

Stars are selected for optimum definition only in connection with the lens distortion calculations.

The geometric properties are defined as follows: not less than two stars are employed on any given radial distance from the fiducial axes. Consecutive stars of any number of stars at a given radial distance from the fiducial axes are required to subtend equal angles $\pm 5°$ at the fiducial axes intersection.

The purpose of selecting only stars that fulfill these geometric properties for the calibration equations involving the determination of the coordinates of the plate perpendicular is to cancel out the deleterious effects of lens distortion. It is assumed that the lens distortions on any given radius are approximately equal in magnitude and direction, in which case the asymmetrical effects of lens distortion are cancelled out by selecting star images that conform to the geometric criteria outlined above. The lens distortion differentials in star images not symmetrical with respect to the fiducial axes, produce a tilt effect in the image plane, which in turn displaces the location of the plate perpendicular as a function of the induced tilt. The fiducial axes intersection is chosen as the reference origin on the grounds that the lens distortion differential is negligible for small eccentricities of the plate perpendicular with respect to the fiducial axes.

The general solution is used in this method since the star images do not define a mutually orthogonal system of lines and n images do not define a line where n is greater than 2, and since the optical axis is only approximately oriented on the observer's zenith. The general space calibration equations require an arbitrary origin of known relative angular components. This requires that a star image near the fiducial axes be selected in addition to those fulfilling the geometric properties outlined.

If the calibration is an unique determination, a minimum of four star images are selected consisting of an arbitrary origin and three equidistant from the fiducial axes ± 6 mm. and adjacent pairs subtending an angle of $120° \pm 5°$.

(f) Star identification.

The watch time of the exposure is converted to local sidereal time.

The local sidereal time is the right ascension of the observer at the instant of exposure. The approximate latitude and right ascension of the observer are plotted on a suitable star chart. The distance between the x or y fiducial marks (usually y) on the exposure negative is divided into the linear length of the camera cone angle along the x or y axis as scaled from the star chart.

Suppose, for example, the cone angle along the y axis is 20°, and suppose further that the observer's latitude is 39°. The distance along the meridian given by the right ascension of the observer from $+29°$ to 49° is measured with a scale. This scaled value divided by the linear separation of the y fiducial marks gives the ratio of reduction or enlargement necessary to obtain a one-to-one ratio between the star images and the plotted positions of the stars on the star chart.

Since the y axis of the camera was placed approximately in the observer's geographic meridian, the fiducial axes intersection superimposed over the position plotted and the y fiducial axis coinciding with the hour circle of the observer, make star identification of the images selected a simple matter of recording the catalog numbers of the stars on the chart coinciding with the star images directly above. The catalog numbers are recorded next to the numbers assigned the star images when they were selected.

(g) Coordinate measurement of star images.

If the star exposure consists of a single long trail, a dot of ink, or the identifying number, is placed on the same end of each star trail image. This is to prevent the observer from measuring the wrong end of the image following rotation of the plate. If the star exposure images have a long and short trail, the identifying number is placed anywhere near the image. The location of the short trail with respect to the long trail provides the key to which end is common to all positions. Generally, both ends are measured, providing two sets of data whereby the computations may be done in duplicate.

Calibration values obtained from two sets of data, differing only in the coordinates of opposite ends of the star trails, will agree within the probable error of the measurements. In fact, the difference in the calibration values from two sets is a measure of the precision of the coordinate measurements.

2. *Computation Procedure.*

There are three computations that precede the calibration computations: (1) correction of image coordinates for refraction; (2) apparent places of stars from the mean epoch of 1950 to the time of the exposure; and (3) standard coordinates of stars referred to an arbitrary star imaged near the center of the exposure plate.

(a) Refraction correction.

The star images are displaced radially toward the zenith point (same as the nadir point) by atmospheric refraction. This displacement increases as the zenith angle increases.

A small error in the zenith angle results in a negligible error in the refraction correction. For this reason the fiducial axes intersection may be used for the zenith point, inasmuch as the camera optical axis has been oriented approximately on the observer's zenith. For the same reason the approximate focal length may be used with the measured camera coordinates to compute approximate tangent functions of the zenith angles. These factors are useful in that the refraction correction, like the coordinate measurements, may proceed independently of the star identification and reduction to apparent places.

