NUMERICAL RELATIVE ORIENTATION

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Abstract

A simplified European method of relative orientation using an arithmetic analysis is explained for American operators of stereoscopic mapping instruments. The method applies to nearly every photogrammetric plotting instrument that is equipped to indicate the *by* setting of one projector. The ideas of least squares and weighted means are included to obtain a logical orientation even if the model is faulty. The application of the method is described at operator level, and the essential items of the mathematical basis are explained in sufficient detail for use with models of different shapes and sizes.

Order of Contents: 1. Introduction, 2. Brief outline of the Method, 3. Detailed Instructions, 4. Theoretical Basis, 5. Application to the Y-Swing Method, 6. References

1. Introduction

The procedure of adjusting the two projectors of a photogrammetric instrument so that common images exactly coincide is called relative orientation. Under favorable conditions, the two image areas can be made to coincide completely by adjusting five motions according to a standard, conventional, systematic routine of successive approximations. The routine is based on the recognition and removal of any lack of coincidence (parallax) at six standard locations in the overlap area (model). It can be shown that the five motions are theoretically sufficient to remove all the parallax at the first five points, after which no parallax will be present at any other place in the model. The model is then said to be "clear" and subsequent mapping operations can proceed.

Frequently in practice, however, a significant and disturbing amount of parallax remains at the sixth point as well as throughout the model, whence the operator sometimes spends an objectionably long amount of time in attempting to clear the model further or to distribute the remainder. The remaining parallax may be due to an unavoidable lack of symmetry between the projector and the aerial camera lenses, or to differential film shrinkage present before making the reproduction on glass, as well as to other known and unknown factors.

This discussion demonstrates a simplified European method (1 . . . 7) for removing or distributing logically the remaining parallax in an effort (1) to reduce the time required for relative orientation and (2) to improve the quality of the relative orientation. Experience of users of this general method indicates that vertical and horizontal errors in the models are minimized and significantly reduced if this method is applied. The method incorporates the principle of least squares to correct for six observed discrepancies by means of five adjusting motions while giving double weights to the two points beneath the projectors. The method results in a final distribution of residual parallaxes that is independent of the operator or of the degree of refinement of the initial orientation. The general method applies to any instrument that has the required measuring and adjusting motions, the only difference being that the table of products may be different than that shown herein.

The demonstration first considers the "swing-swing" method of relative orientation where both projectors are adjusted, and finally the system is also applied to the "*Y*-swing" method where only one projector is moved.

2. Brief Outline of the Method

The general routine used in applying this numerical method is briefly outlined by enumerating a series of steps. The method requires the use of an indicator (graduated in any convenient units) attached to the *by*-motion of one projector, as shown in Figure 1 with regard to the Kelsh plotter.

2.1 Relative orientation is begun as usual, employing the conventional procedure and repeating only as necessary to reduce the parallaxes to the order of about one-half millimeter. This can be done in projection-type instruments without using the anaglyphic spectacles. No effort need be made to refine the results.

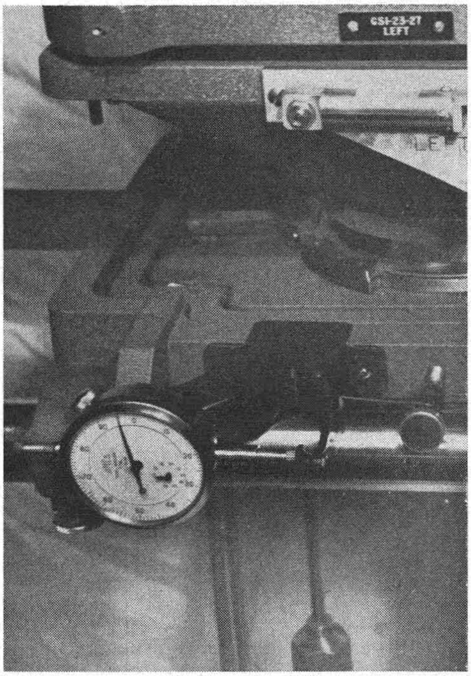

FIG. 1. Dial indicator attached to one projector of a Kelsh plotter to register a small translational movement in the y-direction.

2.2 The y-motion is adjusted until no parallax is evident at one of the six standard positions (measuring station, Figure 2), and the value shown on the y-indicator is recorded. The operation is repeated at each of the other five positions.

2.3 The computation follows, consisting largely of addition and subtraction as indicated on the form (Figure 2), and obtaining values from a set of tables (Figure 3). The computation involves only simple numbers of two digits ranging up to about 100. Strict observance of algebraic signs is mandatory, but is simplified as much as possible. The computations result in five parallax corrections to be applied by the proper orientation adjustment screws.

2.4 The parallax corrections are applied* by combining the y-motion indicator

* This method of adjusting a motion a given amount and direction may be new. It has the advantage of requiring only one graduated measuring device (the y-motion indicator, which need not be calibrated) for all five adjustments, and is free from any mechanical inaccuracy of indicators on the adjustments themselves.

