DETERMINATION OF SPATIAL POSITION AND ATTITUDE OF A BOMBING AIRCRAFT BY AN AIRBORNE PHOTOGRAMMETRIC CAMERA*

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ABSTRACT

It is proposed to determine the conditions of bomb release by the use of a single photogrammetric camera, mounted in the bombing aircraft and photographing a suitable system of reference points on the ground.

In addition to giving the position of the aircraft as a function of time, the method allows the determination of the attitude of the aircraft as well.

I. INTRODUCTION

INSTEAD of measuring the position of a spatial object with several ground-based instruments, such as cinetheodolites or ballistic cameras, it is possible to determine the spatial position and attitude of the object from a single photograph of ground control points, taken by a photogrammetric camera which is installed in the object, provided it can be recovered after the experiment. The over-all accuracy of the result of such a method is comparable to that obtained from methods based on ground-established instruments.

Proposals for using airborne cameras for trajectory determination have been made before by others, for example (1), those concerned with the application of photogrammetry to non-topographical problems. These proposals have been based on the use of analogue computers. The spatial resection is simulated either by using a bundle of rays identical to the bundle which produced the photograph, or a bundle different from the original one. The first principle is used in the universal plotting machines; the second method is applied in the automatic rectifiers.

The reasons for disregarding this method until now may well have been:

(a) The limited number of high precision universal evaluation machines or rectifiers available for non-topographical use;

(b) The need for highly skilled operators and the subjectivity of the reduction method.

The analytical method until now has been considered entirely impractical. This view has been based less on the complexity of the analytical solution as compared with the ground-based intersection method, than on the amount of computing necessary for a least squares adjustment in the spatial resection problem. However, this point, which was significant in the past, has become of secondary importance with the availability of electronic computing devices. Consequently, the application of the method of the spatial resection now offers a practical, independent trajectory measuring method. The method is especially suited for determining angular orientations in space.

In the following paragraphs, the resection method is discussed in its application to high altitude bombing. The statements are general and, therefore, can be applied directly to similar problems.

II. APPLICATION TO RANGE BOMBING

In high altitude bombing it is necessary to determine the spatial position and velocity vector of the airplane at the moment of release. In addition a measurement of the attitude of the airplane is desirable. The first of these objectives is obtained by determining a series of points of the trajectory of the airplane as a function of time, symmetrically arranged with respect to the release point.

Instead of triangulating the spatial position of each of these points by intersecting two or more lines of sight originating from ground based instruments, the triangulation may be done by spatial resections based on a series of photographs of ground control points taken from the

* The contents of this paper will be published as BRL-Memo Report No. 787.

airplane. The advantage of such an approach is especially significant in terms of the total test effort. Ground operations, originally requiring skilled crews to handle measuring instruments, synchronization and communication equipment are reduced to the maintenance of a certain number of signals used to mark the control points which are being photographed from the air.

In the airplane carrying out the bombing mission, a precision surveying camera must be installed, and the operation of this camera requires one additional skilled operator. In general, neither the additional weight of the camera (250 lbs.) nor its space requirements (10 sq. ft.) will create a serious problem. The moments of exposure are recorded directly in the airplane. Often there will arise the necessity to correlate the aerial photographs with the other phases of the experiment as, for example, the trajectory of the bomb. The necessary ground-air time reference may be established by recording photoelectrically on the ground a short duration, high intensity flash on the airplane, triggered automatically by the surveying camera at the moment of each exposure.

III. Instrumentation

An automatic plate changing camera, the Wild RC-7, is available which performs the changing of 80 plates at an adjustable rate with a minimum interval of about 3 seconds. The Wild RC-5 with the Aviotar or Aviogon lens (60 degree and 90 degree with f=210 and 115 mm, respectively) appears to be, at the present time, the camera best suited for the purpose under discussion. Here the changing of plates must be accomplished by hand and, therefore, the necessary time interval between consecutive photographs will be larger. This fact, however, may not be serious because it should be possible to record photographically a sufficiently large series of images on a single plate in the daytime, provided the control points are marked by targets in high contrast against the surrounding area (e.g. white painted aluminum plates). The establishment of some kind of signals equipped with active light sources in connection with filters on the camera may prove advantageous for both day and night work.

