REPORTS OF EXPERIMENTS ON VOLUME DETERMINATION BY THE AID OF STEREO INSTRUMENT AND A PLANIMETER*

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IN THIS paper a description is given of a method for measuring volumes automatically in a stereo instrument with a connected planimeter. The experiments concerned swellings on the face of dental patients and were conducted at the Division of Photogrammetry at the Royal Institute of Technology, Stockholm.

The experiments were based on the following considerations.

If a stereo model is defined by a closed surface, it is possible to determine the volume within this surface, provided the contours are traced with the moving point of the planimeter connected to the drawing pencil of the stereo instrument as illustrated in Figure 1. The advantages of directly connecting the planimeter to the stereoscopic instrument, instead of the usual practice of first drawing the contours and then measuring with the planimeter, are the following.

1. By a suitable adjustment of the length of the planimeter arm, the total volume is made directly readable on the planimeter. This is the difference between readings before and after the contouring. Drawing paper and numerical calculations are unnecessary; time is thereby gained.

2. By reducing the contour interval below the practical limit for contouring on paper, the approximation error is lessened, and a smoothing effect on unsystematic contour errors is at the same time obtained.

3. The error in working the planimeter by hand is eliminated, that is deviations from the contour do not occur.

4. Time is also gained by making the contouring and planimeter operations simultaneous.

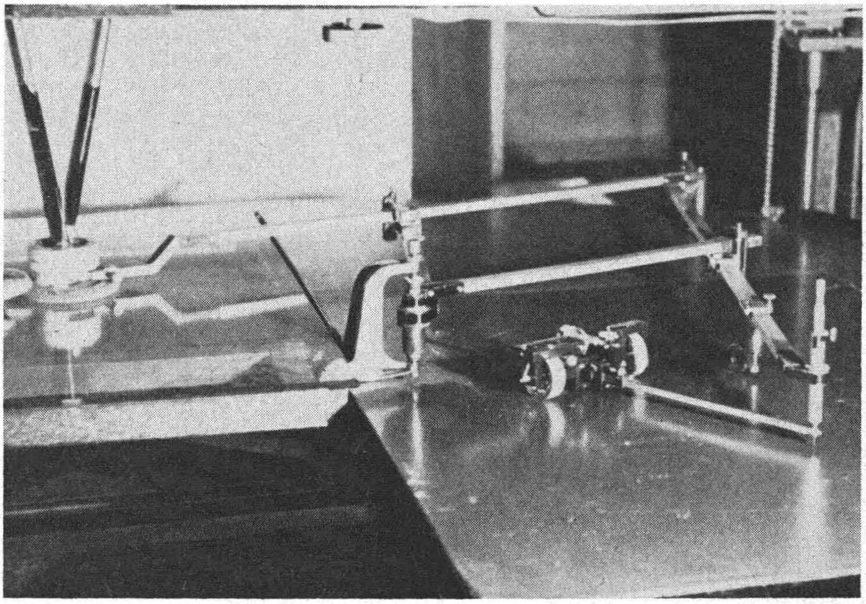

FIG. 1. Planimeter, coupled to the stereo instrument.

* This paper is a part of the Symposium on Non-Topographic Photogrammetry, See the Sept. 1953 issue. It was decided to postpone printing of this paper until illustrations could be obtained.

The planimeter computes the volume by means of numerical integration. Different formulas give different measuring programs, but the general form of the approximate volume is some weighted mean of the contour areas, and therefore, it is possible to perform integration according to any suitable formula. The ordinary average multiplied by the contour interval is the most simple formula, and since the interval can be chosen arbitrarily small (no plotting), this formula was used in the present application. The fact that only differences in volume were required; made a refinement of the approximation to true volume unnecessary. The choice of this simple formula was intended to lessen the risk of gross errors.