5 6
1, k_2 — 2, k_1
3, ϕ_2, w 4, ϕ_1, (w)

Computation Form
RELATIVE ORIENTATION
Kelsh Plotter

Model No. ______
Date ______
Operator ______

Initial Observation						Final Observation					
Pt	y	p	Pt	y	p	Pt	y	p	Pt	y	p
1	180	−9	2	176	−13	1	176	+3	2	176	+3
1	180	−9	2	176	−13	1	176	+3	2	176	+3
4	167	−22	3	180	−9	4	180	+7	3	177	+4
6	230	+41	5	210	+22	6	165	−6	5	160	−13
		+1	Mean	189	−13			+7	Mean	173	−3

Pt	Parallax p	Swing I k_1	Swing II k_2	Tip I ϕ_1	Tip II ϕ_2	Tilt w
1	−9	(−1.75) +16	(−2.25) +20			(+2) −18
2	−13	(−2.25) +29	(−1.75) +23			(+2) −26
3	−9	(+0.875) −8	(+0.625) −6		(−½) +4	(−1) +9
4	−22	(+0.625) −14	(+0.875) −19	(−½) +11		(−1) +22
5	+22	(+0.875) +19	(+0.625) +14		(+½) +11	(−1) −22
6	+41	(+0.625) +26	(+0.875) +36	(+½) +20		(−1) −41
Totals		+68	+68	+31	+15	−76

FIG. 2. Computation form for numerical relative orientation of any instrument like a Kelsh plotter using 90° photography and the swing-swing method.

and the proper orientation motion. A specific one of the six standard positions is designated as a "measuring station" for each of the five adjustments. The parallax is first removed at a measuring station with the y-motion as in *Step 2*, and

TABLE OF PRODUCTS

p	1.75	2.25	0.875	0.625
2	4	5	2	1
3	5	7	3	2
4	7	9	4	3
5	9	11	4	3
6	11	14	5	4
7	12	16	6	4
8	14	18	7	5
9	16	20	8	6
10	18	23	9	6
11	19	25	10	7
12	21	27	11	[illegible]
13	23	29	[illegible]	
14	25	[illegible]		
15	[illegible]			
				26
			37	26
		97	38	27
	77	99	39	28
45	79	101	39	28
46	81	104	40	29
47	82	106	41	29
48	84	108	42	30
49	86	110	43	31
50	88	113	44	31
51	89	115	45	32

p	1.75	2.25	0.875	0.625
52	91	117	46	33
53	93	119	46	33
54	95	122	47	34
55	96	124	48	34
56	98	126	49	35
57	100	128	50	[illegible]
58	102	131	[illegible]	
59	103	[illegible]		
60	[illegible]			
				[illegible]
			[illegible]	54
		[illegible]	77	55
	[illegible]	200	78	56
90	158	203	79	56
91	159	205	80	57
92	161	207	81	58
93	163	209	81	58
94	165	212	82	59
95	166	214	83	59
96	168	216	84	60
97	170	218	85	61
98	172	221	86	61
99	173	223	87	62
100	175	225	88	63
101	177	227	88	63

FIG. 3. Multiplication table for use with the computation form of Figure 2.

the indicator setting is noted. The correction is then applied numerically to the indicator setting with y-motion screw. Next, the proper orientation screw is adjusted until the parallax is again removed. This installs the parallax correction. These three substeps are repeated for each of the five orientation adjustments.

2.5 The operation is checked by repeating *Step 2* and recording the six final observations in their places on the form. The weighted mean value of the final observations should be the working value set on the y-indicator. The entire procedure must be repeated if the final residual parallaxes are unduly large or concentrated, but a single procedure is usually sufficient.

3. Detailed Instructions

(1) Set the diapositive plates in the projectors in the usual manner and perform relative orientation using the conventional method until the residual parallaxes are reduced to the order of about 0.5 mm.

(2) Remove all the parallax at Point 1 with the y-motion only and record the indicator setting y in the Initial Observation space at the top of the Computation Form (Figure 2). Note in the sample that the value is recorded twice.

(3) Repeat *Step 2* at Point 2, using only the y-motion to remove all the parallax, and reading the indicator setting and recording it twice on the Form.

(4) Repeat *Step 2* at Points 3, 4, 5 and 6, recording the settings (once only) in the indicated spaces on the Form.

(5) Add the eight recorded values and divide by 8 to obtain the weighted mean, which is recorded as shown in the sample computation.

(6) Subtract the mean from each of the eight y-values and record the difference in the p or parallax column. If the mean is larger than the y-value, the algebraic sign of p is minus; otherwise, plus.

(7) As an arithmetic check, add algebraically the two columns of four p-values each and enter the sums on the form. Theoretically the two sums should be small, numerically equal and their sum should be zero if no mistakes are made in averaging, subtracting and adding. Present experience indicates that the two values in the initial computation are usually almost numerically equal, of opposite sign, and about 10 to 20 in magnitude. In the final computation, they are usually 1 or 2.

(8) Copy the p-values into the lower part of the Computation Form, including their algebraic signs.

