SIMPLIFIED GRAPHS AND TABLES FOR USE WITH OPTICAL RECTIFIERS

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ABSTRACT

The author presents the standard rectification computations in a form intended to be convenient for operational use. Analytical procedures outlined are applicable to the Bausch and Lomb autofocus and the Fairchild RP-II rectifiers. Limiting values of the elements of rectification recoverable in these instruments may be interpolated from the graphs and tables.

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THIS material is presented in order to show the simplicity with which one can answer the agelong questions "How much tilt will the rectifier accommodate?" and "How can computations be simplified for use in production?"

Nothing new is included in this paper because the derivations and initial applications were presented long ago by such men as Abbe, Taylor, Scheimpflug, von Gruber, and many others who are well known in the field of photogrammetry. However, the use of modern equipment, such as the typewriter and calculator, justifies the rearrangement of the available material for production use.

In some quarters nomograms are considered the simplest form of presenting material involving numerous variables. But since this paper is not intended to make a comparison between methods, this and other comparisons are left to the reader.

To define some of the terms herein used the well known diagram showing the relationship of the negative plane to the lens and map planes is given in Figure 1.

Where:

- p, i, n, are the principal point, iso-center, and nadir points respectively on the negative plane,
- P, I, N, are the principal point, iso-center, and nadir points respectively on the map plane,
- f is the effective focal length of the negative,
- h is the flying height above the map plane, and
- t is the tilt of the negative along the principal line (the principal line connects, p, i, n, and can be projected as a line through the lens to P, I, N).

In most cases the scale, or its component h is difficult to determine for the tilted photograph. Also, the large numbers one must deal with in using any map, chart or photograph prove to be somewhat clumsy to handle and the procedures more difficult to follow. For these reasons it is advisable to use the ratios of scales in working with photographs for rectification.

So in Figure 1, it can be seen that the ratios Rp, Ri, and Rn for the points on the negative p, i, n, are as follows:

$$Rp = \frac{h/\cos t}{f} = \frac{h}{f \cos t}$$

$$Ri = \frac{\dfrac{h}{\cos t/2}}{\dfrac{f}{\cos t/2}} = \frac{h}{f}$$

$$Rn = \frac{h}{f/\cos t} = \frac{h \cos t}{f}.$$

These ratios can be found for the three definite and most important points on the photograph. However, the principal point is easily found by merely using the fiducial marks on the photograph. Therefore, it is advisable to use the ratio Rp at the principal point when working with photographs.

Using the formulas which satisfy the Scheimpflug condition, simplifications can be made which expedite computations. Thus the formulas

$$\frac{F}{f} \sin t = \sin \beta$$

$$\frac{F}{h} \sin t = \sin \alpha$$

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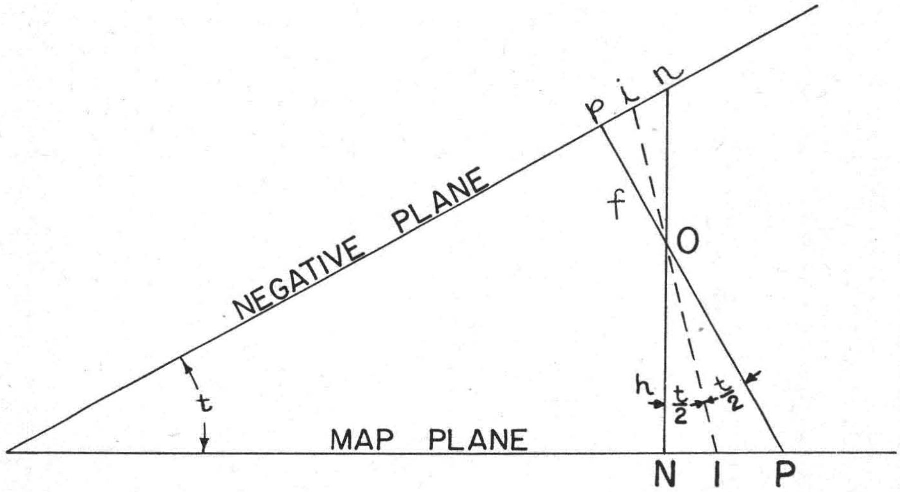