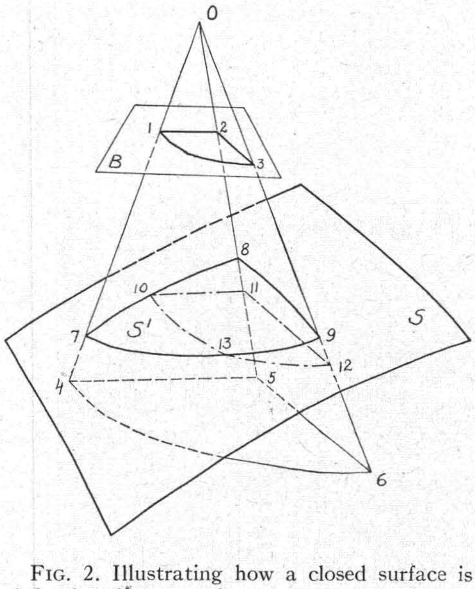

FIG. 2. Illustrating how a closed surface is defined and measured.
O = Perspective center (corresponds to camera lens).
B = Plate holder.
S = Surface of the stereo model.
$7\cdot 10\cdot 8\cdot 9\cdot 13\cdot 7 = S'$ = Part of S. The deformation lies within S'.
$1\cdot 2\cdot 3\cdot 1$ Contour drawn on plate holder. This contour and the point O define the reference cone in the following manner: O is the top of the cone and the generatrix describes the contour.
$4\cdot 5\cdot 6\cdot 4$ Bottom surface of the reference cone. The part of the reference cone between S' and the bottom surface $(4\cdot 5\cdot 6\cdot 4)$ constitute the closed surface ,the volume of which is to be measured.
$10\cdot 11\cdot 12\cdot 13\cdot 10$ levelling contour, followed by "the floating mark." The planimeter measures the area of this closed contour and adds the value obtained to the sum of all previous contour areas, beginning with "the bottom surface" (or the top surface).

The difficulties which arise when using this method are the following.

1. A stereoscopic view of the closed surfaces must be possible. To produce a stereoscopic model of an arbitrary form of surface, the space within the surface must be transparent, and the front surface must also be partly transparent so that the back surface will not be entirely hidden.

This requirement was not met. However even though the conditions of transparency do not exist, the direct method for volume determination is still applicable provided certain limitations with reference to the generality of the form of the surface are allowed.

The most simple case is that of the back surface being plane and limited by a closed curve, which at the same time forms the limitation of the front surface. When contouring on planes parallel to this "bottom surface," the "lower curve" will thus consist of the limitation curve. As an example, this would be the case if a heap of non-transparent material, placed on a flat plate, were stereoscopically photographed, so that, on the one hand, the whole heap is stereoscopically covered and, on the other hand, the surface of the heap is not hidden on any of the photographs. By absolute orientation on points on the plate, the contouring would be more easily executed in planes parallel to this plate.

An estimate of the volume variations in a locally deformable closed surface is somewhat more complicated when not all of the front surface is stereoscopically covered. This case can be brought back to the most simple one, described above, if the orientation is left untouched and the whole of the deformed part is stereoscopically covered. This procedure, which is diagrammed in Figure 2, is based on a contour, 1.2.3.1., drawn in one of the plate holders so as to include the deformed surface, S'. When drawing contour lines this contour is followed wherever it lies beneath (10.8.9.13) the deformed surface, S'; elsewhere, the surface contour, 13.10, is traced to form a closed plane curve whose area is measured by means of the planimeter. It is evident from Figure 2 that the solid whose volume is to be measured is the truncated portion of a cone whose vertex is the center of projection, 0, and whose generatrix is the contour, 1.2.3.1, drawn on one of the plate holders. The top of this solid is the deformed surface, S', and the bottom is the plane surface. 4.5.6.4. placed at least as low as the lowest curve of the top surface. Thus, if it is possible to draw the contour on the picture holder as above described, the total volume variation, due to surface deformation of the space within the surface originally closed, but not stereoscopically covered, manifests itself as a variation of the volume measured.