(9) Affix the proper algebraic signs in the squares of the lower part of the Form, considering one row at a time. The normal sign is indicated in parenthesis in each square. If the sign of p is plus, copy the normal sign immediately below the parenthesis in each square throughout the row for that particular p-value. If the sign of p is minus, reverse the sign in each square throughout the row. Fox example, the sign of p of the first row is minus in the sample, whence the signs of the first two squares are reversed from minus to plus, and in the last square the sign is reversed from plus to minus.

(10) Place numbers in each of the squares of the lower part of the Form, obtaining the numbers from the Table of Products (Figure 3) (which is independent from the particular units of the dial gage) with regard to the value of p in each row and the factor ($1\frac{3}{4}$, $2\frac{1}{4}$, etc.) in parenthesis in each square. In the sample, the value 16 in the first square of the Form is found in the Table in the $1\frac{3}{4}$-column opposite the p-value of 9; 20 is found in the $2\frac{1}{4}$ column opposite 9; and 18 is obtained by multiplying 9 by the 2 in the parenthesis.

(11) Add algebraically all the numbers in each of the five columns of the lower part of the Computation Form, entering the sums as Totals in the bottom row. These are the parallax corrections to be applied.

(12) Install the Total correction for Swing 1 (κ_1), which is +68 in the sample, as follows:

a. At Point 2 (which is the measuring station for κ_1 as shown in the diagram at the top left margin of the Computation Form) remove all the parallax with the *y-motion*. This has the effect of setting the indicator at a starting position for measuring.

b. Note the indicator setting.

c. Increase or decrease the indicator setting equal to the correction shown on the bottom row of the Form *with the y-motion*. In the sample, the setting is increased 68 units. If the algebraic sign had been minus, the setting would have been decreased.

d. At the same Point 2, remove all the parallax again, but this time *with the swing motion of the left projector* (κ_1 motion). This installs the first of the five corrections.

(13) Install the Total correction for Swing II (κ_2), which is also +68 in the sample, similar to Step (12), by:

a. Removing the parallax at Point 1 (which is the designated measuring station) *with the y-motion.*
b. Noting the indicator setting.
c. Increasing or decreasing the indicator setting the required amount using the y-motion.
d. Removing the parallax again *with the swing motion of the right projector* (κ_2 motion). This installs the second parallax correction.

(14) Install the remaining corrections as indicated in the bottom row of the Computation Form, one at a time, using the general system of *Steps 12 and 13,* and observing the proper measuring stations, adjustment motions, and algebraic signs.

(15) Repeat *Steps 2 through 7,* recording the Final Observation in the upper right part of the Computation Form. The solution is ordinarily satisfactory if all the newly computed values of p are less than 0.05 mm. in the Kelsh plotter, whose model is five times the size of the photograph. If it is considered desirable to reduce the parallaxes further, copy the new p-values in the lower part of a new Computation Form, and repeat the computation and adjustment. A Final Observation should then be repeated also.

(16) Set the indicator at the Mean y-value of the Final Observation. This completes the relative orientation, after which the model is ready for scaling, leveling and contouring.

In extremely mountainous terrain, it may be necessary to repeat the entire solution three or four times as the parallaxes may not be reduced sufficiently by fewer corrections, or may be overcorrected.

Where extreme film shrinkage or other difficulties are present, it may not be possible to reduce all the parallaxes to the order of 0.05 mm. (for the Kelsh plotter).

If the model is to be used in bridging either horizontally or vertically, extra solutions should be repeated if necessary in an effort to reduce the parallaxes as much as possible.

4. Theoretical Basis

A set of practical numerical equations are sought expressing the amounts and directions of the required orientation adjustments of an instrument in terms of the y-parallaxes, which can be measured, such that the parallaxes at several places in the model are reduced to a minimum. Rigorous derivations of such expressions occur in the literature [1 . . . 8] and herein are not repeated.

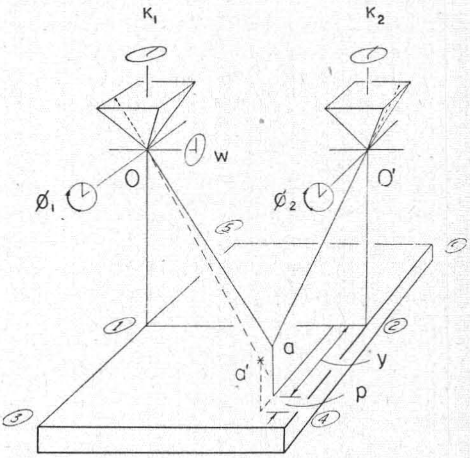

FIG. 4. Diagram showing the motions and nomenclature of the swing-swing method of relative orientation.

The amount of y-parallax p at a given place in the model (Figure 4) can be considered a function of five independent variable angular adjustments of the projectors: κ_1, ϕ_1, ω of the first projector, and κ_2, ϕ_2 of the second.

$$F(p) = f_1(\kappa_1) + f_2(\kappa_2) + f_3(\phi_1) + f_4(\phi_2) + f_5(\omega). \tag{1}$$

These five adjustments are known to be sufficient to eliminate the parallaxes at five places in the model, and, if the two projectors reproduce faithful perspectivities, parallaxes will also be non-existent elsewhere in the model. Theoretically, each of these functions is circular or quadratic, but p is always sufficiently small to permit the functions being considered linear.

