time computers busy, before electronic computing would be profitable. Not many private operations will have so large a demand. But it is still possible for those whose requirements do not justify installing their own machines to gain the advantages of electronic calculation. Most government agencies have, or can arrange, access to machines in other departments. Many universities now operate computing laboratories and the number and scope of these is rapidly increasing. There also exist a number of commercial concerns where computing time can be purchased.

5. Conclusions

The primary advantage of electronic computing is that it permits the solution of problems whose scope would make them unreasonable or impossible on desk machines. A second advantage is speed since even the slowest machines are several times faster than the best operator on a desk machine. The third advantage is accuracy. Although electronic computers do make random errors these are easily checked and corrected, whereas in most desk machine operations, errors may only be found by repeating the whole calculation with no guarantee that the second attempt is any better than the first. Furthermore the ease with which results can be presented in the most convenient form is a considerable advantage.

The big disadvantage of electronic calculation on parallel machines is the fact that they are only suitable for repetitive work. A one shot computation, regardless of its complexity, might just as well be performed on a desk machine. However this is not necessarily true of computation on series machines. Another point to be considered is that machine costs continue whether they are operating or not, and they require personnel with skills which may not contribute directly to the work in an organization.

Automatic computation will not bring on the millennium but it will certainly do much to expedite the handling of large masses of data.

It is by no means necessary nor even expedient that professional photogrammetrists become skilled practitioners of electronic computation. But it is highly desirable that they become cognizant of the capabilities and limitations of these machines, so that they may be aware of where they can use this important new tool, and may design their problems accordingly. The practical questions of programming and machine operation are far better left in the hands of those who have made electronic computing their profession.

The author expresses his appreciation to Mr. Robert A. Mosier, mathematician in charge of the computing section at the Mapping and Charting Research Laboratory, who gave valuable assistance in the technical aspects of this paper.

MEDIUM-CAPACITY ELECTRONIC COMPUTERS IN PHOTOGRAMMETRY*

Arthur J. McNair, Professor and Head, Surveying Department, School of Civil Engineering, Cornell University

IN DISCUSSING the subject of this paper the author is aware that there is no well-defined or accepted concept of exactly what is meant by medium-capacity electronic computers. The factors which must be stipulated would include the rate at which the computer performs the mathematical operations, the storage

* This paper is a part of the Symposium on Computing Trends in Photogrammetry held on March 7, 1955 during the Society's Annual Meeting.

capacity or ability of the machine to record and recall a number for further use, and the type of computations which can be performed by the machine.

Analogue Computers

At the outset it is necessary to say that at the present time, for the solution of photogrammetric problems, it does not seem that an electronic analogue computer will satisfactorily solve any great number of problems. In most surveying operations involving distances, such as would appear on one photograph or a strip of several photographs, it is necessary to carry computations to at least six significant figures. An analogue computer solves problems by setting up and solving analogies. Usually these are electrical analogies although a multiplex can be considered an optical-mechanical analogy. Analogue computers generally are not capable of producing results on problems as complex as those encountered in photogrammetry beyond about three significant figures. This means that if an electronic analogue computer were devised for photogrammetry, it would produce results comparable with those obtained by field methods equivalent to pacing and pocket compass, or by computation methods equivalent to a slide rule. Existing methods for mapping —or more properly for charting—with this degree of accuracy are already quite highly developed. Consequently, at present, no great demand for an analogue computer exists. It will not be discussed further in this paper. It appears that the digital computer alone will provide the precision for the usual analytical photogrammetric problem.

Medium-Capacity Digital Computer

Because of lack of any well-defined range of capacity for "medium-capacity" digital computers, the following will be adopted as a basis for discussion.

Rate of computing: 70,000 mathematical operations per hour. This value of 70,000 must be understood as being only an average rate since it obviously takes longer to multiply two numbers than to add them, and much longer to extract a square root than to subtract.