The mean value of the measured image coordinates are referred to the fiducial axes. The zenith angle for each image is computed using these coordinates and the approximate focal length as given.

$$\frac{x^2 + y^2}{f^2} = \tan^2 \gamma.$$

Then

$$dx_r = x(1 + \tan^2 \gamma) \cdot k_r \tag{1}$$

$$dy_r = y(1 + \tan^2 \gamma) \cdot k_r \tag{2}$$

where

$$k_r = \frac{983 \cdot b \cdot \tan 1''}{460 + t}$$

and b is the barometric pressure in inches

t temperature in Fahrenheit degrees.

Since atmospheric refraction moves the image toward the zenith point, the plate coordinates of refraction are always applied so as to numerically increase the absolute value of the camera coordinates. The correction is additive, omitting the sign of the coordinates.

$$x_c = x + dx_r, \qquad y_c = y + dy_r.$$

(b) Apparent places of stars at the time of exposure.

Two computations are involved in the reduction of the stars' celestial coordinates from the mean epoch of 1950 to the apparent place at the time of observation. The first computation is the reduction from the mean epoch of 1950 to the mean place of the beginning of the year of the observation. The second computation is the reduction from mean to apparent place for the time of observation.

(1) Computation of mean place for year of observation. Formulas:

$$\alpha_0 = \alpha m + (t_0 - 1950)AV_\alpha + 1/2(t_0 - 1950)^2 1/100SV_\alpha + (t_0 - 1950)^3 3dt_\alpha$$

$$\delta_0 = \delta m + (t_0 - 1950)AV_\delta + 1/2(t_0 - 1950)^2 1/100SV_\delta + (t_0 - 1950)^3 3dt_\delta$$

where

α_0 is mean right ascension for beginning of year,
α_{m1} catalog right ascension,
δ_0 mean declination for beginning of year,
δ_{m1} catalog declination,
t_0 year of observation,
AV_α, AV_δ annular variation of star in right ascension and declination,
SV_r, SV_s secular variation in right ascension and declination,
$3dt_\alpha$, $3dt_\delta$, third term in right ascension and declination.

Each star identified has a catalog number, and all constants used in the reduction to mean place may be found in the general catalog on the same line to the right of the number. For observations made in the year 1950 $\alpha_0 = \alpha_m$ and $\delta_0 = \delta_m$.

(2) Reduction from mean to apparent place.

Two solutions are generally given in textbooks on astronomy: one uses Besselian star numbers and Bessel's star constants, while the other uses independent star numbers. The solution employing independent star numbers is the more practical where a multitude of stars that are not repeated with each calibration are being reduced. Formulas:

$$\text{(time)} \quad \alpha = \alpha_0 + f + f' + \mu\tau + \frac{g\cdot\sin(G + \alpha_0)\tan\delta_0}{15} + \frac{h\cdot\sin(H + \alpha_0)\sec\delta_0}{15}$$

$$\text{(arc)} \quad \delta = \delta_0 + \mu'\tau + g\cdot\cos(G + \alpha_0) + h\cdot\cos(H + \alpha_0)\sin\delta_0 + i\cdot\cos\delta_0$$

where

$$g = \frac{B}{\sin G}, \qquad h = \frac{C}{\sin H}.$$

The quantities $f, f, \tau, B, C, G, H,$ and i are found in the tables for independent star numbers in the *American Ephemeris;* μ, and μ' are found in the *Boss Star Catalog.* The fractional part of a G.C.T. day is used to interpolate these values from the tables. Computation of the mean places for the year of the observation is done on form No. 1 and the reductions from the mean to apparent places are done on form No. 2. Only form No. 2 is shown with the numerical data supporting the example calibration.

(c) Standard coordinates.

The standard coordinates of the selected star images referred to an arbitrarily selected star image near the fiducial axes' intersection are computed with the general transformation equations described in section II. Here declination δ replaces altitude h, and the difference in right ascension converted to arc t corresponds to the horizontal angle ω. That is,

$$\tan \xi = \frac{\tan \delta - \tan \delta_0 \cos t}{\cos t + \tan \delta \tan \delta_0}$$

$$\sin \eta' = \cos \delta \sin t$$

$$\tan \eta = \frac{\tan \eta'}{\cos \xi}.$$

(d) Calibration equations.

