DETERMINATION, BY THE AID OF AN X-RAY STEREOSCOPIC METHOD, OF VOLUME VARIATIONS OF THE LIVER OF ANIMALS

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THIS paper describes the method used in determining the volume variations. The animal to be experimented on is operated on and pieces of metal are fixed in its liver in such a way that a polyhedron is obtained—the metal objects constituting its corners—and so that this polyhedron confines that part of the liver, which is to be used as a volume of reference.

Stereoscopical X-ray photographs of the animal are then taken by simultaneously releasing two X-ray tubes with known position in relation to the X-ray film. There will be a double reproduction of each piece of metal. The double exposure will of course make an interpretation of the rest of the picture impossible. However, it is only the configuration in space of the pieces of metal that is of interest, and this can be determined by measuring the image co-ordinates and by knowing the position of the X-ray tubes.

The volume of the polyhedron of reference is then computed from the co-ordinates of the metal pieces. Variations in volume between successive exposures can also be directly determined from differences in the co-ordinate computations between different X-ray photographs, the variations in the liver and the deplacements of the animals in relation to the X-ray apparatus are moderate. In practice, this can be done as is described below.

Two X-ray tubes which can be simultaneously released are firmly joined by a mechanical connection, so that the distance between the foci is horizontal and known. Under these a table is set upon which the animal to be experimented on is placed. The table top must be penetrable by X-rays. Under the table top, there is a frame for the X-ray magazine. The frame is provided with two marks, which are reproduced on the X-ray photograph. The dictance between these marks is equal to the distance between the foci. By the aid of weights, hanging from the X-ray tubes, the table is placed with the marks vertically under the foci. At the same time the table can be levelled according to a water-level by means of foot-screws, so that the X-ray magazine is horizontal.

Knowledge of the height of the tube over the magazine is required. This is roughly obtained by measuring the distance externally. The value obtained is corrected by means of small balls of metal, whose height over the magazine is known. The balls are fixed in the part of the table that carries the magazine frame. The position in space of these balls can be computed from the X-ray photographs and compared to known data, and from the value thus obtained, the position of the X-ray tubes in relation to the image is determined.

Owing to the perspective properties of the reproduction, the following formulas are obtained for the computation of the position in space of an object point (a ball or other object of metal): A co-ordinate system, orthogonal and with three axes, is assumed as placed with the origin at one focus, one axis through the other focus, one axis perpendicular to the image, and the third axis perpendicular to the other two. In this system, the position of one object point is given by means of radius vector M, leading from origin to the object point in question. (See Figure 1.) As the reproduction is thought to be made by straight lines from focus (so-called central projection), the radius vector to the image point, S, must be found on the same straight line as M. Consequently, M can be written $=\mu S$, where μ is a value which can be called scale factor. This value is obtained by knowing the distance b between the foci, and the distance p, measured on the picture, between the same object point on the two stereoscopic pictures. From Figure 1 the following formula is obtained by uniformity:

$$\mu = \frac{b}{b+p}.$$

M is determined by measuring the distance p on the picture, and μ is obtained as b is known. The component of S, perpen-

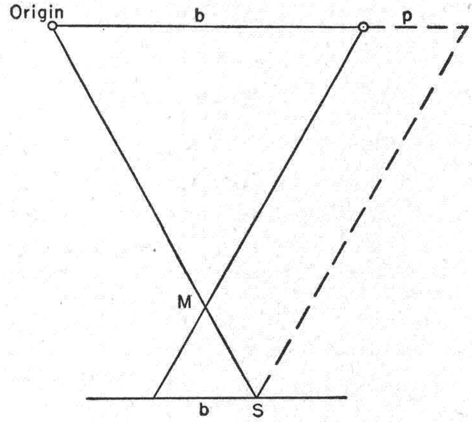

FIG. 1

dicular to the image, is known (it equals the height of the tube), and the component of S, situated on the image plane, is measured on the image. With that operation S is determined and M is obtained by multiplication with μ.

In this way are determined all points desired (pieces of metal or control balls), and the volume of the polyhedron, formed by the pieces of metal, is directly computed from the co-ordinates. This is executed after a subdivision into tetrahedrons. If the co-ordinate system is moved to one of the corners in a tetrahedron, whose other corners have the co-ordinates (x_i, y_i, z_i) where $i=1, 2, 3$, it has been shown that the volume of the tetrahedron is

$$\frac{1}{6}\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

The change of volume due to small changes in the co-ordinates is obtained by differentiation of the determinant, and consists of the sum of the three determinants, which arise if the co-ordinates in the rows, one after the other, are exchanged for the respective co-ordinate changes. The computation is easily made automatic by working out all of the second order determinants and multiplying them with their μ-values so that the volume difference is obtained as a linear function of the co-ordinate differences on the plane of the image. At moderate differences of height, the multiplication with μ can be omitted if only percental values of the volume variations are desired. The approximation error in this determination of difference is of the same percental magnitude as the percental part of the respective co-ordinates in the co-ordinate differences. The mean error of the result can of course be conveniently computed from mean errors in the co-ordinates as the coefficients of the linear expression above are available. In the experiments made, the mean error of the volume variations was somewhat below 1 per cent of the total volume.

The investigations were performed at the Division of Photogrammetry at the R. Institute of Technology, Stockholm.