The resultant change in parallax dp created at a point in the model by small changes $d\kappa_1$, $d\kappa_2$, etc., in each of the five adjustments can be expressed as

$$p = \frac{\partial F(p)}{\partial k_1} d\kappa_1 + \frac{\partial F(p)}{\partial k_2} d\kappa_2 + \frac{\partial F(p)}{\partial \phi_1} d\phi_1 + \frac{\partial F(p)}{\partial \phi_2} d\phi_2 + \frac{\partial F(p)}{\partial \omega} d\omega. \tag{2}$$

It is required to express each of the unknown differentials $d\kappa_1$, $d\kappa_2$, etc., in terms of dp's (measured at various places in the model) and a set of fixed constants.

To simplify the work, the dp's are considered to be observed and measured always at the same positions in the model. The partial derivatives such as $\partial F(p)/\partial\kappa_1$, are functions of position only, whence their numerical values do not change from model to model, and need be evaluated but once. In other words, the partial derivatives of Equation 2 can be regarded as fixed numerical constants. It remains to express their numerical values at the six locations in the model.

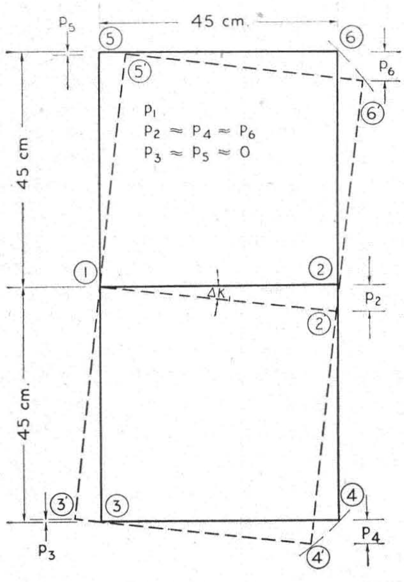

FIG. 5. Plan view of the model showing the effects of the swing (κ_1) motion of the left (first) projection on the parallaxes at the six positions.

4.1 VALUES OF THE COEFFICIENTS

Figure 5 shows the effect of a small change $\Delta\kappa_1$ in the swing adjustment at the points 1 . . . 6 on a plan of the model. The dimensions of the model are taken as 45 cm. in line of flight, and 90 mm. perpendicular to the direction of flight, corresponding to current wide-angle photography. It should be readily apparent without formal proof that: (1) the parallax created at point 1 is zero because it is the center of rotation; (2) the y-components of the total parallaxes at points 2, 4 and 6 are exactly equal; and the parallaxes at points 3 and 5 are essentially zero because the image motion is very nearly perpendicular to the y-direction, the discrepancy being a term of higher order which is of no practical consequence if the parallax at point 2 is itself small. It may also seem logical that point 2 would be an appropriate place to measure the effect of $\Delta\kappa_1$.

If it be assumed that the effect of one unit adjustment of the $\Delta\kappa_1$ motion is one unit of parallax at point 2, then the effect of $\Delta\kappa_1$ at each of the six standard positions is, respectively, (0, +1, 0, +1, 0, +1). It may already seem logical that these six values are the values of the partial differentials $\partial F(p)/\partial\kappa_1$ in six different instances or locations. In other words, the change in the value of p in this instance is that caused by changing $\Delta\kappa_1$ only, considering momentarily that all the other adjustments are unchanged or constant.

Similarly, the partial differentials $\partial F(p)/\partial\kappa_2$ are (+1, 0, +1, 0, +1) for the six places, respectively.

Figure 6A is an elevational view in the xz-plane showing the effect of a small change in the ϕ adjustment at points 1 and 2, the change appearing only in the x-direction. (Note that the projection distance is taken as 75 cm.) Transferring

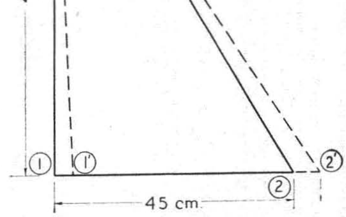

FIG. 6A. Elevation view in the XZ-plane showing the y-tilt (tip, ϕ_1) motion of the left projector.

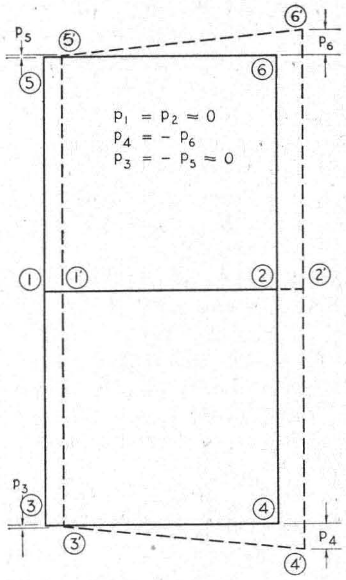

FIG. 6B. Plan view showing the effects of the y-tilt (tip, ϕ_1) motion of the left projector on the parallaxes.

