A SIMPLIFIED ANALYTICAL METHOD FOR THE ADJUSTMENT OF AERIAL TRIANGULATION

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ABSTRACT

An analytical method of adjustment of individual aerotriangulated strips is presented. The assumption is made that the errors in the x, y, and z coordinates of points as established by the photogrammetric instrument are second degree functions of x and y only. Based on this assumption a series of condition equations is obtained and by the application of the method of least squares, the values of coefficients which give the closest agreement with the data are computed. Checks on the computation are provided throughout the solution. The exactness of the assumption is indicated by the magnitude of the residual error in coordinates of control points. Similarly, unreliable points of control are indicated by excessively large residuals. The solution of an example is included.

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THIS method, which has been used in connection with Wild A5 and A6 stereo-plotting machines, was developed in an endeavor to retain as many as possible of the advantages of the analytical method without incurring the corresponding disadvantage of excessively lengthy computation. It is capable of being quickly learned by even an inexperienced computer, and adequate checks on the accuracy of the numerical work are easily applied where not automatically provided. It has the advantage over the graphical method formerly used, that no control point is accepted as more reliable than another, equal weight being given to all; and an indication of the relative reliability of the individual control points is furnished by the table of residual errors obtained.

When the stereo-plotting machine is used to measure co-ordinates and heights of control points in a strip of photographs, it is first set up by means of the control points on the first overlap of the strip. This initial set-up, because of small errors in control co-ordinates, identification of points, or photograph distortion, may not be exactly correct; and as the measurement proceeds along further overlaps in the strip, various sources of error combine to cause the measured positions of points to depart from their true values. It is the aim of this method of adjustment to compute a set of coefficients which, upon being applied to the position of any point as measured in the machine, will convert it as nearly as possible to the true value in terms of the control points supplied as data.

The co-ordinates of the control points are specified in three dimensions—easting, northing, and height above datum, so that it is convenient to resolve the errors in machine readings into the same three components. As the strips usually run from west to east, the west-east line is taken as the axis of x, with eastings positive; in the y direction northings are positive, and heights are denoted by the symbol h.

The assumption is made that errors in the machine readings are functions of x and y only, and of the second degree; which may be expressed symbolically as:

$$\text{correction} = ax^2 + bxy + cy^2 + dx + ey + f.$$

No term in h has been included in this equation as it has been found by trial that, under the conditions obtaining in this country, the inclusion of such terms has materially increased the computational work without any significant improvement in results. If, however, under different conditions, it is desired to in-

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Control Point	Eastings (x)					Northings (y)					Heights (h)				
	Control	Plot	Error $P-C$	Corr	Residual	Control	Plot	Error $P-C$	Corr	Residual	Control	Plot	Error $P-C$	Corr	Residual
88/4	353236	353237	+1	+2	+3	465925	465926	+1	−1	0	1523	1523	0	−2	−2
XV	353262	353262	0	+1	+1	468714	468714	0	+2	+2	1201	1200	−1	0	−1
88/5	353275	353274	−1	+2	+1	465850	465849	−1	0	−1	1535	1532	−3	0	−3
88/1	353355	353353	−2	+2	0	467043	467042	−1	−4	−5	1342	1354	+8	−14	−6
8572	353976	353974	−2	0	−2	466096	466103	+7	−6	+1	1939	1934	−5	+9	+4
88/3	354385	354380	−5	0	−5	466758	466775	+17	−10	+7	1406	1408	+2	+11	+13
8575	356656	356649	−7	+6	−1	467909	467926	+17	−18	−1	1709	1600	−109	+109	0
8574	357152	357150	−2	+5	+3	467278	467294	+16	−22	−6	2181	2042	−139	+134	−5
94/1	359803	359770	−33	+32	−1	468763	468776	+13	−19	−6	1004	655	−349	+357	+8
94/4	360127	360121	−6	+5	−1	465558	465591	+33	−33	0	1802	1416	−384	+384	−2
85810	360158	360144	−14	+16	+2	466574	466610	+36	−33	+3	1737	1365	−372	+371	−1
94/2	360494	360463	−31	+33	+2	467992	468032	+40	−27	+13	1022	599	−423	+408	−15
94/3	361225	361192	−33	+31	−2	467106	467131	+25	−33	−8	881	414	−467	+477	+10

