SOME ANALYSIS AND ADJUSTMENT METHODS IN PLANIMETRIC AERIAL TRIANGULATION

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Introduction and Purpose

In the paper, "A Résumé of Aerial Triangulation Adjustment at Army Map Service," which appears in Photogrammetric Engineering, December, 1951, Mr. R. S. Brandt in part of his discussion describes some practical procedures for the adjustment of horizontal aerial triangulation. Since the publication of this paper, these methods have been somewhat altered at the Army Map Service.

Some variations in the treatment of the computations for the X error and azimuth error curves have been adopted. These variations have led to a new mathematical approach, which provides a more rigorous solution to the problem of separation of the components of total X error and total azimuth error for three or more noncollinear points in a triangulated strip. As used here, the terms, "total X" and "total azimuth" errors, are defined as the algebraic sum of the X scale and swing errors and the algebraic sum of the azimuth and Y scale errors, respectively.

The problem of component separation is common to the analysis of either graphical or coordinate horizontal triangulation; generally it is encountered when field control is distributed at random or asymmetrically within a rectangular coordinate system. This paper purports to present and briefly explain these newer technical methods for the determination of compensatory values in horizontal aerial triangulation.

X and Azimuth Curves

There has been no significant departure from the O. Von Gruber[1] theoretical equations for the accumulation of X and Y scale error. Professor Von Gruber's equation, $\Delta x = kX^2/2$, has been written, $\Delta x = kX^2$. Since $k/2$ is no more or less a constant than is k, the represented ordinate remains identical. Von Gruber, however, was concerned with the actual scale-change accumulation and assumed a true scale in the starting model with a resultant zero slope of the X curve at the zero abscissa. In practice, such an assumption is frequently not correct; since $\Delta x = kX^2$ is the equation of a true parabola, the assumption is not necessary. If three or more control points along the line of flight can be obtained, then the point-slope equation of a parabola $\Delta x = kX^2 + hX$ (where h is the slope of the curve at the zero abscissa) will, by simultaneous solution at the two control points, $X \neq 0$, give the proper curve for any angle of tangency at the starting point. Adhering to Von Gruber's conclusion that the Y scale error is the first derivative of the equation of the X error curve, then by differentiation the equation for Δy becomes $\Delta y = 2kX + h$. Here, Δy refers to the Y scale error in units per unit, and must of course be multiplied by the Y distance of the point being considered, from the chosen instrument X axis. As defined in this treatise, the instrument X axis is any line of constant ordinate in the instrument coordinate system. The X scale and azimuth error accumulations are assumed to be functions of the directed X distance parallel to this line.

A significant economy of computation has been achieved by treating the azimuth error with a parabolic equation identical to that of the X scale error. This treatment is not an argument against the conclusion of Mr. Brandt[2] that the azimuth error curve is probably a circle. The fact is that in practice the infinitesimal quality of the curvature constant is such that there is no significant difference between the ordinate of a parabola and that of a circle at any point. The assumption that the azimuth

[1] Von Gruber, O., Beitrag zu Theorie und Praxis von Aeropolygonierung und Aeronivellment, *Bildmessung und Luftbildwesen*, Number 3, 1935.

[2] Brandt, R. S., Résumé of Aerial Triangulation Adjustment at Army Map Service, Photogrammetric Engineering, Vol. XVII, Number 5, December, 1951.

error curve is the parabola $\Delta Az = AX^2 + BX$ leads to the use of its first derivative, $\Delta Sw = 2AX + B$ as the equation of the Swing curve.

The Component Separation Problem

While the accumulation of the X scale and the azimuth errors are functions of X, the error components of Y scale and Swing are functions of both X and Y. More specifically, the errors of Y scale and Swing are directly proportional to their Y distance from the instrument X axis. Therefore, when Δy and ΔSwing exist at any abscissa, the only point at which they are zero is a point zero distance from the X axis, or more simply, on the axis itself.

In the past the separation of the four components of horizontal error (see Figure 1), has been accomplished, where possible, by reducing ΔSwing and ΔY to zero by the method of interpolation of total X and total azimuth error values to their proper values at the axis. This can be done with a reasonable degree of approximation when there are control points of nearly equal abscissae, located on opposite sides of the instrument X axis. Under practical conditions, however, numerous cases are encountered where this method cannot be used. The practical problem is more generally represented in Figure 2 wherein a random distribution of control is given in an area of sparse ground control.

With points A and B distributed as in Figure 2, the axis may be placed at any ordinate between the two points by simple interpolation. However, the method of reducing ΔY and ΔSwing to zero by interpolating the proper total X and total azimuth error values to points on the axis at control points M and N, obviously cannot be applied. Under such conditions in the past, an approximation method has been employed, wherein a tentative X curve was drawn to the total X error values of the mean point A-B, point M and point N, considering the ΔSwing component to be zero, as if the points were actually located on the instrument axis. A correction for ΔY could then be computed from this tentative X curve and applied to points M and N, after which the final azimuth, Swing, X and Y curves could be