Inasmuch as no rigorous orientation properties are imposed on the camera at the moment of exposure, the space calibration equations described in section II are employed in the computation of the camera constants ($\Delta x, \Delta y, f$). The plate coordinates of the images corrected for refraction and referred to the image origin, and the corresponding standard coordinates, are the given data. With these data for a minimum of three star images plus the image origin, the following equations previously described are formed and solved.

(1) General space calibration equations.

(a)

$$x_1(\tan s_x) + y_1(a) - \tan \theta_1 y_1(b) = \tan \theta_1 x_1$$

$$x_2(\tan s_x) + y_2(a) - \tan \theta_2 y_2(b) = \tan \theta_2 x_2$$

$$x_3(\tan s_x) + y_3(a) - \tan \theta_3 y_3(b) = \tan \theta_3 x_3$$

(b)

$$Q_1 = \frac{x_1 \tan s_x + y_1(a)}{\tan \eta_1(1 + \tan^2 s_x)^{1/2}}.$$

$$Q_2 = \frac{x_2 \tan s_x + y_2(a)}{\tan \eta_2(1 + \tan^2 s_x)^{1/2}}$$

$$Q_3 = \frac{x_3 \tan s_x + y_3(a)}{\tan \eta_3(1 + \tan^2 s_x)^{1/2}}.$$

(c)

$$u + x_1\mu + y_1\nu = Q_1$$

$$u + x_2\mu + y_2\nu = Q_2$$

$$u + x_3\mu + y_3\nu = Q_3$$

(d)

$$z_x = \frac{u}{1 + \mu^2}, \qquad z_y = \frac{u}{1 + \mu^2}(1 + \mu^2 - \nu^2)^{1/2}$$

$$\Delta x = \frac{u\mu}{1 + \mu^2}, \qquad \Delta y = \frac{u\nu}{1 + \mu^2}$$

$$f = (z_x^2 - \Delta y^2)^{1/2}, \qquad f = (z_y^2 - \Delta x^2)^{1/2}$$

$$z_0 = (z_x^2 + \Delta x^2)^{1/2}, \qquad z_0 = (z_y^2 + \Delta y^2)^{1/2}$$

$$\cos t = \frac{f}{z_0}.$$

Check on solution

$$\cos \alpha_x \cos \alpha_y + \cos \beta_x \cos \beta_y + \cos \gamma_x \cos \gamma_y = 0$$

where

$$\cos \alpha_x = \frac{\tan s_x z_x}{(1 + \tan^2 s_x)^{1/2} z_0}, \qquad \cos \alpha_y = \frac{\tan s_y z_y}{(1 + \tan^2 s_y)^{1/2} z_0}$$

$$\cos \beta_x = \frac{z_\alpha}{(1 + \tan^2 s_x)^{1/2} z_0}, \qquad \cos \beta_y = \frac{z_y}{(1 + \tan^2 s_y)^{1/2} z_0}$$

$$\cos \gamma_x = \frac{\Delta x}{z_0}, \qquad \cos \gamma_y = \frac{\Delta y}{z_0}$$

and

$$\tan s_x = \frac{(a)}{(b)}.$$

Lens distortion

$$dx_1 = \left(\frac{f}{f_1} - 1\right)(x_1 - \Delta x), \qquad dy_1 = \left(\frac{f}{f_1} - 1\right)(y_1 - \Delta y)$$

$$f_1 = x_1\left(\frac{\cos \alpha_x}{\tan \eta_1} - \cos \gamma_x\right) \cos t + y_1\left(\frac{\cos \alpha_y}{\tan \eta_1} - \cos \gamma_y\right) \cos$$

$$f_1 = x_1\left(\frac{\cos \beta_x}{\tan \xi_1} - \cos \gamma_x\right) \cos t + y_1\left(\frac{\cos \beta_y}{\tan \xi_1} - \cos \gamma_y\right) \cos t.$$

C. DESCRIPTION OF A CAMERA CALIBRATION BY THE STAR EXPOSURE METHOD

Thirty zenithal star exposures were made with a camera transit on Tri X type B plates, for the purpose of developing field procedures for photographic register of the fiducial marks, without appreciable deterioration of the star images, and for the purpose of evaluating the star exposure method of camera calibration.