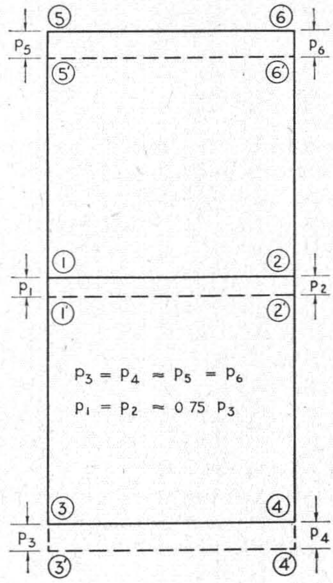

FIG. 7B. Plan view showing the effects of the X-tilt ("tilt," ω) motion of either projector on the parallaxes at the six positions.

these data to a plan view (Figure 6B) it is seen that point 4 is moved to 4′, 6 to 6′, etc. However, it is also evident that the y-parallax effects at points 4 and 6 are equal in magnitude but opposite in direction; that at 1 and 2 they are exactly zero, and at points 3 and 5 the effect is essentially zero if that of 4 is small. Considering a unit of $\Delta\phi_1$ as that which produces a unit linear y-parallax at point 4, then the partial derivative $(\partial F(p)/\partial\phi_1)$ at the six places in numerical order are (0, 0, 0, +1, 0, −1). Similarly the effects of $d\phi_2$ are (0, 0, +1, 0, −1, 0).

Figure 7A is an elevational view in the yz-plane showing the actual y-parallax effect of a small change in the ω adjustment $\Delta\omega$. Figure 7B is a plan view. It may be apparent that the effects of points 3, 4, 5 and 6 are equal, and that at points 1 and 2 are somewhat less.

Consider Figure 6A in deriving the relation of the parallax p_1 in terms of the parallax p_3. Right triangles Onn', Oca are es-

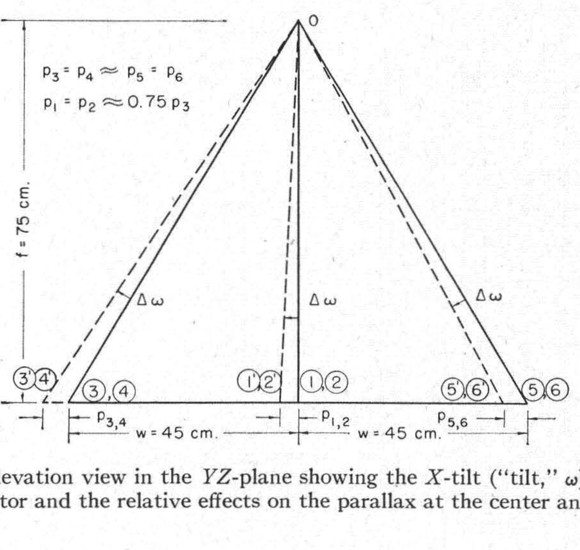

FIG. 7A. Elevation view in the YZ-plane showing the X-tilt ("tilt," ω) motion of either projector and the relative effects on the parallax at the center and the edges.

sentially similar and

$$ac/Oa = nn'/On; \quad ac = Oa \times nn'/On. \quad (3)$$

Also, right triangles *acb* and *Ona* are similar, and essentially,

$$ac/ab = On/Oa$$

$$ac = On \times ab/Oa \quad (4)$$

Equating the right hand members of Equations 3 and 4 and solving for nn',

$$nn' = \overline{On^2} \times ab/\overline{Oa^2}.$$

As $nn' = p_1$, $On = f$, $ab = p_3$, and $Oa = \sqrt{f^2 + w^2}$; then

$$p_1 = f^2 p_3/(f^2 + w^2). \quad (5)$$

But $f = 75$ cm., $w = 45$ cm., whence

$$p_1 = 0.735 p_3. \quad (6)$$

In this analysis it is assumed that $p_1 = 0.75 p_3$, first for convenience in computing and secondly—because operators are inclined to select points nearer the center where the ratio is large and images are clearer, rather than farther out where the ratio is smaller and images are poorer.

Consequently, if a unit change of ω is considered a unit change in parallax at point 3, say, it causes a change of 0.75 at points 1 and 2 and the values of the partial derivatives $(\partial F(p)/\partial\omega)$ at the six places are, in numerical order, (+0.75, +0.75, +1, +1, +1, +1).

The effects of each of the motions at the six places is summarized in Table I.

4.2 THE CONDITION EQUATIONS

Equation 2 can now be expressed with numerical coefficients. For example, the parallax p_1 at point *1* is

$$p_1 = 0\kappa_1 + 1\kappa_2 + 0\phi_1 + 0\phi_2 + 0.75\omega$$
$$= 1\kappa_2 + 0.75\omega$$

where the coefficients consist of the first values from the first column of Table I. The "*d*" and "delta" have been omitted from the differentials for brevity.

This equation is a condition equation in that the values of the variables are desired to create an observed value of parallax such that, if the adjustments are made in the opposite direction, the remaining parallaxes will be zero. One equation exists for each of the six points (Table II).

As these equations are linear, there are six simultaneous linear equations in five unknowns, where the values of p are obtainable through physical measurement.

