Anderson methods. This information was published in the December 1954 issue of PHOTOGRAMMETRIC ENGINEERING.

Mr. Randall D. Esten, from the Engineer Research & Development Laboratories, Fort Belvoir will speak on The Development of Photogrammetric Computing Systems. Mr. Esten received a B.S. in Physics from M.I.T. in 1945, and an M.S. in Photogrammetry from Syracuse University in 1948. He is presently the Chief, Map Compilation Techniques Section, Map Compilation Branch, Topographic Engineering Department, Engineer Research and Development Laboratories. He formerly was a Research Associate at the Mapping and Charting Research Laboratory of the Ohio State University Research Foundation and a Physicist at Corning Glass Works. His talk in the area of computational systems I'm sure will include new thoughts for us to consider.

ANALYTICAL PHOTOGRAMMETRY AS APPLIED TO FLIGHT-TESTING*

Dr. Hellmut Schmid, Geodesist, Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland

ABSTRACT

After a few remarks on the mission of the Ballistic Research Laboratories with respect to the determination of accurate trajectory information of airborne test vehicles, the application of photogrammetric measuring methods is singled out. It is shown how the analytical methods applied serve basically the same purpose as those in conventional fields of photogrammetry concerned with the determination of spatial coordinates of individual points, and how these methods, on the other hand, lead to an approach to a certain extent different from the treatment of the corresponding stereoscopic evaluation on universal plotters. The discussion is supplemented by some remarks on equipment used in measuring plate coordinates, and on the phase of digitalizing the comparator readings.

ONE of the missions of the Ballistic Research Laboratories of Aberdeen is the development of full-scale trajectory measuring methods for problems assigned to Ordnance, especially in connection with the development of guided missiles and related projects. In general terms this task may be characterized as the problem of determining the space-time coordinates of the trajectories of airborne targets. Both electronic and optical measuring methods are in use and their further development is under investigation. In the Optical Measurements Branch, one of the methods studied is concerned with the application of ground-based and airborne precision photogrammetric cameras, for the purpose of providing an independent trajectory measuring method, and to serve as a calibration standard for other optical and electronic measuring methods. The scope of our work and the basic concept underlying our approach are more fully appreciated if it is understood that the measuring methods under consideration must be applicable to the determination of the trajectories of various types of guided missiles, of small and large airborne test vehicles, directly or remotely controlled, as well as of free flight ballistic missiles flying over short, moderate and long ranges. The problem presents a challenge because the accuracy requirements of position and time are comparable to those encountered in classical geodetic measuring methods.

Although this introduction might lead one to expect a very special treatment of the photogrammetric problem, this actually is not required. After removing the time-coordinate and disregarding the synchro-

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nization of the shutters of sometimes widely separated camera stations to within 10–50 microseconds and recording accurately the corresponding moments of exposure, a basic problem in photogrammetry remains. This is the problem of the triangulation of spatial coordinates of certain target points from photographs taken by cameras stationed at the ends of one or more base lines. The fact that the cameras are ground-based does not simplify the problem because the elements of orientation obtained from circle readings and level settings may not be accurate enough for use in the evaluation. In addition, one can not always depend upon the metric values of the cameras as determined by calibrations performed at a time and location different from the actual field operation. Therefore, in such cases the calibration becomes a part of the triangulation problem.

In addition to the problem so far mentioned, our work involves the determination of the spatial position and attitude of aircraft from photographs of ground-based control points taken from the airborne test vehicle. Geometrically speaking, this problem is essentially the same as the conventional mapping problem of interpolating stereoscopic models between independently determined control points. Although such absolute control is usually available for our purposes, the economy requirement of flight testing is the same as in topographical photogrammetry, that is, to reduce the amount of ground control to a minimum. In our problem the determination of the absolute parameters of the camera orientation is a requirement which is in excess of the requirements of the basic aerial triangulation in which only the photographed object has to be reconstructed. Therefore, the calibration of the cameras and their maintenance during the flight become of paramount importance in our case. Since the flight may differ considerably from an ideally flown strip in terms of pattern and angle of attack of the aircraft, it is necessary to consider both the single and double-point resections in space, for both normal and oblique photography. Other tasks that are similar to conventional problems in aerial photogrammetry, even to the extent of serving the same purpose, could be mentioned; for example, the problem of determining the impact pattern of relatively widely spread fragments of certain weapons.

