AN ANALYTICAL METHOD FOR THE CALIBRATION OF A VARIABLE RATIO PANTOGRAPH

Albert S. Knight, Jr., Bausch & Lomb Optical Co., Rochester, New York

ABSTRACT

To the author's knowledge, there has never before been published a mathematical approach to the calibration of a Variable Ratio Pantograph of the parallelogram type, the basic form of which is shown in Figure 1. Most textbooks on Kinematics[1] give good descriptions of the theory and correct mathematical proof, but the actual calibration of one to function correctly within 0.1 mm. (.003″) requires very careful adjustment of a number of variable design members. This precision is needed for a pantograph used in map scale reduction, and is certainly desirable in other applications of the instrument. The method described herein is applicable to variable ratio pantographs found in various industries including the machine tool industry in which very sturdy pantographs are used.

MANY mechanisms can be laid out on paper and their principle of operation can be definitely proven. However, actual performance of a manufactured instrument is a very different story. One of these is the Variable Ratio Pantograph. The basic form of the instrument (Figure 1), is drawn on a rectangular

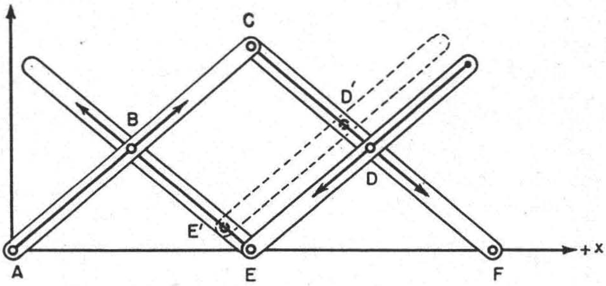

FIG. 1.

xy-coordinate system, of which point A, the axis of the polar bearing, is the origin, and AF, the line between the pole A and the stylus point F, is the abscissa. The pencil or tool, point E, is also on the abscissa when the pantograph has been adjusted correctly.

In Figure 2 the solid lines represent a theoretically correct setting of the pantograph members, while the broken lines represent the positions of the arms BE and DE when errors exist in the setting of point B on arms AC and BE. These errors may be designated as e_{AC} and e_{BE}. Each is considered to be negative, since the length AB becomes AB' which is less than AB (A being the zero end) of the respective setting scale, and length BE becomes $B'E$, which is shorter than BE (E being the zero end of the setting scale on member BE). Corresponding errors of point D will then be designated as positive, since the zero ends of the respective scales on members CF and DE are at the opposite ends from those of AC and BE.

A length ΔBE is subtracted from the distance BE. This rotates member BE into the position BE'. EE' is an arc of a circle and approaches a straight line perpendicular to DE as EE' approaches zero.

Let angle CAF be θ. Then angles AEB and DEF also equal θ, and angles CBE and CDE equal 2θ.

[1] Sloane—"Engineering Kinematics."

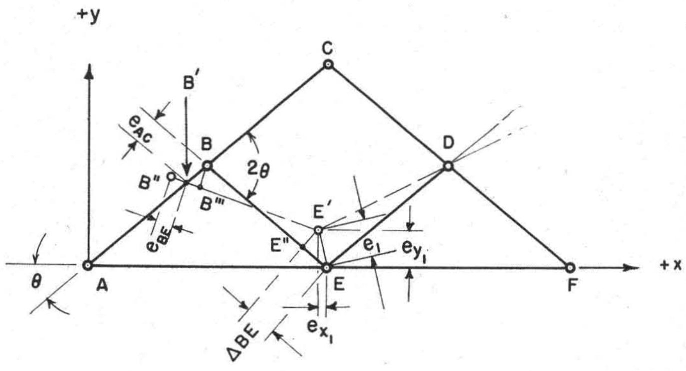

FIG. 2.

A perpendicular upon BE is dropped from E', intersecting BE at E''. Then $EE'' = \Delta BE$. Angle $E'\ EE'' = 90° - 2\theta$. Let EE' equal the error e_1 in location of point E. Then

$$\Delta BE = EE' \cos (90° - 2\theta) = e_1 \sin 2\theta. \tag{1}$$

Errors in the location of point D will cause an error e_2 similar to e_1 and

$$\Delta DE = e_2 \cdot \sin 2\theta. \tag{2}$$

Errors e_{AC} and e_{BE} place point B to positions B' and B'' respectively. B'' is then the correct point B, which is in error. If a perpendicular BB''' upon $B'E'$ is dropped from point B, it is seen that $\Delta BE = B'''B' + B'B'' = BB' \cdot \cos 2\theta + B'B''$ with sufficient approximation.