Initial $x=353000$, $y=465000$

y	y^2	x	xy	x^2	$\Sigma+1$
0.93	0.86	0.24	0.22	0.06	+ 5.31
3.71	13.76	0.26	0.96	0.07	+ 18.76
0.85	0.72	0.27	0.23	0.07	− 1.86
2.04	4.16	0.35	0.71	0.12	+ 13.38
1.10	1.21	0.97	1.07	0.94	+ 6.29
1.78	3.17	1.38	2.46	1.90	+ 25.69
2.93	8.58	3.65	10.69	13.32	− 58.83
2.29	5.24	4.15	9.50	17.22	− 85.60
3.78	14.29	6.77	25.59	45.83	−271.74
0.59	0.35	7.12	4.20	50.69	−295.05
1.61	2.59	7.14	11.50	50.98	−275.18
3.03	9.18	7.46	22.60	55.65	−315.08
2.13	4.54	8.19	17.44	67.08	−374.62

	1	2	3	4	5	6	A.T. (*x*)	A.T. (*y*)	A.T. (*h*)	Σ
	13	26.77	68.65	47.95	107.17	303.93	− 135	+ 203	− 2244	− 1608.53
1	+ 3.6056	68.65	200.94	107.17	287.44	680.22	− 355.23	+ 423.47	− 5051.84	− 3612.41
2	+ 7.4247	+ 3.6775	363.41	287.44	850.69	1833.34	−1041.32	+1059.30	−13735.04	− 9839.59
3	+ 19.0401	+ 16.1993	+ 3.3863	303.93	680.22	2123.42	− 911.04	+1241.48	− 15722.50	−11841.93
4	+ 13.2989	+ 2.2922	− 0.8577	+ 11.0036	1833.34	4711.37	−2416.49	+2607.59	− 35016.50	−26355.17
5	+ 29.7236	+ 18.1512	− 2.7425	+ 21.8992	+11.5449	15343.95	−6608.35	+8547.55	−113237.56	−86302.13
6	+ 84.2950	+ 14.7804	− 3.2712	+ 87.7625	+ 0.5749	+ 17.5087				
A.T. (*x*)	− 37.4423	− 21.0013	+ 3.4821	− 32.8959	−16.6678	− 13.3502				
A.T. (*y*)	+ 56.3021	+ 1.4803	−10.8311	+ 43.6258	− 6.7436	− 4.6022				
A.T. (*h*)	−622.3736	−117.1727	+ 3.8778	−651.9418	− 8.9089	−103.3126				
Σ	−446.1259	− 81.5931	− 6.9563	−520.5466	−20.2005	−103.7563				
x	+ 1.5280	+ 2.2198	− 0.6450	− 5.8898	+ 1.4058	+ 0.7625				
y	+ 7.5777	− 9.0053	+ 2.0918	− 7.1979	+ 0.5710	+ 0.2629				
h	+ 21.3011	− 35.5182	+ 7.7875	+ 11.2351	+ 0.4778	+ 5.9006				

clude further terms in the equation, it is a simple matter to increase the number of normal equations and expand the solution accordingly.

By substituting in the above equation the known values of the corrections and of x and y for each control point a series of condition equations is obtained, and by the application of the method of least squares the values of the six coefficients which give the closest agreement with the data are computed. It is, however, found that in this particular case the computation is facilitated by rearranging the order of the terms in the condition equations as follows:

$$\text{correction} = A + By + Cy^2 + Dx + Exy + Fx^2.$$

There will of course be a value of the correction, and a set of six coefficients, in each of the three dimensions, x, y, and h.

If the assumption, mentioned above, were exactly valid, the corrections at each control point would be equal and opposite to the error. In practice this is not so, and the magnitudes of the residuals (i.e. the algebraic sum of the error and correction at a control point) indicate how far this assumption departs from the truth. If a small number of points show a large residual error while the remainder are in reasonably good agreement, it will be obvious that these points are unreliable, and unless investigation reveals an error in the data, should be rejected. A fresh adjustment based on the remaining points should then be made, and should yield satisfactory results.