FIG. 1. Relationship of Aerial Negative to Map Plane.

where, F is the calibrated focal length of the rectifier, f is the effective focal length of the negative, t is the air tilt angle, β is the easel tilt angle, α is the negative tilt angle, and h is the flying height above the map plane in the same units as F and f (i.e. inches, mm., ft., etc. See Figure 1) can be simplified by substituting K for Ff when working with constant focal lengths and using $Rp = h/(f \cos t)$ in the form $h = f(Rp) \cos t$.

These two formulas then become

$$K \sin t = \sin \beta$$

and

$$\frac{K \sin t}{Rp \cos t} = \frac{K}{Rp} \tan t = \sin \alpha.$$

It should be noted that h is eliminated and that $(K/Rp) \tan t$ is a simple three step calculator problem for finding $\sin \alpha$. Also since h/f equals Ri, $(K \sin t)/Ri = \sin \alpha$. For high obliques then, it may be simpler to use $(\sin \beta)/Ri = \sin \alpha$. The choice is left to the individual.

AUTO-FOCUS AND FIXED LENS RECTIFIER

Since angles α and β are used for the internal geometry of all rectification problems let us look at two different rectifiers and see what other simplifications may apply. Figure 2 is a schematic diagram of the conditions necessary to rectify and preserve focus in any auto-focus or fixed lens rectifier.

It can be readily seen that the object distance divided by the image distance is the ratio (Rr) on the rectifier lens axis. Also, in Figure 2 $B/\overline{O'S}$ and $A/\overline{O'S}$ are the tangents of the angles β and α respectively, and the following is true:

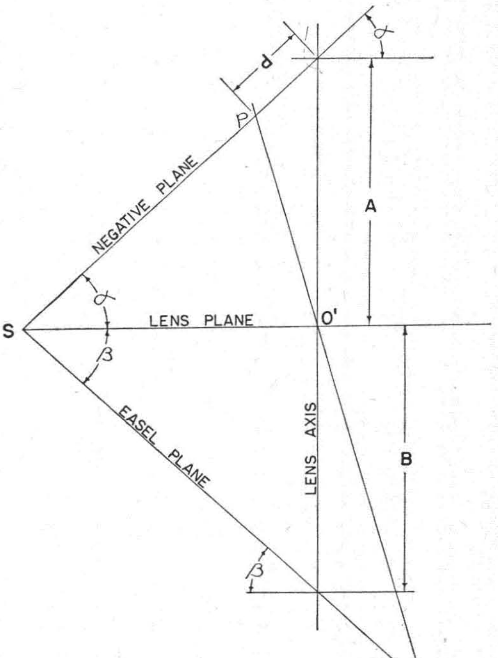

FIG. 2. Fixed-Lens and Auto-Focus Rectifier Diagram.

$$Rr = \frac{B}{A} = \frac{B/\overline{O'S}}{A/\overline{O'S}} = \frac{\tan\beta}{\tan\alpha}.$$

In using this formula the lens law

$$\left(\frac{1}{F} = \frac{1}{B} + \frac{1}{A}\right)$$

is rearranged for convenience and simplified. It can be shown that

$$A = F + \frac{F}{M} + F\left(1 + \frac{1}{M}\right)$$

and $B = F + FM = F(1+M)$ where M is the magnification B/A.

Because measurement is made on the lens axis, M equals Rr; and the formulas to use for computations are $A = F(1+1/Rr)$ and $B = F(1+Rr)$. Having found sin β and sin α previously, trigonometry tables will provide the tangents.