IV. Accuracy of the Method

Methods of reduction are treated in "An Analytical Treatment of the Orientation of a Photogrammetric Camera" (2). The error theory of the general resection method has proved to be rather complex. For our purpose, the problem is simplified by assuming vertical or approximately vertical photographs. In addition, it is assumed that the differences in elevation between the control points are small compared with the flying height of the aircraft. Omitting the derivation, there are the following mean errors of the elements of exterior orientation denoted by m_α, m_ω, m_κ for the three rotations α, ω, κ, and m_{X_0}, m_{Y_0}, m_{Z_0} for the coordinates X_0, Y_0, Z_0 of the nodal point, respectively.

Formulas (1) to (6) are rigorous expressions if the control points are chosen in sets of four, arranged symmetrically with respect to the nadir point in the X, Y-coordinate system on the ground, or correspondingly, symmetrically with respect to the principal point in the oriented x, y-coordinate system on the plate.

$$m_\kappa = \frac{\mu_0 \rho}{\sqrt{[r^2]}} \tag{1}$$

$$m_\alpha = \frac{\mu_0 \rho c \sqrt{n}}{\sqrt{n([x^2 r^2]) - ([x^2])^2}} \tag{2}$$

$$m_\omega = \frac{\mu_0 \rho c \sqrt{n}}{\sqrt{n([y^2 r^2]) - ([y^2])^2}} \tag{3}$$

$$m_{Z_0} = \frac{\mu_0 Z_0}{\sqrt{[r^2]}} \tag{4}$$

$$m_{X_0} = \frac{\mu_0 Z_0}{c} \sqrt{\frac{[x^2 r^2] + c^2([c^2 + 2x^2])}{n([x^2 r^2]) - ([x^2])^2}} \tag{5}$$

$$m_{Y_0} = \frac{\mu_0 Z_0}{c} \sqrt{\frac{[y^2 r^2] + c^2([c^2 + 2y^2])}{n([y^2 r^2]) - ([y^2])^2}} \tag{6}$$

where

- c is the focal length of the surveying camera
- Z_0 the flying height above the plane of the control points
- x, y the oriented plate coordinates with the principal point as origin ($r^2 = x^2 + y^2$)
- μ_0 the mean error of unit weight of a single plate measurement, before adjustment, and
- $\rho = 206265''$

[ ] denotes summation of the specific term for the n control points

Assuming certain limits for the maximum values of the oriented plate coordinates x and y, i.e. assuming a fixed size of the photographic plate in the surveying camera, the following conclusions may be drawn from the formulas (1) to (6):

(a) The minimum mean error of the swing angle κ (formula (1)) is obtained from maximum radial distances r, and consequently is independent of the opening angle of the lens, the flying height and the distribution of the control points in azimuth.

(b) The minimum mean error of the flying height Z_0 (formula (4)) is obtained from the maximum radial distances r. In addition, the mean error is directly proportional to Z_0, but is independent of the opening angle of the lens and the distribution of the control points in azimuth.

(c) The minimum mean errors of the rotational components α and ω (formulas (2) and (3)) are obtained from maximum values of x and y, or from points with maximum differences in the absolute values of the x and y coordinates, respectively. In addition, the mean errors are directly proportional to the focal length c, thus calling for a camera of maximum opening angle.

(d) The same conditions that lead to the minimum mean errors of the rotational components α and ω, as given in (c), apply for the minimum mean errors of the X_0 and Y_0 coordinates of the nodal point (formulas (5) and (6)). In addition, the mean errors are directly proportional to the flying height and inversely proportional to the focal length.

