A PHOTOGRAMMETRIC METHOD FOR THE TRIDIMENSIONAL MEASUREMENT OF SAND GRAINS*†

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INTRODUCTION

THE quantitative analysis of many of the physical properties of sediments involves the measurement of individual particles. Since the latter part of the nineteenth century a multitude of techniques has been developed for measuring sedimentary particles and for using these measurements to describe various physical properties of the particles.[7] However, the methods which employ direct measurements of particles cannot be used for small grains, and the techniques which utilize projections or single photographs of grains have many shortcomings due to the use of two-dimensional concepts or other oversimplifications. One possible solution, described in this paper, is to measure sand grains tridimensionally with a stereophotogrammetric method.

PHOTOGRAMMETRIC METHOD

Stereophotogrammetry is the science of obtaining reliable measurements by means of photography with the aid of stereoscopic equipment and methods.[2] The non-topographic application of this art to the study of small objects, such as sand grains, can also be classed as photomicrogrammetry, a term which denotes the use of photography for the measurement of small two- or three-dimensional objects. Beginning with Zeller's study of the corroded surface of an iron pipe in 1937,[12] photomicrogrammetry has been used extensively, especially in European countries, in such diverse fields as dentistry[11,13] intra-oral radiography,[3] electron microscopy, biology,[1] and others.[4,5,6] In the laboratories of the Belgian l'Association pour l'Etude de la Paleontologie et de la Stratigraphie, stereotechniques are used in micropaleontological research,[9,10] which, although highly refined, are in principle similar to the method described in this paper.

PRINCIPLES

For the measurement of the third dimension of sand grains, i.e., their height, conventional stereophotogrammetric techniques in their simplest form have been used in this work. Two photographs of a group of sand grains strewn on a surface are taken from different camera stations. The difference in the distances from the camera stations to the top of individual grains and to the surface expresses itself on the photographs as the measurable parallax difference Δp. A parallax bar, also called stereometer, is used to measure Δp by setting under a stereoscope the floating mark first on the top of a sand grain and then on the surface near that particular grain. Two readings are obtained and their difference is Δp. Figures 1 and 2 show the principle of determining the parallax difference and the operation of the parallax bar. The parallax difference has a direct relation to the height of the sand grain measured, and since the measurements of these small grains are within the range of simple differential formulae, the height can be expressed as

$$\Delta h = \frac{\Delta p}{\theta m}$$

where

Δh = height of grain in mm.
Δp = parallax difference in mm.
θ = base/distance ratio (a camera constant) = $\frac{1}{8}$
m = magnification of the object.

* Based on a portion of a thesis submitted to the Graduate School, Southern Methodist University, Dallas, Texas, in partial fulfillment of the requirements for the degree of Master of Science, August, 1954.

† This is a condensation of the paper submitted in competition for the Bausch & Lomb Award and which was awarded first place. The paper was entitled "Tridimensional Shape Analysis of Sand Grains with Photogrammetric Methods."