Thus Rr is readily found using $\tan\beta/\tan\alpha = Rr$. ($Rr$ is the ratio set up on the rectifier and should not be confused with Ri, Rn, or Rp which are the negative (or positive) to map plane ratios.)

Having satisfied the conditions for focus on the lens axis using $B = F(1+Rr)$ and $A = F(1+1/Rr)$, and also having satisfied the Scheimpflug condition of focus by intersecting the planes as shown in Figure 2 using the formulas $K \sin t = \sin\beta$ and $K/Rp\ (\tan t) = \sin\alpha$, a third condition must be satisfied. This is not a condition of focus but of perspective around the rectifier lens. It is accomplished by off-setting the negative in fixed lens rectifiers.

In Figure 2 it is shown that the principal point p is moved down the negative plane and thus down the easel plane and that this offset is denoted by a distance d. When the negative is offset down the easel it is computed as a minus number when using the familiar formula

$$d = f/\tan t - F/\cos\alpha \tan\beta.$$

Using Figure 3 which represents the upper part of Figure 2, it can readily be seen that a simpler form of the displacement formula is as follows:

$$d = \frac{A - A'}{\tan\alpha}.$$

It can also be seen that tan α had been previously found to determine Rr, A had been previously computed using $A = F(1+1/Rr)$, and that A' can be found using $A' = F(1+1/Rp)$. Placing A in a calculator and subtracting A' leaving the re-

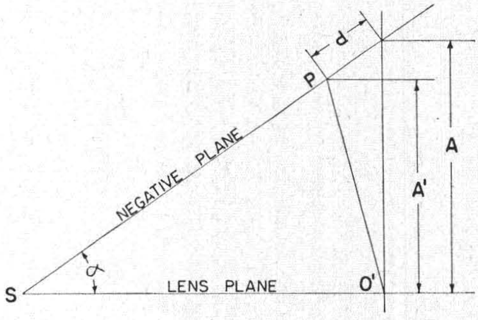

FIG. 3. Tilting-Lens Rectifier Diagram.

mainder in the machine and dividing it by the tan α gives the result d. However, care should be exercised in rounding off the two image distances to avoid serious errors, especially for low α (negative tilt) angles.

Close observation may bring a belief that nothing is saved in this off-set formula because there must first be computed the separate image distances A and A'. However, if one works consistently with a rectifier and must do volume work, it is a time saver to compute tables for that rectifier, such as shown in Table 1. Once these tables are computed, they can be typed in a standard form which can be bound in a permanent folder. Then no image or object distance need ever be computed again for that particular fixed-lens rectifier.

To summarize this material for autofocus and fixed lens rectifiers, the equipment and material required for the minimum of time and effort expended are as follows:

Calculator (or slide rule depending upon the accuracy desired)

Trigonometry Tables (6 or 7 place depending on accuracy desired)

Typewritten Tables for Image and Object Distances (one set for each rectifier so that individual adjustments can be made throughout to compensate for the variations within the linkages of the machine. The same table can be used for Rp and Rr).

Formulas:

$$K \sin t = \sin\beta$$

$$\frac{K}{Rp} \tan t = \sin\alpha$$

$$d = \frac{A - A'}{\tan\alpha}$$

$$\frac{\tan\beta}{\tan\alpha} = Rr.$$

TABLE 1. PORTION OF A PAGE OF TYPEWRITTEN TABLES
B & L
$F = 138.5$ mm.