Polaris was used as an approximate azimuth mark in orienting the camera y axis on the observer's meridian. Two of the thirty plates exposed were selected for calibration purposes. The fiducial marks were pre-exposed with a hand lamp light source subdued with a yellow filter and two layers of thin translucent paper on the two plates selected.

The east end of each star trail was made distinguishable in all positions by a dot of ink and an identifying number. The identifying number was later keyed to the star catalog numbers. Three readings were taken on both ends of the star trails in x and y in the direct and reverse positions of the plate, with a Mann measuring machine. The mean values of the plate coordinates were rounded off to the nearest micron. The apparent places and the subsequent standard coordinates of the stars were rounded off to the nearest second. An example of each distinct computation leading up to the calibration computations, a list of the mean plate coordinates and corresponding standard coordinates, the complete calibration computation, and the lens distortion computations are presented in the order listed in the numerical example.

Filter Technique 1

FORM NO 2

REDUCTION TO APPARENT PLACE WITH INDEPENDENT STAR NUMBERS

DATE 14 NOV 50 PROJECT Calibration COMPUTER Ostberg, T.E.

STAR NO.	334	918	1159	1394
MAG ~ ϵ_a ~ ϵ_δ	4.44 ~ .09 ~ .08	5.99 ~ .20 ~ .18	6.22 ~ .28 ~ .27	4.28 ~ .16 ~ .12
a_o (ARC)	3-37-04	10-51-02	13-52-21	16-38-50
G (ARC)	333-11-01			
H (ARC)	35-34-42			
$(G + a_o)$	336-48-05	344-02-02	347-03-22	349-49-51
$(H + a_o)$	39-11-46	46-25-44	49-27-03	52-13-32
τ	.8709			
B	-8.89			
C	+11.55			
i	+5.01			
sin G	-.451 13284			
sin H	+.581 81545			
g = B/sin G	+19.706			
h = C/sin H	+19.852			
$\sin (G + a_o)$	-.393 91963	-.275 06875	-.223 99672	-.176 55985
$\sin (H + a_o)$	+.631 97670	.724 51948	.759 84838	.790 42831
$\cos (G + a_o)$	+.919 14489	.921 42456	.974 58990	+.984 28991
$\cos (H + a_o)$	.774 98739	.689 25432	.650 10033	.612 55456
$\tan \delta_o$	.792 70865	.985 74195	.666 46890	1.071 46229
$\cos \delta_o$	.783 64828	.712 16560	.832 12624	.682 30678
$\sin \delta_o$	.621 20477	.702 01151	.554 58626	.731 06598
μ	-.070	+.036	+.056	+.009
μ'	-.018	-.007	-.058	-.008
a_o ~~(time)~~	3-37-04.380	10-51-01.650	13-52-20.955	16-38-49.770
f	+40.200			
f'	+0.270			
$\tau\mu$	-0.061	+0.031	+0.049	+0.008
m	-6.153	-5.343	-2.942	-3.728
n	+16.010	+20.196	+18.128	+22.998
a	3-37-54.646	10-51-57.004	13-53-16.660	16-39-49.518
δ_o	38-24-14.790	44-35-18.560	33-40-56.330	46-58-32.690
$\tau\mu'$	-0.016	-0.006	-0.051	-0.007
m'	+18.113	+18.946	+19.205	+19.396
n'	+9.557	+9.606	+7.157	+8.890
$i \cdot \cos \delta_o$	+3.926	+3.568	+4.169	+3.418
δ	38-24-46.370	44-35-50.674	33-41-26.810	46-59-04.387

FORMULAE CHECKED BY Kowalski, J.A.

$$m = \frac{g \cdot \sin (G + a_o) \tan \delta_o}{\not{15}} \qquad n = \frac{h \ \sin (H + a_o)}{\not{15} \cos \delta_o}$$

$$m' = g \ \cos (G + a_o) \qquad n' = h \cdot \cos (H + a_o) \sin \delta_o$$

Filter Technique 1

FORM NO. 3 STANDARD COORDINATES

DATE 14 NOV. 50 COMPUTER Ostberg, T.E. CHECKED BY Kowalski, J.A.