In essence, our general task may be defined as the problem of reconstructing, for each camera station, the bundle of rays of projection and the geometrical conditions under which the corresponding photograph was taken, using images of absolute and/or relative control points. Geometrically speaking, nine degrees of freedom must be either known or determined for each camera station; they are:

1) the elements of interior orientation, consisting of
   a) two translations of the plate in its own plane, corresponding to the location of the calibrated principal point
   b) one translation normal to the plane of the plate, corresponding to the calibrated focal length
2) the elements of exterior orientation, consisting of
   a) three translations, corresponding to the X, Y, Z coordinates of the center of projection
   b) three rotations, corresponding to the X-tilt (ϕ), Y-tilt (ω) and the swing angle (κ).

Thus, the simultaneous orientation of two cameras located at the ends of a base line and oriented so that, at least to a certain extent, they photograph the same objects, may be considered as the fundamental configuration on which all other problems, as extensions or multibase triangulations, can be based.

In analytical terms, we are thus confronted with the simultaneous determination of 18 unknown parameters, which are the elements of orientation of two consecutive camera stations, and with the triangulation of X, Y, Z coordinates of certain target points. Any parameter of the orientation elements which is independently determined may be enforced in the solution. This, of course, reduces the number of parameters to be determined, but obviously does not invalidate the general concept of our approach. Such parameters could be the calibrated elements of interior orientation, independently measured differences of flying heights by statoscope registration, or the geodetic coordinates of the positions of ground based cameras. Without subdividing photogrammetry into an often confusing nomenclature of highly specialized fields of application, the general solution as just defined can be considered adequate

a) to cover the whole range of application for photogrammetric measuring methods,
b) to incorporate all possible spatial orientations of the camera axes, thus eliminating the need for special treatments of normal and oblique cases.

From this point, the practical phase of data reduction may obviously follow two courses. We may consider the use of a universal plotter such as the Wild A7 or Zeiss C8, which are essentially analogue computers, or an electronic computer utilizing suitable plate coordinate measuring devices known as comparators.

To avoid any misunderstanding it must be understood that we deal in the phase of triangulation with individual points and their photographed images only, and not with the problem of compiling information by plotting continuous lines from stereoscopic models. In our work the determination of individual points is by far the most important task, and, in fact, the development of photogrammetric methods in topographic fields is characterized by an increasing interest in the same problem.

Using conventional photogrammetric equipment, including first-order instruments, the stereoscopic model is created by a process of parallax elimination in the model itself—relative orientation—followed by the absolute orientation, which is established by a similar process, namely, the compensation of differences in the coordinates of certain control points within the stereoscopic model. Consequently, the basic idea underlying our stereoscopic evaluation instruments is obviously the application of a process of interpolation. The ingenuity of this idea is apparent as it does not require absolute precision in the individual orientation movements of the evaluation machine, but only a means of identifying and eliminating parallaxes in the model. Aerial triangulation, on the other hand, is basically a process of extrapolation and, consequently, in order to avoid an unfavorable accumulation of systematic errors, requires a rigorous simulation of the geometric conditions existing during the process of taking the photographs. Corresponding demands on the universal plotting machines brought pressure upon the manufacturers who in turn tried to convert the interpolation equipment into analogue computers, thereby sacrificing freedoms of movements in order to increase the internal stability of the first order machines. This trend led to the adding of scales in the orientation movements and the supplementing of the spatial cross slide systems by coordinate reading and recording devices. Tests with grid plates give adequate proof of the remarkable degree of precision obtainable in present first order instruments, but at the same time show clearly that it is impossible for the instrument to work rigorously over the full range of the orientation movements and the X, Y, Z motions. However, an analytical method which would eliminate the shortcomings of the universal plotters could not be considered in the past because the automatic desk computers are entirely inadequate for an economical solution.

The availability of high speed electronic computers at the Computing Laboratory at Aberdeen has made feasible carrying out the analytical treatment of the general photogrammetric problem in a practical manner. The separation of the orientation problem into the two fundamental processes of relative and absolute orientation predominates today's practical photogrammetry. However, the justification for the separation of these two processes has been based strictly upon the practical methods of restitution dictated by the physical properties of the restitution equipment. It has been stated before that during the process of restitution with such equipment, parallaxes of the coordinates on the stereoscopic model itself have to be dealt with and, consequently, the photogrammetric evaluation deals primarily with the creation and orientation of the stereoscopic model. It is the author's opinion that an independent analytical treatment should not be necessarily an analytical copy of the restitution technique employed in the stereoscopic compilation equipment, using the optical or mechanical projection methods.