(e) Conclusions (c) and (d) both call for maximum x and y coordinates in order to obtain minimum mean errors. These requirements automatically include maximum radial distances r as required under (a) and (b). Arranging the control points with regard to optimum geometrical conditions, formulas (1) and (4) reduce to a form in which each mean error is expressed by a certain constant divided by the square root of the number n of control points carried in the solution. For additional control points, the law governing the mean errors becomes more unfavorable because the x and y values of the additional control points are incompatible with the optimum arrangement. The center point is without benefit for the determination of κ and Z_0, because r equals zero. However, the center point is the best choice as a fifth control point for determining the rotations α and ω, or the corresponding translations X_0 and Y_0, respectively.

TABLE I

n number of points	$\frac{1}{\sqrt{n}}$
1	1
2	0.707
3	0.577
4	0.500
5	0.447
6	0.408
7	0.378
8	0.354
9	0.333

From Table I, it is obvious that more than six control points do not increase the accuracy of the result sufficiently to warrant the increase of the computational effort of setting up additional observational and normal equations. One may conclude from the above statements that the images of four control points in the corners of the plate plus the center point provide the most economical solution. Consequently, a square pattern of ground control points will provide an optimum geodetic reference system.

(f) Formulas (1) to (6) show that the mean error of any one of the elements of orientation is directly proportional to the mean error of unit weight μ_0 of the plate measurements. Therefore, it may be advisable to establish ground targets, each of which is composed of several symmetrically arranged markers. The center of symmetry serves as the geodetic reference point. The corresponding plate coordinates are obtained by taking the arithmetic average of the coordinate measurements of the individual markers. Because the random setting and measuring errors are subject to the $1/\sqrt{n}$ law, it may be concluded again from Table I that a single target should be composed of not more than four or five individual markers.

(g) The mean errors of the geodetic

TABLE II

Control Points		m_κ	m_{Z_0} °/°° of Z_0	Narrow angle 30°		Normal angle 60°		Wide angle 90°		Hyper-wide angle 120°		Ultra-wide angle 150°	
				$c=422$ mm.		$c=210$ mm.		$c=115$ mm.		$c=65$ mm.		$c=30$ mm.	
No. n	Pattern	"		$m_\alpha=m_\omega$	$m_{X_0}=m_{Y_0}$ °/°° of Z_0	$m_\alpha=m_\omega$	$m_{X_0}=m_{Y_0}$ °/°° of Z_0	$m_\alpha=m_\omega$	$m_{X_0}=m_{Y_0}$ °/°° of Z_0	$m_\alpha=m_\omega$	$m_{X_0}=m_{Y_0}$ °/°° of Z_0	$m_\alpha=m_\omega$	$m_{X_0}=m_{Y_0}$ °/°° of Z_0
4	∷	±4.6	±.022	±34.0	±.170	±16.9	±.095	±9.3	±.070	±5.2	±.075	±2.4	±.127
5	⁙	±4.6	±.022	±31.0	±.155	±15.4	±.084	±8.5	±.060	±4.8	±.062	±2.2	±.140
9	⁝⁝⁝	±3.7	±.018	±27.8	±.138	±13.8	±.074	±7.6	±.051	±4.3	±.049	±2.0	±.078

The mean error of unit weight of a measured plate coordinate, μ_0, is assumed to ±5 microns.

reference coordinates are propagated like the mean errors of the corresponding plate measurements. In order to suppress sufficiently the influence of the mean errors of the reference coordinates, it is necessary that the component of the geodetic mean error, which influences the determination of a specific orientation element, is no greater than one third of the mean error of a plate measurement of unit weight multiplied by the scale factor Z_0/c.

Table II gives the values of the various errors calculated by means of formulas (1) to (6), for a plate size of 180×180 mm. and maximum x and y coordinates of ±80 mm. The mean error of unit weight of a measured plate coordinate, μ_0, is assumed to ±5 microns. n denotes the number of points, for optimum geometrical conditions.

Figure 1 shows the mean errors of a component of translation and the corresponding component of rotation as a function of the opening angle of the lens cone for $n=4$ control points arranged in the corners of the plate. The curves indicate a minimum for the mean error of a translation for an opening angle of the lens cone of 100 degrees. Because of the flatness of the minimum, the highest overall accuracy for the problem under consideration is obtained with a cone with an opening angle of about 120 degrees.