CAT. NO.	918	1159	1394	1400
σ	10-51-57	13-53-17	16-39-50	16-44-49
$-\sigma_c$	3-37-55			
$\sigma - \sigma_c = t$	7-14-02	10-15-22	13-01-55	13-06-55
ψ	44-35-51	33-41-27	46-59-04	35-21-52
sin t	.125 92002	.178 04850	.225 49427	.226 91101
$\cos \psi$	.712 05668	.832 04288	.682 19689	.815 48712
$\tan \psi$	.986 04788	.666 68597	1.071 78517	.709 72945
cos t	.992 04040	.984 02171	.974 24450	.973 91550
$\tan \xi'$	.993 95940	.677 51144	1.100 11929	.728 73822
ξ'	44-49-36	34-07-05	47-43-46	36-04-56
$-\psi_c$	38-24-46			
$\xi' - \psi_c = \xi$	6-24-50	-4-17-41	9-19-00	-2-19-50
$\cos \xi$	.993 74087	.997 19204	.986 80867	.999 17285
$\tan \xi$	.112 41344	-.075 09774	.164 05498	-.040 69832
$\sin \eta'$	.089 66219	.148 14399	.153 83149	.185 04301
η'	5-08-21	8-31-10	8-50-56	10-39-49
$\tan \eta'$	.090 02467	.149 79797	.155 68202	.188 29426
$\tan \eta$	.090 59170	.150 21978	.157 76312	.188 45014
η	5-10-35	8-32-35	8-57-55	10-40-20
$\tan \Theta$	.805 87962	-2.000 32358	.961 64786	-4.630 41570

Equatorial System: $\sigma = \alpha \quad \psi = \delta$ Horizon System: $\sigma = Az \quad \psi = h$

Subscript "c" denotes value at the arbitrary origin.

Formulae:- by Napiers Rule

$$\tan \xi' = \tan \psi / \cos t \qquad \sin \eta' = \sin t \ (\cos \psi)$$

$$\tan \eta = \tan \eta' / \cos \xi$$

$$\tan \Theta = \tan \eta / \tan \xi$$

PLATE COORDINATES WITH RESPECT TO FIDUCIAL AXES' INTERSECTION

(East end of star trail)

Star	x	y
1	11.605	− 1.367
2	−40.554	14.625
3	44.390	−15.486
4	−56.640	9.570
5	52.210	− 7.731
6	−27.934	−24.150
7	29.394	23.430
8	−47.138	−38.912
9	43.073	35.236

CORRECTION OF ABOVE COORDINATES FOR REFRACTION

$$\text{Kr}=\frac{983\cdot b\cdot \tan 1''}{460+t}=\frac{.142835798}{500}=.00029$$

b = barometric pressure = 29.96 inches

t = temperature = 40°F

$$\tan^2\gamma=\frac{x^2+y^2}{f^2}$$

$$x_c=x[(1+\tan^2\gamma)Kr+1] \qquad y_c=y[(1+\tan^2\gamma)Kr+1]$$

Star	$\sec^2\gamma$	$(\sec^2\gamma Kr+1)=m$	$x\cdot m$	$y\cdot m$
1	1.00308276	1.00029089	11.608	− 1.367
2	1.04195956	1.00030217	−40.566	14.629
3	1.04990146	1.00030447	44.404	−15.491
4	1.07449642	1.00031160	−56.658	9.573
5	1.06289140	1.00030824	52.226	− 7.733
6	1.03078430	1.00029893	−27.942	−24.157
7	1.03190053	1.00029925	29.403	23.437
8	1.08435034	1.00031446	−47.153	−38.924
9	1.06991748	1.00031028	43.086	35.247

Data Sheet

STANDARD COORDINATES AND PLATE COORDINATES WITH RESPECT TO STAR 1 (AFTER CORRECTION FOR REFRACTION)

Star	$\tan\eta$	$\tan\zeta$	x	y
2	−.24631013	.09109023	−52.174	15.996
3	.15021978	−.07509774	32.796	−14.124
4	−.32622581	.07142750	−68.266	10.940
5	.18845014	−.04069832	40.618	− 6.366
6	−.19533900	−.09838371	−39.550	−22.790
7	.09059170	.11241344	17.795	24.804
8	−.29269279	−.16467239	−58.761	−37.557
9	.15776312	.16405498	31.478	36.614