In an analytical treatment, the absolute orientations of the two projective bundles of rays under consideration are of primary concern. The accomplishment of these orientations automatically includes the condition for the relative and absolute orientation of the model. The absolute orientation of the two corresponding photographs are functions of the positions of the photographed images on the plates. Therefore, the photogrammetric problem may be simulated with a set of conditional equations, explicit in terms of the measured plate coordinates, expressing the absolute orientation of the two photographs under consideration. A presentation of this approach is presently being prepared for publication. The result is obtained by applying a few simple geometrical conditions based on the colinearity of three and the coplaniarity of four points, respectively. Such an approach leads to a few somewhat unconventional conclusions. For instance, in an analytical method absolute control data may be used outside the field common to both photographs. This introduces a more extended base for the double point intersection in space, thus introducing the favorable geometry which is characteristic of the single point resection.

With respect to the corresponding theory of errors, it is interesting to note that in an analytical treatment, if there are only absolute control points present, the orientation of the two bundles of rays automatically reduces to two independent camera orientations. Further, it is obvious that the method of observation does not influence the formulas expressing the rigorous geometry. In other words, monocularly and/or stereoscopically observed coordinates may be introduced into the computations as long as the condition is satisfied that the sum of the squares of the weighted residuals of the original plate measurements is minimized. Thus, without going into detail, we may conclude that the superiority of applying the stereoscopic principle in the determination of individual points lies merely in that it provides for better identification and, therefore, for more precise measurements.

After the geometry of the general photogrammetric problem is expressed in suitable condition equations between the plate measurements on the one hand and the given control data, together with the given unknown elements of orientation, on the other, the analytical solution must provide for a rigorous least squares adjustment. Combinations of certain trigonometric functions of the rotational components—essentially the introduction of direction cosines—and the introduction of a combination of these rotational auxiliaries with the measured plate coordinates as linear auxiliary parameters proved to be especially suitable for the numerical treatment. Further, it is possible to arrange the least squares solution in such a way that the final set of normal equations, from which the unknown parameters are computed, are formed as the algebraic sum of as many partial normal equations as there are condition equations involved. Thus, the analytical solution provides for the incorporation of any number of absolute, partially absolute and relative control points and the requirements in the memory capacity of the computer are only slightly affected by the number of points carried in the solution.

The usefulness of this method has been established for airborne and ground based cameras from sample computations carried out by desk computers. A code for this general approach is at present under preparation and will be added to the electronic computing procedures which are presently in routine use for single camera orientations and the corresponding independent triangulation procedures. We have begun to investigate the evaluation problem with both analogue and electronic computers in order to compare their individual merits and expect to have qualitative and quantitative results of these tests at a later date.

The advantages of the analytical method are summarized as follows:

1) In addition to the most probable X, Y, Z coordinates of the triangulation result, the most probable values for the orientation elements, the mean errors of both of these, the mean error of an observation of unit weight, and all the residuals are obtained.

2) Corrections to compensate for systematic errors, such as asymmetrical distortion, differential shrinkage of the emulsion carrier, refraction, etc., are easily applied.

3) In the process of evaluation only the errors of the measured plate coordinates are propagated.

4) Comparators designed in accordance with Abbe's comparator principle are less affected by systematic errors than the three dimensional universal plotters.

5) The process of measuring is reduced from the complex operation of a universal plotter to the measuring of plate coordinates on a relatively simple instrument, as a monocular or stereocomparator.

It is obvious that the coordinate measuring device is a keystone in any analytical evaluation method. The superiority of the method of stereoscopic observation makes it obligatory to employ a precision stereocomparator. The performance of postwar precision surveying lenses, when used with sufficiently flat glass plates, indicates the urgent need for such a coordinate measuring machine with an internal accuracy of ±1 to 2 microns. Continuous efforts on our part to initiate such a development did not meet with success, and it seems that the leading domestic finemechanical optical industry will not undertake such a development. However, indications are that the Wild Company, Switzerland may undertake the development of a modernized version of the classic stereocomparator in order to fill this critical gap in the field of precision photogrammetric evaluation equipment.

In addition to the problem of precision, that of automatically recording the comparator settings should be considered. In a conventional universal stereoscopic plotter, the determination of the coordinates of individual points, after the model has been set up, consists of two or three operations:

1) setting the measuring mark on the target, by three nearly simultaneous motions of the base carriage,
2) printing out the *X*, *Y*, *Z* machine coordinates,
3) a coordinate transformation, if required.

For the corresponding analytical treatment four operations are necessary:

1) setting the measuring mark on the target by four linear motions on the stereocomparator.
2) tabulating automatically the readings and simultaneously preparing records suitable for the input in a high speed electronic computer,
3) computing the result,
4) printing out the final coordinates and additional desired information such as mean errors, residuals, etc.