Ratio	Image	Object	Ratio	Image	Object	Ratio	Image	Object
3.940	174.0	682.4	3.990	173.6	689.2	4.040	173.1	696.1
1	174.0	682.5	1	173.6	689.4	1	173.1	696.2
2	174.0	682.6	2	173.6	689.5	2	173.1	696.4
3	174.0	682.8	3	173.6	689.6	3	173.1	696.5
4	173.9	682.9	4	173.5	689.8	4	173.1	696.7
5	173.9	683.1	5	173.5	689.9	5	173.1	696.8
6	173.9	683.2	6	173.5	690.0	6	173.1	696.9
7	173.9	683.3	7	173.5	690.2	7	173.1	697.1
8	173.9	683.5	8	173.5	690.3	8	173.1	697.2
9	173.9	683.6	9	173.5	690.5	9	173.1	697.4
3.950	173.9	683.8	4.000	173.5	690.6	4.050	173.1	697.5
1	173.9	683.9	1	173.5	690.7	1	173.0	697.6
2	173.9	684.0	2	173.5	690.9	2	173.0	697.8
3	173.9	684.2	3	173.5	691.0	3	173.0	697.9
4	173.9	684.3	4	173.5	691.2	4	173.0	698.0
5	173.9	684.4	5	173.5	691.3	5	173.0	698.2
6	173.9	684.6	6	173.4	691.4	6	173.0	698.3
7	173.8	684.7	7	173.4	691.6	7	173.0	698.5
8	173.8	684.8	8	173.4	691.7	8	173.0	698.6

Rp can be computed on the contact print simultaneously with the ratios computed for the tilt of the plane of the photograph.

Figure 5 is a graph for the Bausch and Lomb auto-focus 20° rectifier. It shows the limits of rectification for a number of different focal length negatives. Some of these focal lengths do not represent "calibrated" camera focal lengths but rather "effective" or "equivalent" focal lengths of reduced negatives. In Table 2—a supplement to Figure 5—are given the absolute limits of reductions.

TILTING LENS RECTIFIERS

The tilting lens rectifiers, which are not auto-focus, require material and consideration similar to that for the auto-focus rectifiers because the internal geometry is identical. That is, the negative and easel planes are tilted the same way with respect to the lens plane as they are in the auto-focus set-up; but because the lens axis is tilted off the measuring axis, different distances and angles must be set on the rectifier.

The similarity can best be seen when Figure 4 is compared with Figure 2.

Angles β and α are found in the same way as before using

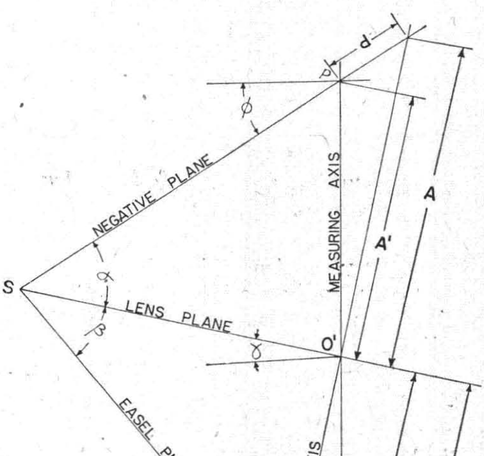

FIG. 4. Relationship of Aerial Negatives and Rectifier Lens Diagram.

TABLE 2. BAUSCH & LOMB RECTIFYING PROJECTION PRINTER
SUPPLEMENT TO FIG. 5
TABLE OF ABSOLUTE LIMITS OF REDUCTIONS

t°\f	6″	8.25″	11.1″	12″	14.9″	18″	20″	24″	36″	48″
1	.550	.550	.550	.550	.550	.550	.550	.550	.550	.550
2	.551	.550	.550	.550	.550	.550	.550	.550	.550	.550
3	.552	.551	.551	.551	.551	.551	.551	.551	.551	.551
4	.554	.553	.552	.552	.552	.552	.552	.552	.551	.551
5	.556	.554	.553	.553	.553	.553	.553	.553	.552	.552
6	.559	.556	.555	.554	.554	.554	.554	.554	.553	.553
7	.562	.558	.556	.556	.555	.555	.555	.555	.554	.554
8	.565	.561	.558	.558	.557	.557	.556	.556	.555	7°17′ .555
9	.569	.564	.560	.560	.559	.558	.558	.558	.557	
10	.574	.567	.563	.563	.561	.560	.560	.559	9°31′ .558	
11	.579	.570	.566	.566	.563	.562	.562	.561		
12	.584	.574	.569	.568	.566	.565	.565	.564		
13	.590	.578	.572	.571	.569	.567	.567	.566		
14	.597	.583	.576	.574	.572	.570	.570	13°27′ .567		
15	.604	.588	.580	.578	.575	.573	.573			
16	.611	.593	.584	.582	.578	.577	15°26′ .574			
17	.619	.599	.588	.586	.582	16°37′ .579				
18	.627	.605	.593	.591	.586					
19	.636	.611	.598	.596	18°39′ .589					
20	.645	.617	.603	.601						
21	.655	.625	.609	20°44′ .605						
22	.665	.632	21°18′ .611							
	22″09′ .667	22°35′ .637								