COMPUTATION OF $\tan s_x$, (a) AND (b)

Observation Equations

Star	(a)	$-\tan\theta$ (b)	$+\frac{x}{y}$ ($\tan s_x$)	$=\frac{x}{y}\tan\theta$
2	1	+2.70402358	−3.261690423	+ 8.819687814
3	1	+2.00032358	+2.322005098	+ 4.644761550
4	1	+4.56722984	−6.240036563	+28.499681193
5	1	+4.63041570	+6.380458687	+29.544176077
6	1	−1.98548113	+1.735410268	+ 3.445624340
7	1	− .80587962	+ .717424609	+ .578157871
8	1	−1.77742480	+1.564581836	+ 2.780926557
9	1	− .96164786	+ .859725788	+ .826753464

NORMAL EQUATIONS

(a)	(b)	$(\tan s_x)$	
+8	+ 8.37155929	− 13.32704827	=+ 79.13976887
+ 8.37155929	+52.28895872	− 79.13976887	=+287.0600997
−13.32704827	−79.13976887	+102.3920331	=−394.4405739
+1	+ 1.04644911	− 1.665881034	=+ 9.89247111
+1	+ 7.440544415	− 9.453408395	=+ 34.29002767
−1	− 5.938281851	+ 7.683024104	=− 29.59699447
	− 4.891836940	+ 6.017143070	=− 19.70452336
	+ 1.502262564	− 1.770384291	=+ 4.69303320
	−1	+ 1.230037539	=− 4.028041736
	+1	− 1.178478605	=+ 3.123976669
		+ .051558934 $\tan s_x$	=− .904065067
		$\tan s_x$	=− 17.53459579
		(b)	=− 17.54016932
		(a)	=− .9632585122

COMPUTATION OF v, μ, AND ν

Observation Equations

Star	(v)	$+x(\mu)$	$+y(\nu)$	$= Q$
2	1	−52.174	+15.996	=+207.9169443
3	1	+32.796	−14.124	=+212.8092369
4	1	−68.266	+10.940	=+207.0812350
5	1	+40.618	− 6.366	=+213.3347303
6	1	−39.550	−22.790	=+208.5388550
7	1	+17.795	+24.804	=+211.1288644
8	1	−58.761	−37.557	=+207.4718392
9	1	+31.478	+36.614	=+211.9319914

$$Q_n = \frac{x_n(\tan s_x) + y_n(a)}{\tan \eta_n (1+\tan^2 s_x)^{1/2}}$$

NORMAL EQUATIONS

(v)	(μ)	(ν)	
+ 8	− 96.064	+ 7.517	=+ 1,680.213696
−96.064	+16,432.35670	+2,398.963813	=−19,350.67552
+ 7.517	+ 2,398.963813	+4,501.304709	=+ 1,679.399512
+1	− 12.008	+ .939625	=+ 210.0267120
−1	+ 171.0563447	+ 24.9725580	=− 201.4352465
+1	+ 319.1384612	+ 598.8166435	=+ 223.4135309
	+ 159.0483447	+ 25.912830	=+ 8.5914655
	+ 490.1948059	+ 523.7892015	=+ 21.9782844
	+1	+ .162920168	=+ .0540179498
	+1	+ 1.272533275	=+ .0448358166
		+ 1.109613107 ν	=− .0091821332
		ν	=− .0082750764
		μ	=+ .0553661267
		v	=+ 210.6993239