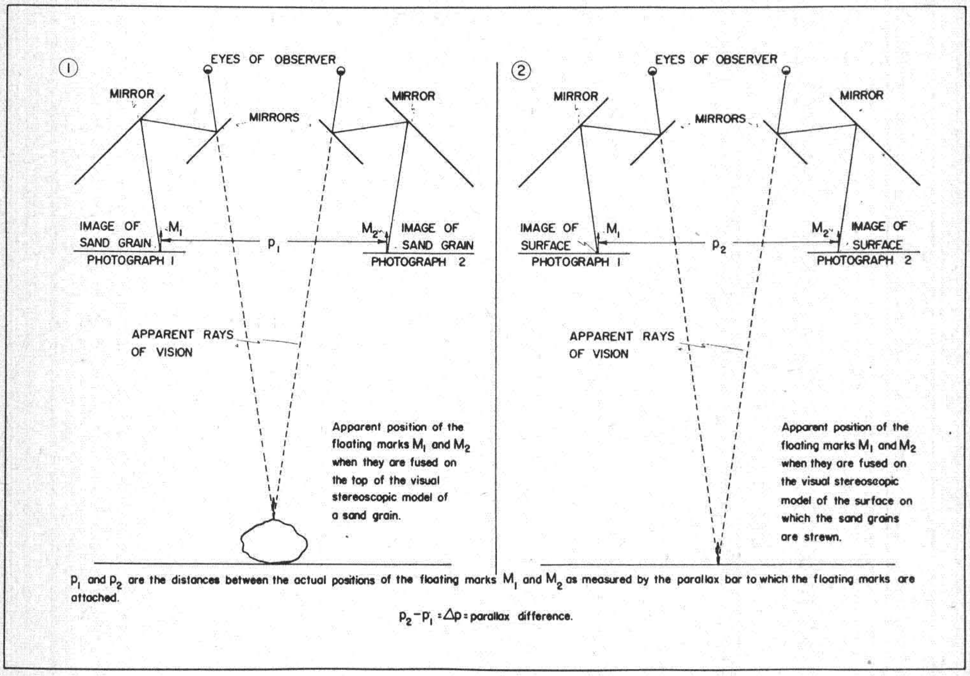

FIG. 1. Principle of determining the parallax difference p.

CAMERA

In order to obtain stereograms suitable for the measurement of sand grains in the size range from 40μ to 2 mm., a camera for stereoscopic photomacrography with a fixed magnification of 20× was built (see Appendix). Figures 3 through 5 show various pertinent details of this camera, the design and dimensions of which reflect the following objectives:

a) to obtain a pair of coplanar photographs, realizing the "normal case" of stereophotogrammetry,

b) to cover with each photograph a sufficiently large and sharp field, suitable for statistical evaluation,

c) to maintain a constant relative orientation of the camera stations to the object by a fixed base/distance ratio θ,

d) to use a base/distance ratio θ which will give a natural impression and little or no height exaggeration when the photographs are viewed stereoscopically.[7]

e) to use reflected light in oblique illumination instead of vertical illumination or of transmitted light in order to get optimum definition of the surface detail of the grains,

f) to maintain a fixed illumination with respect to the object,

g) to get sufficient enlargement into the photographs before measurements are made so that no precision instruments will be necessary for the evaluation,

h) to put an object scale on the surface on which the sand grains are strewn in order to better define this surface and to indicate the magnification of enlarged prints and transparencies,

i) to make use of the lens which was available, namely a Zeiss "Tessar."

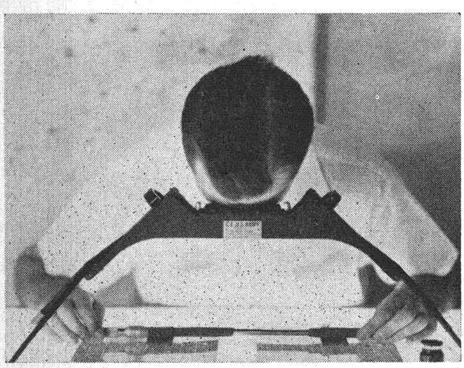

FIG. 2. Operation of the parallax bar (Wild Stereometer).