Operations 2, 3 and 4 are not significantly time consuming since in these operations the relatively long time interval between two consecutive settings can be utilized.

For a screw type comparator a means of automatically measuring the angular positions of the lead screws is necessary. A completely mechanical solution of this problem is employed in the Zeiss C8 coordinate printer. A mechanical-electrical solution based on the method of counting a certain number of contacts by using rotating discs and relays is used on the Wild A7 printer. In both solutions the 10 micron interval is the smallest subdivision recorded. An ingenious solution for an automatic read-out to the nearest micron has been worked out by the David L. Mann Instrument Company, Lexington, Massachusetts and is used on one of its comparators. The method of closing electrical circuits by spring-loaded contacts is combined with a mechanical comblike vernier mounted on the periphery of the reading drum. An electronic solution is available combining components developed by the Telecomputing Corporation, Burbank, California in which the basic idea is to measure with a reading head employing a.c. circuitry and magnetic effects. The system develops a direction-sensing feature and has an accumulator providing means for a digital count which represents the true angular position of the data shaft. The advantage of this approach is that the modification necessary to mount the magnetic head on the lead screw is simple. The possibility to subdivide the count into 1,000, 2,000 and more units per revolution with only slight increase in the size of the heads is another practical feature. The accumulator called the Telecordex tabulates the coordinates and at the same time provides a means of punching cards in connection with the IBM summary punch. The Telecordex records simultaneously 2 coordinates for each read-out operation and is now available for 6 decades. Thus, if one combines a lead screw of 1,000 microns pitch with a head having 2,000 units per revolution, it is possible to cover a distance of 500 mm. in 0.5 micron units. A study made at Aberdeen of a new 1,000 unit head indicates a repeatability within ±1 unit.

Comparators using glass scales as reference for the length measurements and the application of the polar coordinate measuring principle, as used e.g. in the Radial triangulator, present further problems for automatic coordinate recording. It seems that considerable effort is being made by several interested groups to solve the problem of automatically setting mechanical parts relative to precision glass scale graduations and the reverse task of reading automatically the corresponding numerical values on the scales. No doubt the solution of this problem is of paramount importance in connection with the design of a new stereocomparator. Nevertheless, it would seem that satisfactory solutions may be expected for the problems of recording, computing and printing with necessary reliability and speed, since these problems exist in many research and development projects, having even higher requirements. The actual problem in an analytical approach, it is felt, is the economical solution of the measuring process on the stereocomparator. The measuring of plate coordinates of individual points must be carried out without the benefit of a preceding relative orientation, and, therefore, in the general case, specific x and y parallaxes must be eliminated for each individual point during the process of measuring the corresponding plate coordinates. In other words, by scanning over the field common to the two photographs under consideration, the continuity of the stereoscopic sensation is not maintained but must be recovered from point to point. All the necessary means for establishing maximum stereoscopic effect can be applied using image reversing prisms and panchratic systems in both light trains. To what extent advantages inherent in an analytical method can be utilized in practice will therefore depend chiefly on the ingenuity of the designer to combine the well-known basic principles of a stereocomparator with such operational features as will provide an economical observational technique.

PHOTOGRAMMETRIC APPLICATIONS OF SMALL CAPACITY ELECTRONIC COMPUTERS*

Frederick J. Doyle, Research Associate, Mapping and Charting Research Laboratory and Acting Director, Institute of Geodesy, Photogrammetry and Cartography, Ohio State University

ABSTRACT

The operational features, capabilities and limitations of small capacity computers such as the IBM 602 A and its allied equipment are described by analogy to hand computing methods. An evaluation of the potentialities of the equipment shows that it is limited to problems where the same operation must be performed numerous times with variation of some parameters. The computation of graphs of stereo-model deformations is used to illustrate the type of problem suitable for such equipment. Economic considerations make it apparent that a large amount of work is required in order to make the installation of such equipment worthwhile.

1. INTRODUCTION

IF ANY three devices are symbolic of modern technology, they are probably flying saucers, atom bombs, and electronic computers. The amount of misinformation prevalent on these topics is appalling. Most of this misinformation comes from popular articles which tend to be highly sensational in approach. In particular, the reference to computers as "giant electronic brains" causes many people to regard them with an awe which is not otherwise deserved. In addition, the practitioners of electronic computing have developed their own jargon, and delight in throwing around terms like digitizer, readout, nines complement counter, crossfooting, etc., which for the most part leave the average man no wiser than he was before he heard them. One lamentable result of this is

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