According to Finsterwalder (3) each set of three reference points determines for the spatial resection problem a critical surface, which is analytically expressed by a circular cylinder which contains the reference points and is normal to the plane of the three points under consideration. It has become an almost stereotyped phrase in all textbooks that the spatial resection is worthless for practical purposes, because of the fact that in vertical, or approximately vertical, photographs the "critical cylinder" will always affect the result. Actually, there is no substantial reason to consider the existence of the "critical cylinder" as a serious limitation to the practical application of a single vertical photograph for a spatial resection.

The fact is that for a vertical photograph the coefficient determinant of the observational equations, or of the corresponding normal equations, asymptotically approaches zero for a decreasing opening angle σ of the surveying lens. For a unique

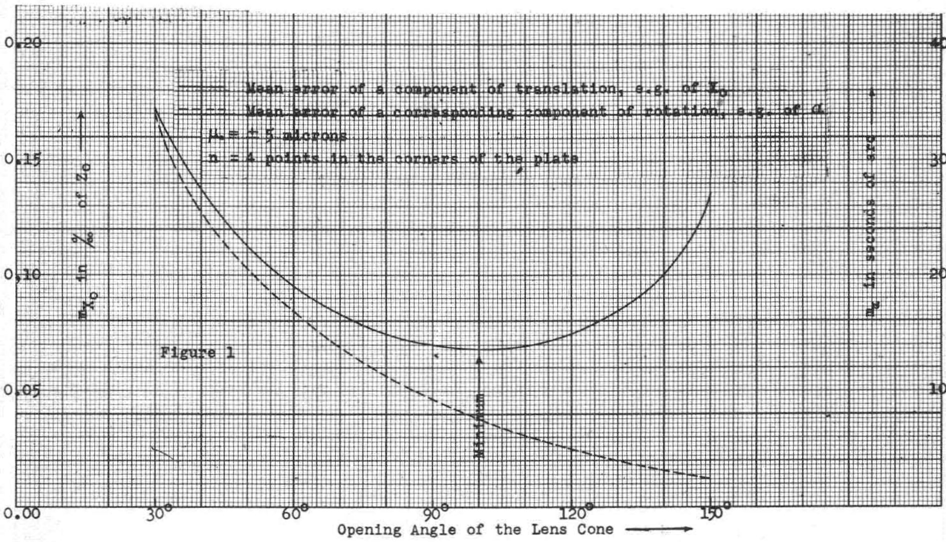

FIG. 1.—Mean error of a component of translation, e.g. of X_0.
Mean error of a corresponding component of rotation, e.g. of α.
$\mu_0 = \pm 5$ microns.
$n=4$ points in the corners of the plate.
m_α in seconds of arc.
m_{X_0} in $^0/_{00}$ of Z_0.
Minimum.
Opening angle of the lens cone.

solution based on an equilateral triangle this determinant equals, for a vertical photograph, $D=2c^6 \tan 9\sigma/2$ indicating the dominant influence of σ on the reliability of the resection solution. For $\sigma=60$ degrees or 90 degrees the value of the determinant D will be sufficiently different from zero. Because the forementioned determinant changes sign as the center of projection is moved from the inside of the "critical cylinder" to the outside, there will be a a zone of unreliability of the solution in the vicinity of the "critical cylinder." However, it is readily seen that for vertical or approximately vertical photographs taken with a 60 degree or 90 degree lens, the topography of the area being photographed will seldom cause the location of the center of projection to be located on, or in the vicinity of, the "critical cylinder."

The mean errors tabulated in Table II indicate the relative accuracy of the spatial resection method for an approximately vertical photograph. In addition, there is a systematic influence of the errors of the elements of interior orientation of the surveying camera. Since the standard for the measurements is supplied by the dimensions of the elements of interior orientation, special care must be taken to calibrate such a camera and to maintain this calibration during the flight. Of special importance is the use of glass plates having a high degree of flatness. Adequate corrections for curvature of the reference datum and for refraction must be made.

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