$$K \sin t = \sin \beta$$

$$\frac{K}{Rp} \tan t = \sin \alpha.$$

Rr is still equal to $\tan \beta / \tan \alpha$ *but not required* and Rp is found on the contact print.

But these angles and ratio cannot be used to set up the rectifier because the lens axis is tilted off the measuring axis on the rectifier.

The next set of formulas seem complex at first glance but a little study shows simplification. Again without deriving formulas the basic equations are:

$$\phi = \tfrac{1}{2}(\alpha + \beta) - x \text{ negative tilt angle}$$

$$\psi = \tfrac{1}{2}(\alpha + \beta) + x \text{ easel tilt angle}$$

where

$$\tan x = \frac{\tan^2 (t/2)}{\tan \frac{1}{2}(\alpha + \beta)}.$$

Finding x is a three-step calculator problem easily computed. The angles ϕ and ψ can then be found. With the angles α, β, ϕ, and ψ the angle γ (lens tilt) can be found using $\gamma = \alpha - \phi$, and $\gamma = \psi - \beta$ as a check in subtracting angles $\alpha - \phi$. This however does not check the angle x computation.

To find the negative to lens and lens to easel distances, it is necessary to find the image and object distances set up as a table (Table 1) and divide them by cosine γ. In formula form

$$O'p = \frac{A'}{\cos \gamma}$$

negative to lens distance and

$$O'P' = \frac{B'}{\cos \gamma}$$

easel to lens distance.

To summarize the formulas for the tilting lens rectifiers it is perhaps best to list them in chronological order as follows:

$$K \sin t = \sin \beta$$

$$\frac{K}{Rp} \tan t = \sin \alpha$$

$$\tan x = \frac{\tan^2 (t/2)}{\tan \frac{1}{2}(\alpha + \beta)}$$

$$\phi = \tfrac{1}{2}(\alpha + \beta) - x$$

$$\psi = \tfrac{1}{2}(\alpha + \beta) + x$$

$$\gamma = \alpha - \phi$$

$$\gamma = \psi - \beta$$

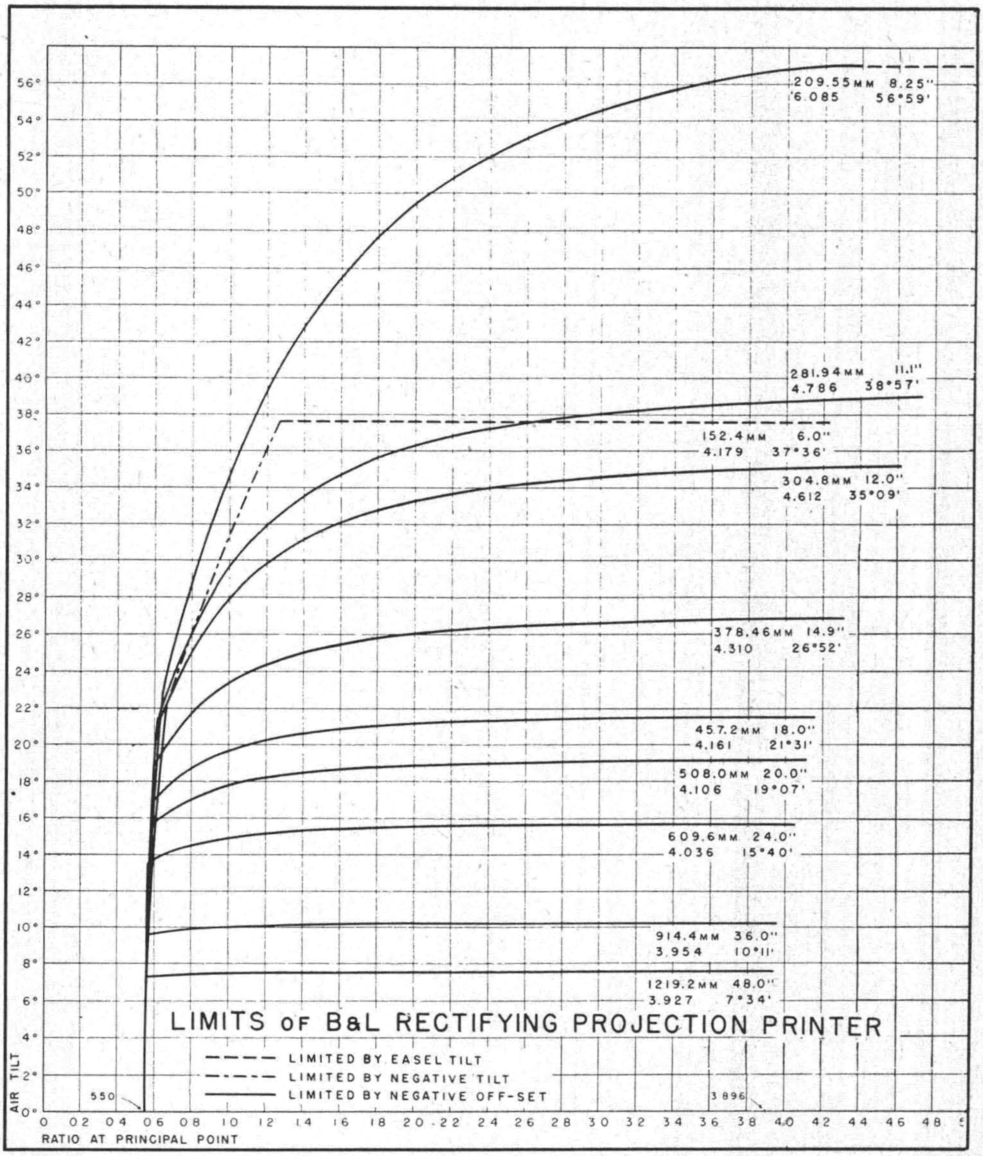

FIG. 5. Limits of Bausch & Lomb Rectifying Projection Printer.

$$O'p = \frac{A'}{\cos \gamma}$$

$$O'P' = \frac{B'}{\cos \gamma}.$$

Figure 6 is a graph for the Fairchild rectifier which can be used in several different ways. Table 3 supplements Figure 6. The limits of only two ways have been graphed to give comparisons both within the Fairchild and with the B & L auto-focus rectifier limits shown in Figure 5.

For those who wish to compare their computations with those graphed in Figures 5 and 6, the B & L rectifier lens focal length is 136 mm. and the Fairchild RP-II focal length is 275.6 mm.

The presentation of this material is just another example of an old problem which can be answered in several ways. For this reason no conclusions will be drawn. But perhaps a few results may be apparent.