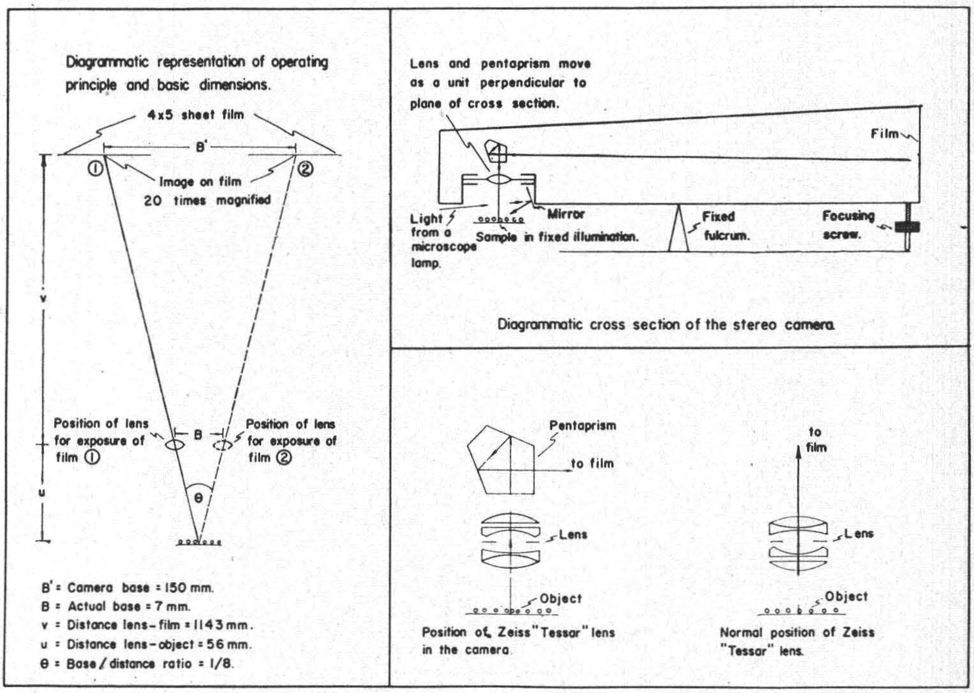

FIG. 3. Design of the stereoscopic camera.

Figure 6 shows the left picture of a pair of stereoscopic photographs of a group of sand grains in order to demonstrate the size and quality of the field and the magnification inherent in the camera.

PROCEDURE

Using the camera, a mirror stereoscope, a parallax bar (Wild Stereometer), and a magnifier with a scale in 0.1 mm. units, the following procedure is employed to measure three mutually perpendicular dimensions of individual sand grains:

A pair of stereoscopic photographs is taken of a group of sand grains, preferably of similar size for better focusing, strewn on the surface bearing the object scale in the form of a grid whose units are 0.1 mm. After enlarging the photographs by a factor of two to a total magnification of 40×, the enlarged prints, or preferably transparencies, are aligned under the mirror stereoscope on a table or light-table. Figures 7 and 8 are stereograms of sand grains. The following dimensions of individual sand grains are then measured:

a) from the stereogram:
   Δh = height of the particle = vertical distance of the highest point of the grain from the grid surface as evaluated from the measurement of the parallax difference
b) from one photograph:
   L = longest apparent intercept of the par-

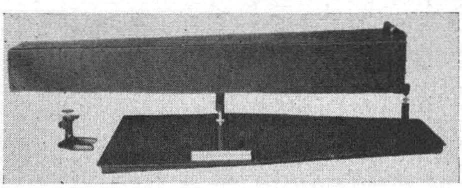

FIG. 4. Sideview of the stereoscopic camera with sample holder. Scale is 6 in. long.

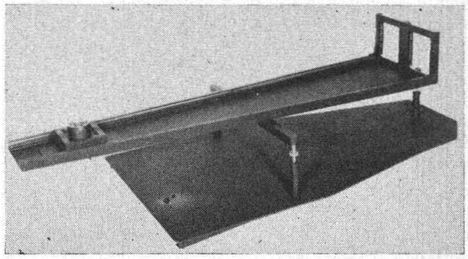

FIG. 5. Stereoscopic camera with housing and prism removed.