High-speed storage capacity: approximately 1,000 to 2,000 "words" or ten-digit numbers. A variation on this is to use eight-digit numbers with the remaining two digits to indicate the location of the decimal point in all operations.

Access time: average 0.002 seconds. This means that it requires an average of 0.002 seconds for the machine to search out a particular "word," or ten-digit number, from the storage pile and to place it in position for operation.

Several of the major business machine manufacturers produce computers in the small-, medium-, and large-capacity range. Although photogrammetrists in general have been very slow to take advantage of modern advances in computing procedures, the techniques have been well proven in accounting, business operations, physics research, and government operations—especially military—to name but a few. Manufacturers of computers are not only willing, but also quite able to assist in the solution of photogrammetric problems.

Computing Times for Photogrammetric Problems

To appreciate what this machine will do in helping the photogrammetrist, it is necessary to consider some of the problems the photogrammetrist has to solve. Perhaps the basic problem for a single aerial photograph is to determine the space resection and orientation problem, namely to determine the X, Y, and Z position occupied by the camera at the instant of exposure, and to determine the tilt and swing of the photograph and the direction on some line on the photograph with respect to survey directions on the ground. A minimum of three points on the ground are required for which the X, Y, and Z positions are known. Three or four methods of solving this problem have been devised, each of approximately equal difficulty, and with the result that about 700 mathematical operations—additions, subtractions, multiplications, divisions, and square roots—must be performed to solve the six unknown quantities in space resection. This problem can be performed on an automatic desk computer, for instance by the Church method, in perhaps four to six hours. A small-capacity computer such as the electronic card program computer (CPC) will solve this problem in about $4\frac{1}{2}$ minutes. The medium-size electronic computer will solve this problem in about 30 seconds. The desk computer can store one or two numbers at a time but each succeeding computation wipes out the numbers from preceding operations so they must be manually recorded. Consequently, a great many intermediate numbers must be recorded so that they can be used in subsequent operations. The above problem requires that nearly 40 numbers will be stored at one time and available for re-use upon call. Thus, the simplest problem, taxes the small-capacity computer to its limit.

The space-resection problem for one photograph is hardly adequate for most photogrammetric purposes. Ordinarily two photographs, namely a stereo-pair, are required for making complete measurements on the ground, that is, to determine the X, Y, and Z positions of points on the ground from images which appear on two overlapping photographs. If adequate control exists for each of the photographs and the effect of earth curvature can be neglected, the position in space for each of the two photographs would first be calculated as above—a total of approximately one minute. This is comparable to having completed both relative and absolute orientation for stereo-plotting as by multiplex. This puts the operator in position to compute the locations of intersecting rays to photographic images of points on the stereo-pair. This intersection problem involves perhaps 80 arithmetic operations per point, and a storage of up to 30 numbers at any one time for reuse. On the medium-sized computer the computation of X, Y, and Z positions of such points requires approximately 5 seconds per point.

What happens when the refinement of considering the effect of earth curvature is to be included? In conventional surveying, the solution of geodetic problems is many times as complex as plane surveying problems. In the same way computation of the geodetic photogrammetry problem is considerably more complex than the above case for a plane. By one method of solution, if the effect of earth curvature is to be included, the single problem of space resection for one photograph involves about 1,015 computations, a storage again of about 40 intermediate results, and requires about $5\frac{1}{2}$ minutes for its solution on a small-size computer or about 45 seconds on the medium-size computer. The nice thing about problems computed this way, however, is that any single photograph will now tie in with any other photograph at any time without further reduction of geodetic position.

Next consider the extension problems involved when sufficient ground control does not exist for each individual photograph in a strip. Assume that three given-control points exist in the area of overlap of the first pair of photographs. The problem of extension of control forward into each successive photograph requires that the position of at least two new points in each new photograph be determined. Simultaneously allowing for earth curvature entails the solution of an average of about 2,560 arithmetic operations and a maximum storage at any one time of nearly 90 numbers. The medium-sized computer can perform these operations in less than two minutes. The computation of X, Y, and Z positions of nine points per photograph in a strip of ten photo graphs would require about 15 minutes.