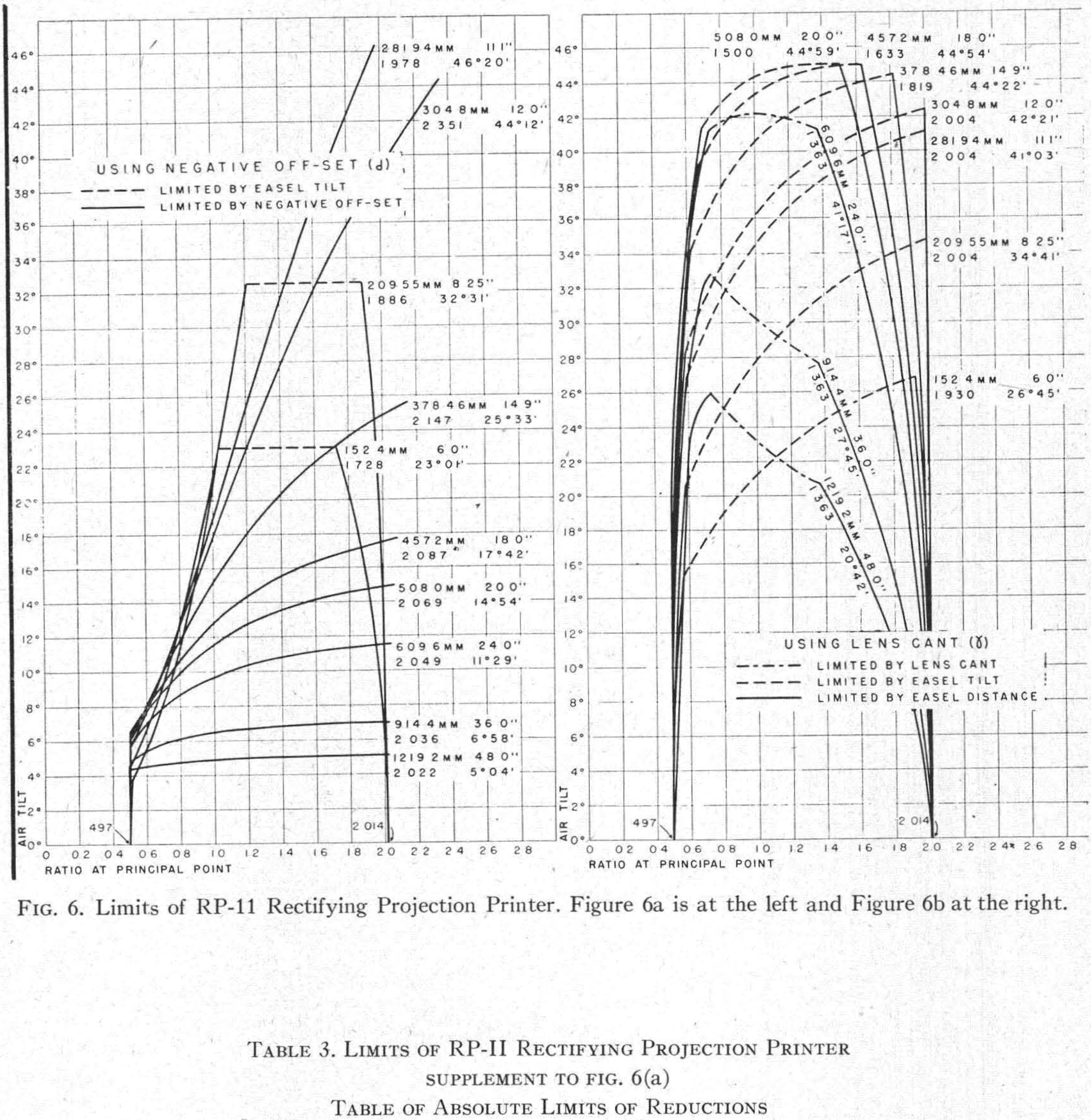

FIG. 6. Limits of RP-11 Rectifying Projection Printer. Figure 6a is at the left and Figure 6b at the right.