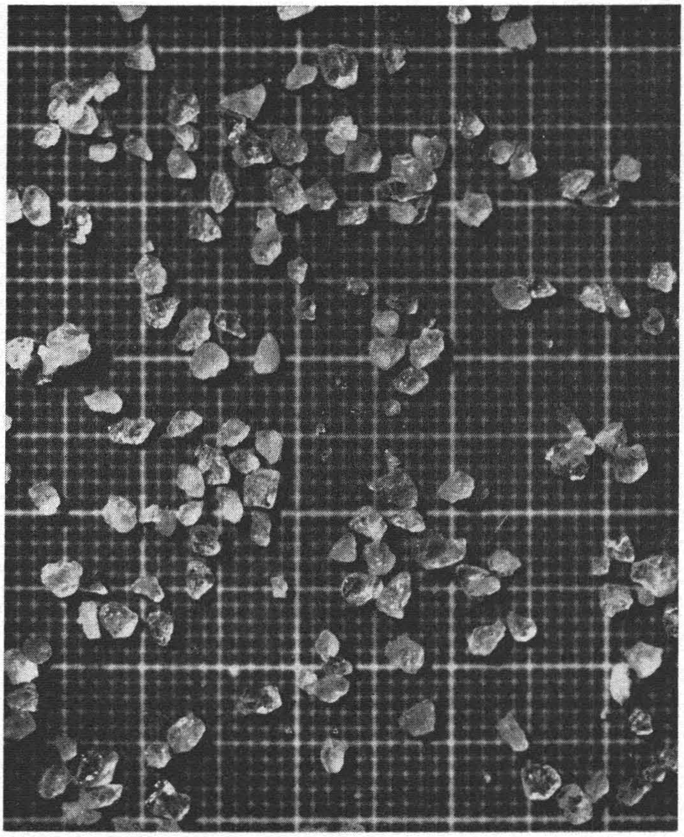

FIG. 6. Field of the stereoscopic camera.

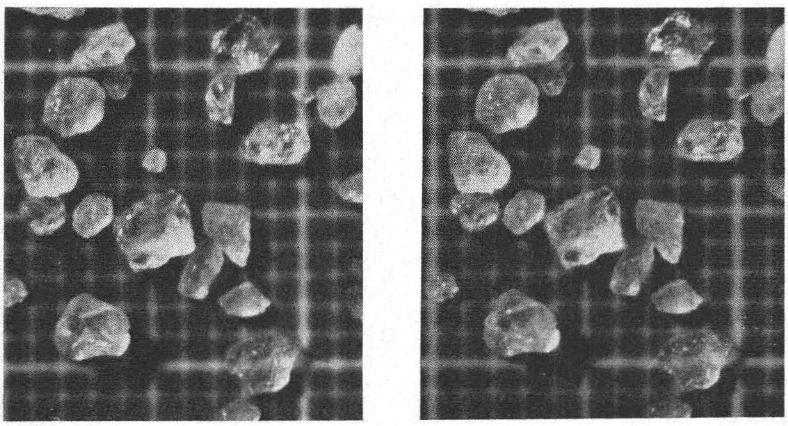

FIG. 7. Stereogram of sand grains showing a small portion of the field.

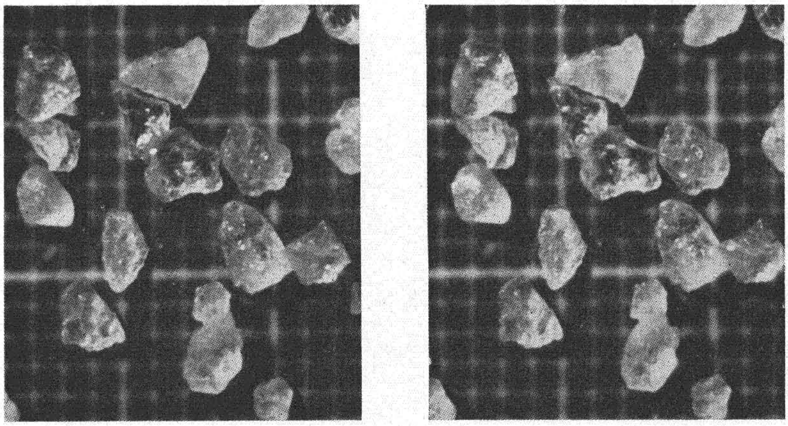

FIG. 8. Stereogram of sand grains showing a small portion of the field.

ticle in the grid plane as measured with the magnifier-scale and divided by the total magnification m

M = longest chord of the grain image in the grid plane perpendicular to L as measured with the magnifier-scale and divided by the total magnification m.