COMPUTATION OF PLATE PERPENDICULAR AND FOCAL LENGTH

$$z_x = \frac{v}{1+\mu^2} = 210.0554183$$

$$\Delta x = \frac{v}{1+\mu^2} \cdot \mu = +11.62995490$$

$$f_x = (z_x^2 - \Delta y^2)^{1/2} = 210.0482262$$

$$z_0 = (z_x^2 + \Delta x^2)^{1/2} = 210.3771247$$

$$z_y = \frac{v}{1+\mu^2} (1+\mu^2-\nu^2) = 210.6849399$$

$$\Delta y = \frac{v}{1+\mu^2} \cdot \nu = -1.738224635$$

$$f_y = (z_y^2 - \Delta x^2)^{1/2} = 210.3637042$$

$$z_0 = (z_y^2 + \Delta y^2)^{1/2} = 210.6921103$$

$$f = 210.2059652$$

$$z_0 = 210.5346175$$

$$\cos t = \frac{f}{z_0} = .998438963$$

$$t = 3°12'07''$$

CHECK ON SOLUTION

$$\cos\alpha_x \cos\alpha_y + \cos\beta_x \cos\beta_y + \cos\gamma_x \cos\gamma_y = 0$$

$$\cos\alpha_x = \frac{\tan s_x \cdot z_x}{(1+\tan^2 s_x)^{1/2} \cdot z_0} = .9961053256$$

$$\cos\beta_x = \frac{z_x}{(1+\tan^2 s_x)^{1/2} \cdot z_0} = -.0568080005$$

$$\cos\gamma_x = \frac{\Delta \cdot x}{z_0} = .0552401075$$

$$\cos\alpha_y = \frac{\tan s_y \cdot z_y}{(1+\tan^2 s_y)^{1/2} \cdot z_0} = .0548738132$$

$$\cos\beta_y = \frac{z_y}{(1+\tan^2 s_y)^{1/2} \cdot z_0} = .9992083769$$

$$\cos\gamma_y = \frac{\Delta y}{z_0} = -.0082562414$$

$$\cos\alpha_x \cos\alpha_y + \cos\beta_x \cos\beta_y + \cos\gamma_x \cos\gamma_y = -.0025590081$$

COMPUTATION OF LENS DISTORTION

$$f_\eta = x_n\left(\frac{\cos\alpha_x}{\tan\eta_n} - \cos\gamma_x\right)\cos t + y_n\left(\frac{\cos\alpha_y}{\tan\eta_n} - \cos\gamma_y\right)\cos t$$

$$f_\xi = x_n\left(\frac{\cos\beta_x}{\tan\xi_n} - \cos\gamma_x\right)\cos t + y_n\left(\frac{\cos\beta_y}{\tan\xi_n} - \cos\gamma_y\right)\cos t$$

Star	f_n	$f\xi$	f_m
2	210.1194	210.6899	210.405
3	210.0538	210.4773	210.266
4	210.1376	210.8663	210.502
5	210.2190	210.3661	210.293
6	209.7507	210.2918	210.021
7	209.5843	210.3735	209.980
8	209.6274	210.2265	209.927
9	209.7207	210.3392	210.030

Average $f_m = 210.178$

$$dx_n = \left(\frac{f}{f_n} - 1\right)(x_n - \Delta x)$$

$$dy_n = \left(\frac{f}{f_n} - 1\right)(y_n - \Delta y)$$

Star	$\frac{f}{f_n} - 1$	$x_n - \Delta x$	$y_n - \Delta y$	dx_n	dy_n
2	−.001079	−40.590	14.680	.044	+.016
3	−.000419	44.380	−15.440	−.019	.006
4	−.001539	−56.682	9.624	.087	+.015
5	−.000547	52.202	− 7.682	−.029	.004
6	.000748	−27.966	−24.106	−.021	−.018
7	.000943	29.379	23.488	.028	.022
8	.001196	−47.177	−38.873	−.056	−.046
9	.000705	43.062	35.298	.030	.025

$\Delta x = +.024$ and $\Delta y = -.051$ are average coordinates of the principal point (with respect to fiducial axes' intersection) taken from computations made for both east and west ends of the star trails.

D. DISCUSSION

The star exposure method is a true field method of camera calibration in that it requires no equipment to make the calibration exposure, and in that the exposure observation can be accomplished in several minutes. The office work, on the other hand, is somewhat more lengthy than any other method, and requires a greater variety of skills than any other method. The method is comparable to the goniometer method in precision. Calibration data are obtained from a single exposure.

The simplicity of the field observation, and the complexity of the office work, recommend that the personnel using cameras in remote places be responsible only for the exposures and exposure data, and that these calibration exposures be submitted with the operational film received by personnel doing the metrical photography. The personnel using the operational photography would accomplish the office aspects of the calibration.

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