Henceforth,

$$\Delta BE = - e_{BE} - e_{AC} \cdot \cos 2\theta. \tag{3}$$

Likewise the error of length DE:

$$\Delta DE = + e_{DE} + e_{CF} \cdot \cos 2\theta. \tag{4}$$

Combining (1) with (3) and (2) with (4) give the following:

$$e_1 \cdot \sin 2\theta = - e_{BE} - e_{AC} \cdot \cos 2\theta \tag{5}$$

$$e_2 \cdot \sin 2\theta = + e_{DE} + e_{CF} \cdot \cos 2\theta. \tag{6}$$

The X- and Y- components of e_1 and e_2 will be designated respectively as e_{x_1}, e_{y_1}, e_{x_2} and e_{y_2}. Then e_{x_1} is seen to be negative and e_{x_2} is positive. e_{y_1} and e_{y_2} are positive. Angle AEE' determines the direction of e_{x_1} and is equal to angle $(E'EE'' + AEB) = 90° - \theta$. Therefore, $e_{x_1} = EE' \cdot \cos AEE'$ and $e_{y_1} = EE' \sin AEE'$ or

$$e_{x_1} = - e_1 \cos (90° - \theta) = - e_1 \sin \theta \tag{7}$$

$$e_{y_1} = + e_1 \sin (90° - \theta) = e_1 \cos \theta. \tag{8}$$

Likewise it can be shown that

$$e_{x_2} = + e_2 \sin \theta \tag{9}$$

$$e_{y_2} = + e_2 \cos \theta. \tag{10}$$

Adding (7) to (9) and (8) to (10) gives:

$$e_x = e_{x_1} + e_{x_2} = (e_2 - e_1) \sin \theta \tag{11}$$

$$e_y = e_{y_1} + e_{y_2} = (e_2 + e_1) \cos \theta. \tag{12}$$

Equations (5) and (6) give the following two expressions:

$$(e_2 - e_1) \sin 2\theta = (e_{DE} + e_{BE}) + (e_{CF} + e_{AC}) \cos \theta \tag{13}$$

$$(e_2 + e_1) \sin 2\theta = e_{DE} - e_{BE} + (e_{CF} - e_{AC}) \cdot \cos 2\theta. \tag{14}$$

Combining (11) with (13) and (12) with (14) give:

$$\frac{e_x \sin 2\theta}{\sin \theta} = e_{DE} + e_{BE} + (e_{CF} + e_{AC}) \cdot \cos 2\theta \tag{15}$$

$$\frac{e_y \sin 2\theta}{\cos \theta} = e_{DE} - e_{BE} + (e_{CF} - e_{AC}) \cos 2\theta. \tag{16}$$

Combining (15) and (16) gives the following two expressions:

$$(e_x \cos \theta + e_y \sin \theta) = (e_{DE} + e_{CF} \cos 2\theta) \tag{17}$$

$$(e_x \cos \theta - e_y \sin \theta) = (e_{BE} + e_{AC} \cos 2\theta). \tag{18}$$

Let $\theta = 30°$. Equations (17) and (18) become:

$$.866e_{x_{30}} + .500e_{y_{30}} = e_{DE} + 1/2e_{CF} \tag{19}$$

$$.866e_{x_{30}} - .500e_{y_{30}} = e_{BE} + 1/2e_{AC}. \tag{20}$$

Let $\theta = 60°$. Equations (17) and (18) become:

$$.500e_{x_{60}} + .866e_{y_{60}} = e_{DE} - 1/2e_{CF} \tag{21}$$

$$.500e_{x_{60}} - .866e_{y_{60}} = e_{BE} - 1/2e_{AC}. \tag{22}$$

Subtracting (22) from (20) and (21) from (19) give:

$$e_{AC} = .866(e_{y_{60}} + e_{x_{30}}) - .500(e_{y_{30}} + e_{x_{60}}) \tag{23}$$

$$e_{CF} = .866(e_{x_{30}} - e_{y_{60}}) + .500(e_{y_{30}} - e_{x_{60}}). \tag{24}$$