Normally the sand grains are oriented by gravity and then the height is the shortest dimension. Sometimes, however, electrostatic forces, generated by friction in the process of strewing the particles on the grid surface, orient the grains differently. In order to avoid problems arising from the orientation of the grains, the three mutually perpendicular dimensions Δh, L, and M are oriented with respect to the orthogonal Cartesian coordinate axes X, Y, and Z as follows: whichever of the three intercepts is longest becomes parameter a along the X-axis, the shortest becomes c along the Z-axis, and the intermediate dimension becomes b along the Y-axis. Figure 9 shows an example of intercept distributions obtained by tridimensional measurements.

Once the intercepts have been obtained and oriented they may be employed in any manner desired. The author used the three mutually perpendicular parameters a, b, and c in order to approximate the shape of sand grains in terms of Wadell's degree of true sphericity[7] and Zingg's shape factor.[7]

CONCLUSIONS

The technique described in this paper introduces a new dimension to many of the quantitative methods which utilize intercept measurements in order to express certain physical characteristics of sedimentary particles. Therefore it is thought that the photogrammetric method for the tridimensional measurement of sand grains may become a useful tool in various phases of stratigraphy and sedimentation. Furthermore, it is hoped that this technique may suggest the application of stereophotogrammetry to other problems in geological research and exploration.

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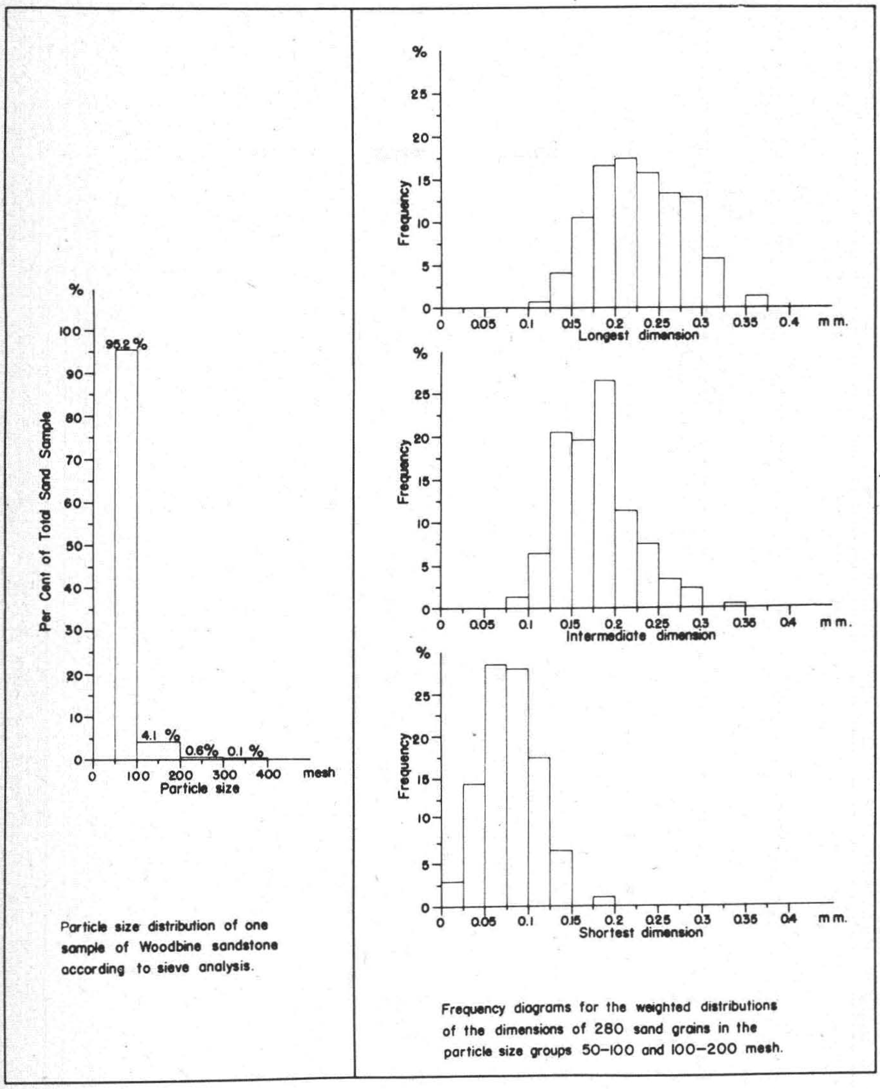