TABLE II

THE CONDITION EQUATIONS

$$\left\{\begin{array}{l} 0 + \kappa_2 + 0 + 0 + 3/4\omega = p_1 \\ \kappa_1 + 0 + 0 + 0 + 3/4\omega = p_2 \\ 0 + \kappa_2 + 0 + \phi_2 + \omega = p_3 \\ \kappa_1 + 0 + \phi_1 + 0 + \omega = p_4 \\ 0 + \kappa_2 + 0 - \phi_2 + \omega = p_5 \\ \kappa_1 + 0 - \phi_1 + 0 + \omega = p_6 \end{array}\right.$$

4.3 THE WEIGHTED EQUATIONS

Both [1] and [2] advocate that the points 1 and 2 should be favored by doubly weighting them. This is proved in [2] and certainly seems logical because of brighter images, better lens distortion correction, etc. The method of doubly weighting a point consists of multiplying the condition equation by the square root of 2 (Table III).

4.4 THE NORMAL EQUATIONS

As there are more weighted equations than there are unknowns, the customary unique solution of five simultaneous equations in five unknowns will probably not satisfy the conditions of the sixth equation. A method of reducing the number of

TABLE I

SUMMARY OF EFFECTS OF UNIT ADJUSTMENTS AT EACH OF SIX POINTS

Adjusting Motion	Numbered Point of the Model					
	1	2	3	4	5	6
κ_1	0	+1	0	+1	0	+1
κ_2	+1	0	+1	0	+1	0
ϕ_1	0	0	0	+1	0	−1
ϕ_2	0	0	+1	0	−1	0
ω	+3/4	+3/4	+1	+1	+1	+1

TABLE III

THE WEIGHTED EQUATIONS

$$\left\{\begin{array}{llllll}
0+\sqrt{2}\kappa_2 & 0 & 0 & +3/4\sqrt{2}\omega & = & \sqrt{2}p_1 \\
\sqrt{2}\kappa_1 & 0 & 0 & 0+3/4\sqrt{2}\omega & = & \sqrt{2}p_2 \\
0+\ \kappa_2 & 0 & +\phi_2 & +\ \omega & = & p_3 \\
\kappa_1 & 0 & +\phi_1 & 0+\ \omega & = & p_4 \\
0+\ \kappa_2 & 0 & -\phi_2 & +\ \omega & = & p_5 \\
\kappa_1 & 0 & -\phi_1 & 0+\ \omega & = & p_6
\end{array}\right.$$

weighted equations to five normal equations is used, which gives a solution that most nearly satisfies all six equations according to the principle of least squares. Sufficient information is given so that the reader may apply these ideas to his own problem, but a proof of the method is not attempted as it is given in textbooks on least squares and geodesy.

$$\begin{aligned} fe &= \tfrac{3}{4}\cdot\sqrt{2}\cdot\sqrt{2}p_1 + \tfrac{3}{4}\sqrt{2}\cdot\sqrt{2}p_1 \\ &\quad + 1\cdot p_3 + 1\cdot p_4 + 1\cdot p_5 + 1\cdot p_6 \\ &= 1\cdot 5p_1 + 1\cdot 5p_1 + p_3 + p_4 + p_5 + p_6. \end{aligned}$$

In like manner, all 30 coefficients of the normal equation are evaluated.

4.5 SOLUTION OF THE NORMAL EQUATIONS

Inasmuch as the normal equations are five simultaneous linear equations in five unknowns, they can be solved by any of the several methods commonly used in algebra. It is noted that the third and fourth equations are immediately solved and the other three equations are independent from their solution. Thus the solution is reduced to three equations in three unknowns. The process of elimination by addition and subtraction is applicable, and the methods of determinants is simplified by the large number of zeros present in the coefficient matrix. The solution of the problem is given as follows where all algebraic signs have been reversed to give the effect of a correction:

TABLE IV

THE NORMAL EQUATIONS

$$\left\{\begin{array}{llllll}
4\kappa_1 & 0 & 0 & 0 & +3\tfrac{1}{2}\omega = & 2p_2+p_4+p_6 \\
0 & +4\kappa_2 & 0 & 0 & +3\tfrac{1}{2}\omega = & 2p_1+p_3+p_5 \\
0 & 0 & +2\phi_1 & 0 & 0 \quad = & p_4-p_6 \\
0 & 0 & 0 & +2\phi_2 & 0 \quad = & p_3-p_5 \\
3\tfrac{1}{2}\kappa_1 & +3\tfrac{1}{2}\kappa_2 & 0 & 0 & +6\tfrac{1}{4}\omega = & 3/2p_1+3/2p_2+p_3+p_4+p_5+p_6.
\end{array}\right.$$