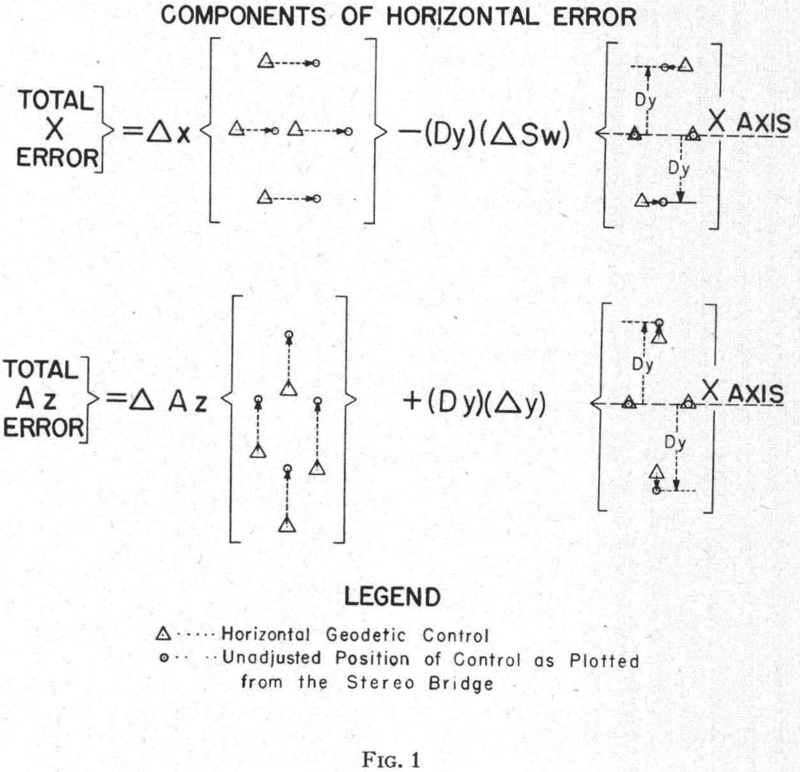

Fig. 1

ERROR EQUATIONS FOR ASYMMETRICAL CONTROL ALIGNMENT

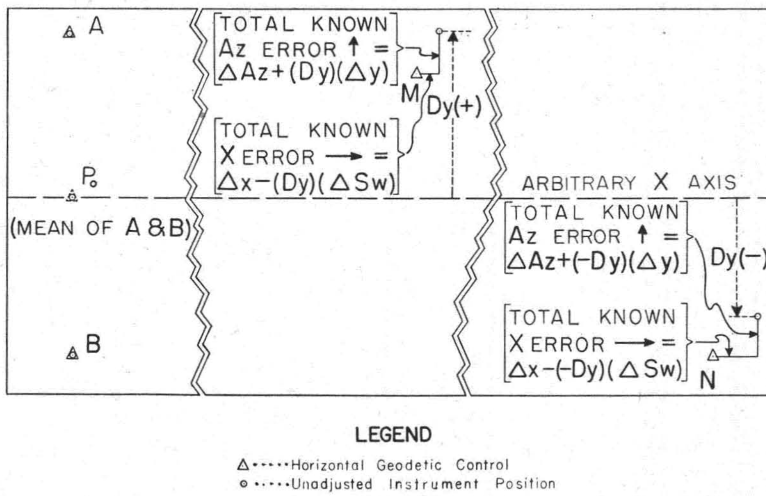

FIG. 2

computed in that order. As stated, this was an approximation method, which limited the precision of the adjustment, and in the case of coordinate triangulation, proved undesirably laborious.

A Solution to the Component Separation Problem

A mathematical method has been developed, which provides a rigorous solution for any case of noncollinearity of control points. From a practical accuracy standpoint, it is desirable that the points be located respectively at the two ends and somewhere near the middle of the triangulated strip, but this requirement is not essential to solution of the problem.

Assuming at the start that the X and azimuth error curves are parabolas, and that the Y scale and Swing curves are the first derivatives of the first two, then for any triangulated strip such as Figure 2, by adding the first derivative of the X curve to the equation of the azimuth curve, and conversely, subtracting the first derivative of the azimuth curve from the X curve equation, one may set up four simultaneous equations, each with four unknown coefficients of X. Using Figure 2 as a working model, this may be done in the following manner:

(1) known total X error at Pt. $M=kX^2+hX-(+Dy)(2AX+B)$

(2) known total X error at Pt. $N=kX^2+hX-(-Dy)(2AX+B)$

(3) known total Az error at Pt. $M=AX^2+BX+(+Dy)(2kX+h)$

(4) known total Az error at Pt. $N=AX^2+BX+(-Dy)(2kX+h)$

where k is the constant of curvature of the X error curve, h is the slope of the curve at zero abscissa, A is the constant of curvature of the azimuth error curve, B is the slope of the curve at zero abscissa, and Dy is the signed Y distance from the ordinate of the mean control point $A-B$, at zero abscissa.

The simultaneous solution of the foregoing equations will give the coefficients of the error curves from which corrections to the coordinates of each photo tie point can be readily computed. It should be noted in this discussion of error components that the algebraic sign of the error rather than that of the correction has been used throughout. For correction, a reversal of signs of the final values is necessary. Also, in coordinate triangulation care must be exercised in handling the equations in cases where the instrument coordinates and the true X and/or Y coordinates increase in opposite directions.

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