FIG. 9. Example of intercept distributions obtained from tridimensional measurements.

APPENDIX

DESIGN OF THE CAMERA

The following constants were either chosen or determined to fulfill the objectives outlined in the text:

a) Chosen constants:
   1) Fixed magnification on the film $= m = 20X$.
   2) Base/distance ratio $= \theta = \frac{1}{8}$.
b) Determined constants:
   1) Equivalent focal length of the Zeiss "Tessar" lens $= f = 53$ mm.
   2) Optical shift due to the pentaprism $= \Delta P = 30$ mm.

Using these constants the basic dimensions of the camera were calculated as follows:

1) Distance from the lens to the film $= v$;
$$v = f(m + 1) + \Delta P = (53 \text{ mm.} \cdot 21) + 30 \text{ mm.} = 1{,}143 \text{ mm.}$$
2) Distance from the lens to the object $= u$;
$$u = f\left(\frac{m + 1}{m}\right) = 53 \text{ mm.} \frac{21}{20} = 56 \text{ mm.}$$
3) Distance between the centers of the two 4×5 in. film holders $=$ camera base $= B'$;
$$B' = \theta(u + v) = \tfrac{1}{8}\,(56 \text{ mm.} + 1{,}143 \text{ mm.}) = 150 \text{ mm.}$$

4) Distance between the positions which the center of the lens takes = actual base = B;

$$B = \theta u = \frac{1}{8} 56 \text{ mm.} = 7 \text{ mm.}$$

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THE DE KONINGH MIRROR STEREOSCOPE AND THE MEASUREMENT OF *Y*-PARALLAXES

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TWO interesting papers* in the March, 1955 issue of PHOTOGRAMMETRIC ENGINEERING mention a procedure for measuring *y*-parallaxes with a mirror stereoscope and parallax bar. The procedure involves a 90 degree rotation of the photographs, one about its principal point, the other about the transferred principal point. With the photos so arranged, *y*-parallaxes appear as *x*-parallaxes which are then measured with the parallax bar. Mr. Doyle remarks that "the effect is exactly the same as that obtained in the first-order instruments by rotation of the dove prisms." The author has had occasion to work with a mirror stereoscope which includes dove prisms in its optical system, the de Koningh Mirror Stereoscope produced by G. de Koningh of Arnhem, Holland. The principal use of the dove prisms in this stereoscope has been for the detection of residual *y*-parallaxes in the transfer of points from one photograph to another. The dove prisms make this the ideal instrument for the precision transfer of points. The instrument should prove particularly adaptable to the orientation studies described by Hallert and Doyle.

* Hallert, Bertil, "Discussion of Mr. Fischer's Paper: Photogeologic Instrumentation in the U. S. Geological Survey."
Doyle, Frederick J., "Photogrammetric Measurement of Spectrograms."