At this point every photogrammetrist immediately begins to make mental calculations as to economics. The operator of a stereo-plotting machine, such as a multiplex, immediately thinks of the time involved in bridging each picture in a strip of photographs and of making subsequent adjustments to this strip. The field man thinks of the field-party time involved in locating forward control points by conventional surveying methods. The comparisons leave each type of operator to draw his own conclusions as to the economics involved in his own operations.

The final problem in most cases is that of adjusting adjacent strips of photographs to fit each other and the ground control. This is the so-called "block" or area adjustment. Without giving any details, it is possible to say that the more complex the problem the greater is the economy in high-speed computers. This problem can be solved with relatively greater ease than is required for a comparable adjustment by optical, mechanical, or graphical methods.

Conclusion

Any of the problems encountered in photogrammetry as in any other field can be solved by analytical methods as well as by graphical or mechanical methods and with greater precision. The mathematic relations involved in three dimensional space are more complicated than those involved in a great many engineering problems which can be limited to two dimensions. This basically is the reason photogrammetric problems, such as mapping, have been solved primarily by analogue instruments such as the multiplex, the Wild autograph, and the stereoplanigraph. With the advent of a new, more powerful tool, namely high-speed electronic computers, it is time to again re-evaluate the processes which have been followed in photogrammetry.

The medium-capacity electronic computer appears to be quite satisfactory for all but the most complex problems involving very large areas and requiring a very large storage capacity or "memory." With a storage capacity of 1,000 or 2,000 ten-digit "words" and performing operations at the rate of approximately 70,000 per hour, it is economically feasible to determine ground positions of any points from photo images with results at least equal to the accuracy required for plotting. If such points can be located at this rate, they may well provide the control which would enable less elaborate machines to be employed by operators with less technical skill for final map compilation. All the techniques of these operations obviously have not been completely worked out to date. However, photogrammetrists are making rapid strides in this direction, and in the near future sufficient data should be available to enable critical comparisons to be made and conclusions drawn profoundly affecting photogrammetric operations.

USE OF LARGE CAPACITY COMPUTERS IN PHOTOGRAMMETRY*

Robert S. Brandt, Chief Development Branch, User Requirements and Development Division, Army Map Service†

THERE seems to be no absolute criteria for defining large or small capacity computers. I have a vague idea that UNIVAC is what is known as a large capacity electronic digital computer. In this paper I will discuss the use of UNIVAC. Specifically, its use for the adjustment of Aerotriangulation at the Army Map Service. The capacity of our UNIVAC is something of which I know very little but I do have knowledge of the uses we have made for the adjustment of Aerotriangulation.

Generally speaking, evaluations of equipment by users are usually influenced by particular environmental factors. One often finds a wide range of efficiencies being obtained, from the same types of equipment. More specifically, the efficiency of equipment is greatly dependent upon how well it is adapted to the various situations in which it is used. So without reference to the many other uses we have for UNIVAC at Army Map Service, I shall attempt to describe our basis for use of the UNIVAC in Aerotriangulation. You may then judge its utility for that operation.

Very early in the course of plotting and adjusting the results of planimetric stereo triangulation, it was recognized that a completely numerical recording and adjusting system was possible. It certainly has been well known for many years that most high order Aerotriangulation instruments are equipped with digital counters for recording X, Y and Z. These digital counters provide the means for completely numerical solutions independent of graphical plotting or mechanical templets. Under such conditions the precision of stereo Aerotriangulation need depend only upon the precision of the numerical data recorded and upon the number of decimal places to which computations are carried out.

Although these possibilities for extensive use of numerical stereo triangulation have been known for a long time, practical use

* This paper is a part of the Symposium on Computing Trends in Photogrammetry held on March 7, 1955 during the Society's Annual Meeting.

† Subsequent to the Annual Meeting, Mr. Brandt entered the employ of Hycon Aerial Surveys —*Editor*.