Adding (20) to (22) and (19) to (21) and dividing each by 2 give:

$$e_{BE} = .433(e_{x_{30}} - e_{y_{60}}) - .250(e_{y_{30}} - e_{x_{60}}) \tag{25}$$

$$e_{DE} = .433(e_{x_{30}} + e_{y_{60}}) + .250(e_{y_{30}} + e_{x_{60}}). \tag{26}$$

When using formulae (23) to (26) to correct the scale settings of the pantograph members it will be remembered that the algebraic signs of the resulting e-values must be reversed to give correction values C_{AC}, C_{CF}, C_{BE} and C_{DE}.

The practical application of this procedure presupposes the following mechanical conditions being fulfilled:

First, the length of the members AC and CF must be equal within .006 per cent of their length. Bearing axes B and D must always be on straight lines connecting axes A and C, and C and F, respectively, within .003 per cent, of the length AC over the entire range of the instrument. The setting scales indicating the reduction ratios must be ruled very accurately. Member DE may take a position such as $D'E'$ to avoid interference of bearing E with the pencil without disturbing the parallelogram relationship of $BCDE$, and the scale along member CF may be offset the same amount to provide readings equal to those of the other scales.

The sequence of practical operations is then as follows: First the pantograph is set approximately at a ratio of $\frac{1}{2}$ to 1. Stylus point F is set so that arm AC makes an angle of 45 degrees with the X-axis, and members BE and DE are adjusted so that point E is located correctly, that is, exactly on the X-axis and bisecting distance AF. Point F is then moved along the X-axis so that member AC makes an angle of 30 degrees with the X-axis. Departures of the point E from its true $X-Y$ position on the abscissa axis are measured and tabulated as $e_{x_{30}}$ and $e_{y_{30}}$, with $e_{x_{30}}$ being plus if the departure of E from the true position, i.e. the bisection point, is toward F and minus if toward A; $e_{y_{30}}$ is plus if E is displaced toward C and minus if away from C with respect to the X-axis. Next, point F is set on the X-axis so that member AC makes an angle of 60 degrees with the X-axis, and the X- and Y-errors of point E designated as $e_{x_{60}}$ and $e_{y_{60}}$ are again measured and tabulated.

From this tabulation the corrections to the scale setting of each member can be calculated, using above set of formulas 23 to 27. $C_{AC}=-e_{AC}$ denotes the corrections for bearing axis B along member AC, $C_{CF}=-e_{CF}$ for bearing axis D along member CF, $C_{BE}=-e_{BF}$ bearing axis B along member BE, and $C_{DE}=-e_{DE}$ for bearing axis D along member DE.

After corrective settings have been applied to the respective scales of the pantograph's arms, the 30° and 60° settings are repeated in the manner described. Residual errors are again measured and secondary corrections are computed. The procedure is repeated until residual errors are reduced to measurable limits. The indices or verniers are then adjusted to give the correct readings for the reduction ratio 1/2:1. The pantograph should now operate accurately at any desired reduction ratio setting.

EXPLANATION OF A RACK-AND-PINION INVERSOR

Wright K. Gannett, Chief Engineer, Mast Development Company, Inc., Davenport, Iowa

Foreword

IN GEOMETRY the process called *inversion*, when related to a configuration on a straight line, results in the construction of Figure 1 where:

C is the center of inversion;
r is the radius of inversion;
B is selected arbitrarily;
and A is determined (from the construction) such that $xx'=r^2$

This relationship can be mechanized by inversor mechanisms, which are well known in the science of Photogrammetry.

The practical application of an inversor is as an autofocus, that is, as a means to establish a controlled focused relationship between lens, object, and image surface.

This relationship is defined by the Newtonian form of the lens equation where the focal length becomes the radius of inversion

$$xx'=f^2$$

and

$x = p - f$ and $x' = q - f$ where
p = object distance
q = mage distance

By substitution:

$$(p-f)(q-f)=f^2$$

This readily converts to the better known form of the lens equation:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$$

Thus, an inversor is a computing mechanism that solves the lens equation as applied to positive lens systems or focusing mirrors.