2. The planimeter must be moved between the different contour lines along the same line. If the floating mark is shifted from one contour to the next, the two separate marks move on straight lines, which in the normal case lead through the principal point. When one of the picture holders is furnished with a contour to limit the portion of the stereoscopic model that is to be contoured, it is advisable to introduce this straight line on the picture holder—if possible as a part of the contour.

Beside these two chief difficulties, certain practical details claimed special attention in making the experiment. The photographs were taken with Wild's stereo camera with a focal length of about 9 cm and a base of 40 cm. Ancillary lenses were used to shorten the working distance. The Wild Autograph allows a minimum focal length of 10 cm, thus making it necessary to modify the inner orientation through affine transformation unless the plates are enlarged. The distortion introduced by the ancillary lenses, on the one hand, causes disturbing parallaxes and, on the other hand, causes deformations of the model. The model deformation and affine transformation were of no importance, because the outer orientation was unvaried and the photographs were taken as in the normal case. The disturbing vertical parallaxes were most easily eliminated with one of the camera constants. It is true that this gives rise to both an incomplete parallax elimination and extra model deformation, but neither of these was a very great disadvantage in making the experiment. The alternative of eliminating the vertical parallaxes with ϕ, thus diverging from the normal case, was tried at an early stage of the experiment but was abandoned.

As the picture holder of the Autograph has no marks for adjusting negatives from the stereoscopic camera, these had to be introduced. Through use of a coördinatograph, this was done by scratching crosses on a layer of ink on the glass of the picture holder.

Certain additional experiments were made to measure the accuracy of the method. The most important of these are described in the following. The behavior of the planimeter was investigated by a series of measurements of known surfaces. The planimeter showed a good consistency for average size of surface, when the angle between the moving arm and the rolling direction was not less than a certain minimum. The standard deviation, computed from the divergences from the average of 10 measurements with different initial angles all surpassing the minimum angle, amounted to $\frac{1}{4}$ pro mille (0.25 per cent) of the surface (25 cm^2). The systematic error, due to sliding, was investigated by changing the direction of rotation, but this error was without importance in the case in question.

The affinitely transformed elevation scale of the stereoscopic model required a special adjustment of the length of the planimeter arm. This was performed after computation and was controlled through separate plane and height measurings. A crude control was effected by contouring and determining the volume of a clod of putty, which had been photographed under the same conditions as the objects of investigation. Variations of volume, caused by weaknesses in the instruments and in the contouring procedure, were studied through repeated plotting of the same pair of stereoscopic pictures. Volume variations caused by variations on the patients and, in general, variations of the outer orientation were investigated by repeated photographing and plotting of the same patient. The average errors derived from these calculations was of the order of magnitude of 1 and 2 cm^3 respectively, the model of reference chosen being 60 to 130 cm^3. To decide the reliability of the approximation, the same model was measured by a series of successively reduced equidistances. In our case 1 mm in the Autograph, that is about 2 mm in reality, appeared to be a division which was small enough.

Concerning the instrumental work the following must be noted: The reciprocal orientation was left untouched, and the absolute orientation was executed on control points attached to the object, by means of a matrix of the jaw of suitable impression compound, which sets to required hardness. Each model was measured twice, since one omitted contour line would introduce an unacceptable error. This procedure naturally admits of argument. The contouring work is increased and of course it would be possible to use this additional work for lessening the approximation error by using half the contour interval.

The importance of each volume determination separately decided us in favor of the first of the above mentioned procedures. When dealing with a large series of experiments with a slow variation, where large errors give rise to inexplicable jumps in the results, the latter method is preferable. Our case was different, and so the measuring was doubled. This lengthened the total time of work by about 20 per cent. The average time used for contouring one model was about 30 minutes. The chief part of the work was executed in the Autograph A6, belonging to the Photogrammetric Division at the Royal Institute of Technology in Stockholm.