The five normal equations are shown in Table IV, and Table V indicates how each coefficient of the normal equations is formed from the Weighted Equations of Table III. The number in the first column of the first row of the Normal Equations is equal to the sum of the squares of all the coefficients in the first columns of the Weighted Equations:

$$aa = 0^2 + (\sqrt{2})^2 + 0^2 + 1^2 + 0^2 + 1^2 = +4.$$

The number in the first column of the second row is the algebraic sum of the two-term products of all the coefficients of the first column times the respective coefficients of the second column of the Weighted Equations:

$$ba = 0\cdot\sqrt{2}+\sqrt{2}\cdot 0+0\cdot 1+1\cdot 0+0\cdot 1+1\cdot 0=0.$$

Term *ba* is exactly equal to *ab*, which is the term in the second row of the first column. The number in the last row of the last column is equal to the algebraic sum of the products of the coefficients of the last columns times the respective coefficients of the second column of Weighted Equations:

$$\Delta\kappa_1 = -1\tfrac{3}{4}p_1 - 2\tfrac{1}{4}p_2 + \tfrac{7}{8}p_3 + \tfrac{5}{8}p_4 + \tfrac{7}{8}p_5 + \tfrac{5}{8}p_6$$

$$\Delta\kappa_2 = -2\tfrac{1}{4}p_1 - 1\tfrac{3}{4}p_2 + \tfrac{5}{8}p_3 + \tfrac{7}{8}p_4 + \tfrac{5}{8}p_5 + \tfrac{7}{8}p_6$$

$$\Delta\phi_1 = -\tfrac{1}{2}p_4 + \tfrac{1}{2}p_6$$

$$\Delta\phi_2 = -\tfrac{1}{2}p_3 ++ \tfrac{1}{2}p_5$$

$$\Delta\omega = +2p_1 + 2p_2 - p_3 - p_4 - p_5 - p_6.$$

Thus, each of the correlated adjustments in any one of the motions required to eliminate all six parallaxes simultaneously is given in terms of a simple series

TABLE V

SYSTEM OF FORMING COEFFICIENTS OF NORMAL EQUATIONS

aa	*ba*	*ca*	*da*	*ea*	*fa*
ab	*bb*	*cb*	*db*	*eb*	*fb*
ac	*bc*	*cc*	*dc*	*ec*	*fc*
ad	*bd*	*cd*	*dd*	*ed*	*fd*
ae	*be*	*ce*	*de*	*ee*	*fe*

of products of constant factors times the respective amounts of parallax observed at the six points. The labor is facilitated by means of a multiplication table (Figure 3). The proper algebraic signs are all confined to the computation form alone.

5. Application to the y-Swing Method of Orientation

The foregoing analysis applies to the "swing-swing" method of relative orientation where both projectors are oriented, as is frequently used with the initial models of many instruments and with all models of the Kelsh plotter. The present analysis deals with the "y-swing" method where only the second projector is adjusted, which is the most frequent method used with some instruments.

Table VI shows the coefficients of the condition, weighted and normal equations. The y-swing method employs the translational adjustments by_2 and bz_2 (y_2 and z_2) in place of the κ_1 and ϕ_1 adjustments of the swing-swing method, whereas the κ_2, ϕ_2 and ω adjustments are used in both methods. Accordingly, the coefficients of the κ_2, ϕ_2, and ω terms of the two sets of condition equations are identical, and it remains to describe the effects of the y_2 and z_2 motions.

It is evident that the y-translation of a projector changes the y-parallax in all six places in the same way (Figure 8). If the parallax at point 1 is changed one unit, then the parallaxes at each of the other five points is changed one unit in the same direction. Thus the coefficients of the y terms of the condition equations are each unity.

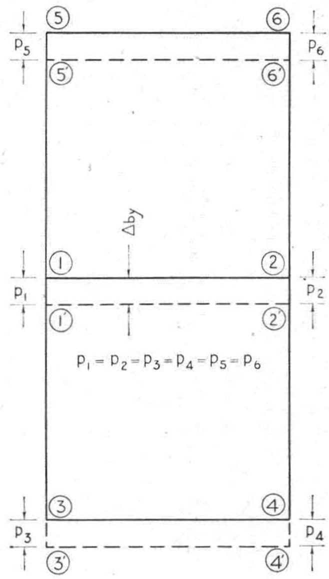

FIG. 8. Plan view showing the effects of the y-translation (by) motion of either projector on the parallaxes.

TABLE VI

THE EQUATIONS OF THE y-SWING METHOD

Condition Equations

$$\left\{\begin{array}{l}
\kappa_2+0\ +3/4\omega_2+y_2+0\ =p_1\\
0\ +0\ +3/4\omega_2+y_2+0\ =p_2\\
\kappa_2+\phi_2+\quad \omega_2+y_2+z_2=p_3\\
0\ +0\ +\quad \omega_2+y_2+z_2=p_4\\
\kappa_2-\phi_2+\quad \omega_2+y_2-z_2=p_5\\
0\ +0\ +\quad \omega_2+y_2-z_2=p_6
\end{array}\right.$$

Weighted Equations

$$\left\{\begin{array}{rrrrr}
\sqrt{2}\kappa_2+ & 0 & 3/4\sqrt{2}\omega_2+\sqrt{2}y_2 & 0 = & \sqrt{2}p_1\\
0\ + & 0 & 3/4\sqrt{2}\omega_2+\sqrt{2}y_2 & 0 = & \sqrt{2}p_2\\
\kappa_2+ & \phi_2+ & \omega_2+ & y_2+z_2= & p_3\\
0\ + & 0\ + & \omega_2+ & y_2+z_2= & p_4\\
\kappa_2 & -\phi_2+ & \omega_2+ & y_2-z_2= & p_5\\
0\ + & 0\ + & \omega_2+ & y_2-z_2= & p_6
\end{array}\right.$$