TABLE 3. LIMITS OF RP-II RECTIFYING PROJECTION PRINTER
SUPPLEMENT TO FIG. 6(a)
TABLE OF ABSOLUTE LIMITS OF REDUCTIONS

t°\f	6″	8.25″	11.1″	12″	14.9″	18″	20″	24″	36″	48″
1	.501	.500	.500	.500	.499	.498	.499	.497	.497	.497
2	.505	.501	.501	.501	.500	.499	.500	.498	.497	.497
3	.510	.503	.503	.503	.501	.500	.501	.499	.498	.497
4	3°42′ .511	.504	.504	.504	.502	.501	.502	.500	.499	.497
5		4°39′ .505	.505	.505	.503	.501	.502	.501	4°46′ .499	4°20′ .497
6			5°38′ .506	.506	.504	.502	.503			
				6°06′ .507	6°03′ .504	6°30′ .502	6°16′ .503	5°36′ .501		

SUPPLEMENT TO FIGURE 6 (b)

TABLE OF ABSOLUTE LIMITS OF REDUCTIONS

t°\f	6″	8.25″	11.1″	12″	14.9″	18″	20″	24″	36″	48″
1	.497	.497	.497	.497	.497	.497	.497	.497	.497	.503
2	.498	.498	.498	.497	.497	.497	.497	.497	.498	.508
3	.499	.499	.499	.498	.497	.497	.497	.497	.499	.513
4	.501	.501	.500	.499	.498	.498	.497	.498	.500	.518
5	.502	.503	.501	.499	.498	.498	.497	.498	.501	.523
6	.504	.505	.502	.500	.499	.499	.497	.499	.502	.528
7	.507	.508	.503	.501	.499	.499	.497	.499	.503	.533
8	.512	.511	.504	.503	.500	.500	.497	.500	.504	.539
9	.517	.514	.505	.504	.501	.501	.498	.501	.506	.545
10	.523	.518	.506	.505	.502	.502	.498	.502	.508	.551
11	.529	.522	.508	.506	.503	.503	.498	.503	.511	.557
12	.537	.526	.510	.508	.504	.504	.498	.504	.514	.563
13	.546	.530	.513	.510	.505	.505	.498	.505	.517	.569
14	.556	.535	.517	.513	.507	.507	.498	.507	.522	.575
15	.567	.540	.521	.516	.509	.509	.498	.509	.527	.580
16	15°11′ .571	.545	.525	.519	.512	.512	.499	.512	.533	.585
17		.551	.530	.521	.515	.515	.499	.515	.539	.591
18		.558	.535	.525	.518	.518	.499	.518	.545	.597
19		.566	.540	.529	.522	.522	.500	.522	.551	.603
20		.574	.546	.533	.526	.526	.501	.526	.557	.609
21		20°34′ .580	.552	.539	.530	.530	.501	.530	.563	.615
22			.558	.544	.535	.535	.502	.535	.569	.621
23			.565	.551	.540	.540	.506	.540	.575	.627
24			.573	.558	.546	.546	.511	.546	.583	.634
25			.581	.564	.552	.552	.516	.552	.590	.672
26			.589	.572	.559	.559	.522	.559	.598	.731
27			26°56′ .597	.578	.566	.566	.529	.566	.608	26°02′ .734
28				.586	.573	.573	.537	.573	.620	
29				28°18 .589	.580	.581	.545	.581	.635	
30					.588	.589	.554	.589	.652	
31					.596	.597	.563	.597	.673	
32					.605	.606	.573	.606	.698	
33					.614	.615	.584	.615	32°46′ .734	
34					.627	.624	.595	.624		
35					34°20′ .631	.633	.606	.633		
36						.642	.619	.642		
37						.651	.632	.652		
38						.660	.646	.662		
39						.669	.660	.682		
40						39°04′ .670	.675	.706		
41							.690	.730		
							41°32′ .705	41°10′ .734		

If this material

helps to expedite production,

may be used as a rule of thumb or as an expeditions training aid,

gives adequate knowledge about a photograph before entering the dark room,

gives results within required tolerances;

then the time required for application is well spent.