The data supplied consist of the correct co-ordinates (in three dimensions) of the control points, and the corresponding values as derived from the machine readings. In the case of the *A6* machine, a plot made by the machine, with the heights as read off written on the plot by the operator, is supplied, so that the horizontal co-ordinates have to be scaled off the plot. When the *A5* machine has been used the readings are supplied as a list on the special form used, together with a list of the respective control co-ordinates. As the machine readings are in millimeters, they must be converted into yards or feet, using the relation

$$1 \text{ mm.} = .0010936152 \text{ yard} = .0032808456 \text{ foot.}$$

Actually the use of an inexact conversion factor would not have an adverse effect upon the result of the adjustment, as the computed constants would be automatically affected and would exactly compensate for any error in scale.

Each line on the adjustment form used (example below) is provided for one control point and is a record of one condition equation. There is a separate section for each of the three dimensions x, y, and h, and in each section are entered (a) the control value for each point, (b) the value derived from the machine (called the plot value), and (c) the plot value minus the control value. This last quantity will be referred to as $(P-C)_x$, $(P-C)_y$, and $(P-C)_h$ respectively.

The values of x and y for each point are then inserted, these being the plot values reduced by arbitrary constants and expressed in units of 1,000 yards; two decimal places are shown. The subtraction of constants has the effect of bringing the axes of co-ordinates near to the points used, and is done to reduce the number of figures dealt with in the numerical work, and to maintain the stability of the normal equations. The constants are made as large as conveniently possible, thus bringing the axes of x and y close to the south-west of the control points, but negative values of x and y are avoided for reasons of convenience in working.

The values of y^2, xy and x^2 are computed and entered, rounded off to two places of decimals.

In the last space in each line the sum for check purposes is entered:

$$\sum+1=1+(P-C)_x+(P-C)_y+(P-C)_h+y+y^2+x+xy+x^2.$$

The six normal equations are now formed, according to the following table, the square brackets indicating summation:

$[1]$	$[y]$	$[y^2]$	$[x]$	$[xy]$	$[x^2]$	$[(P-C)_x]$	$[(P-C)_y]$	$[(P-C)_h]$	$[(\sum+1)]$
	$[y^2]$	$[y^3]$	$[xy]$	$[xy^2]$	$[x^2y]$	$[y(P-C)_x]$	$[y(P-C)_y]$	$[y(P-C)_h]$	$[y(\sum+1)]$
		$[y^4]$	$[xy^2]$	$[xy^3]$	$[x^2y^2]$	$[y^2(P-C)_x]$	$[y^2(P-C)_y]$	$[y^2(P-C)_h]$	$[y^2(\sum+1)]$
			$[x^2]$	$[x^2y]$	$[x^3]$	$[x(P-C)_x]$	$[x(P-C)_y]$	$[x(P-C)_h]$	$[x(\sum+1)]$
				$[x^2y^2]$	$[x^3y]$	$[xy(P-C)_x]$	$[xy(P-C)_y]$	$[xy(P-C)_h]$	$[xy(\sum+1)]$
					$[x^4]$	$[x^2(P-C)_x]$	$[x^2(P-C)_y]$	$[x^2(P-C)_h]$	$[x^2(\sum+1)]$

One set of equations applies to each of the dimensions x, y, and h, but the formation of the normal equations and their forward solution may be done in one operation, with three columns of absolute terms as shown. The last column is the usual check column to detect errors in any of the other columns. In the example the normal equations are solved by the Cholesky method, the three back solutions being done separately; these yield three sets of six coefficients, applying to the x, y, and h dimensions respectively.

The corrections for each control point are now computed from the equation as above:

$$\text{Correction}=A+By+Cy^2+Dx+Exy+Fx^2.$$

As the corrections are computed they are entered in the appropriate columns; the algebraic sum of the two quantities $(P-C)$ and correction in each case is entered as the residual.

The algebraic sum of the residuals in each dimension is taken as a check on the accuracy of the working; if it differs from zero by more than a few units in the last figure, an error is indicated. As the formation and forward solution of the normal equations are checked by the summation columns, any errors in the residuals can only be due to faulty computation of the corrections. It is found advisable to check the values of the constants immediately the back solution is completed, by substitution in the normal equations.

When the values of the residuals have been accepted as satisfactory—if necessary after a recomputation consequent on the rejection of unreliable control points—the constants A to F may be used to compute the corrections to the values of co-ordinates and heights derived from machine readings for other points, and so grid co-ordinates and heights above datum in terms of the control points are obtained.

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