Normal Equations

$$\left\{\begin{array}{l}
4\ \kappa_2+\ 0\ +3\tfrac{1}{2}\omega_2+4y_2+\ 0\ =2p_1+p_3+p_5\\
0\quad +2\phi_2+0\quad +0\ \ +2z_2=p_3-p_5\\
3\tfrac{1}{2}\kappa_2+\ 0\ +6\tfrac{1}{4}\omega_2+7y_2+0\ =3/2p_1+3/2p_2+p_3+p_4+p_5+p_6\\
4\ \kappa_2+0\ \ +7\ \omega_2+8y_2+0\ =2p_1+2p_2+p_3+p_4+p_5+p_6\\
0\quad +2\phi_2+0\quad +0\ \ +4z_2=p_3+p_4-p_5-p_6
\end{array}\right.$$

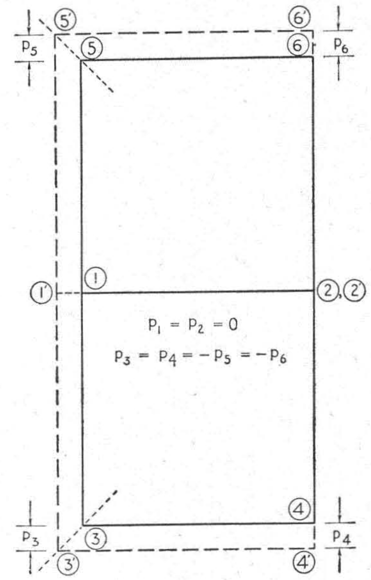

FIG. 9. Plan view showing the effects of the z-translation (bz) of the right projector on the parallaxes.

The bz motion has the effect of enlarging or reducing the pattern of images, causing them to move radially toward or away from point 2 beneath the projector (Figure 9). At point 2, therefore, the parallax is unaffected. At point 1 the parallax is also unaffected because the radial direction of image movement is entirely in the x-direction, the y-component being zero. The effects on the parallax points 3 and 4 are identical to each other, although their resultant radial movements differ. Moreover, the parallaxes at 5 and 6 are numerically equal to that at 3 and 4 but are in opposite directions. Thus the column of coefficients of the z_2 term of the condition equations can be expressed as (0, 0, +1, +1, −1, −1).

The solution of the equations for the y-swing method does not involve any different principles. The solution, with signs reversed so that the results will be corrections, is as follows:

$$\begin{aligned}
\Delta\kappa_2 &= -\tfrac{1}{2}p_1 + \tfrac{1}{2}p_2 - \tfrac{1}{4}p_3 + \tfrac{1}{4}p_4 - \tfrac{1}{4}p_5 + \tfrac{1}{4}p_6\\
\Delta\phi_2 &= -\tfrac{1}{2}p_3 + \tfrac{1}{2}p_4 + \tfrac{1}{2}p_5 - \tfrac{1}{2}p_6\\
\Delta\omega &= +2p_1 + 2p_2 - p_3 - p_4 - p_5 - p_6\\
\Delta y_2 &= -1\tfrac{3}{4}p_1 - 2\tfrac{1}{4}p_2 + \tfrac{7}{8}p_3 + \tfrac{5}{8}p_4 + \tfrac{7}{8}p_5 + \tfrac{5}{8}p_6\\
\Delta z_2 &= -\tfrac{1}{2}p_4 + \tfrac{1}{2}p_6
\end{aligned}$$

6. REFERENCES

[1] Weele, van der, "La Method Numerique d'Orientation Relative Appliquee a la Triangulation Aerienne." leaflet, Delft, Netherlands, 1951.

[2] Hallert, Bertil, "Contribution to Theory of Errors for Double Point Intersection in Space," Ab. Henrik Lindstahls Bokhandel I Distribution, Stockholm, 1950.

[3] Zeller, M., "Text Book of Photogrammetry," H. K. Lewis & Co., Ltd., London, page 189, 1952.

[4] Brandenberger, A., "Die Genauigkeit der gegenseitigen Orientierung," *Schweizerischen Zeitshrift für Vermessung und Kulturtechnik*, No. 8, January 1948.

[5] Bachmann, W. K., "Theorie des Erreurs de l'Orientation Relative," These Ecole d'ingenieurs de Lausanne, 1943.

[6] Krames, M. Josef, "About a New Graphic Method for Orienting a Pair of Aerial Photographs," PHOTOGRAMMETRIC ENGINEERING, vol. XVI, No. 4, page 556, September 1950.

[7] Poivilliers, Georges, "Method de Formation de l'Image Plastique, etc., *Proceedings of Academie des Sciences*, Paris, seances des 4, 9, 19, 23, June 1952.

[8] American Society of Photogrammetry, MANUAL OF PHOTOGRAMMETRY, Second edition 1952.

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