member CF may be offset the same amount to provide readings equal to those of the other scales.

The sequence of practical operations is then as follows: First the pantograph is set approximately at a ratio of $\frac{1}{2}$ to 1. Stylus point F is set so that arm AC makes an angle of 45 degrees with the X-axis, and members BE and DE are adjusted so that point E is located correctly, that is, exactly on the X-axis and bisecting distance AF. Point F is then moved along the X-axis so that member AC makes an angle of 30 degrees with the X-axis. Departures of the point E from its true $X-Y$ position on the abscissa axis are measured and tabulated as $e_{x_{30}}$ and $e_{y_{30}}$, with $e_{x_{30}}$ being plus if the departure of E from the true position, i.e. the bisection point, is toward F and minus if toward A; $e_{y_{30}}$ is plus if E is displaced toward C and minus if away from C with respect to the X-axis. Next, point F is set on the X-axis so that member AC makes an angle of 60 degrees with the X-axis, and the X- and Y-errors of point E designated as $e_{x_{60}}$ and $e_{y_{60}}$ are again measured and tabulated.

From this tabulation the corrections to the scale setting of each member can be calculated, using above set of formulas 23 to 27. $C_{AC} = -e_{AC}$ denotes the corrections for bearing axis B along member AC, $C_{CF} = -e_{CF}$ for bearing axis D along member CF, $C_{BE} = -e_{BF}$ bearing axis B along member BE, and $C_{DE} = -e_{DE}$ for bearing axis D along member DE.

After corrective settings have been applied to the respective scales of the pantograph's arms, the 30° and 60° settings are repeated in the manner described. Residual errors are again measured and secondary corrections are computed. The procedure is repeated until residual errors are reduced to measurable limits. The indices or verniers are then adjusted to give the correct readings for the reduction ratio 1/2:1. The pantograph should now operate accurately at any desired reduction ratio setting.

EXPLANATION OF A RACK-AND-PINION INVERSOR

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Foreword

IN GEOMETRY the process called *inversion*, when related to a configuration on a straight line, results in the construction of Figure 1 where:

- C is the center of inversion;
- r is the radius of inversion;
- B is selected arbitrarily;
- and A is determined (from the construction) such that $xx' = r^2$

This relationship can be mechanized by inversor mechanisms, which are well known in the science of Photogrammetry.

The practical application of an inversor is as an autofocus, that is, as a means to establish a controlled focused relationship between lens, object, and image surface.

This relationship is defined by the Newtonian form of the lens equation where the focal length becomes the radius of inversion

$$xx' = f^2$$

and

$$x = p - f \text{ and } x' = q - f \text{ where}$$

p = object distance

q = mage distance

By substitution:

$$(p - f)(q - f) = f^2$$

This readily converts to the better known form of the lens equation:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

Thus, an inversor is a computing mechanism that solves the lens equation as applied to positive lens systems or focusing mirrors.

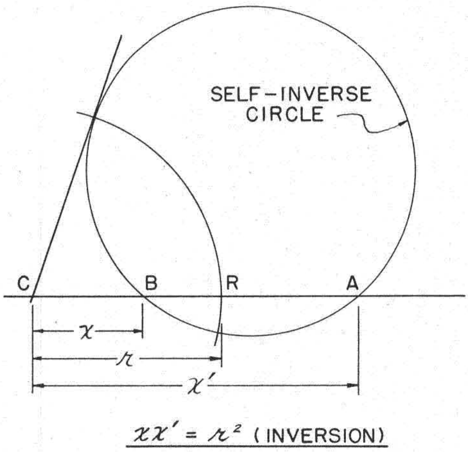

FIG. 1.—Construction for inversion.

The following analysis establishes the adaptability of the lens equation to mechanization. Consider Figure 1A in which points *CBRA* of Figure 1 are repeated. Point *D* is added such that $DC = CR = f$. Now perform the quadrangle construction of geometry such as to divide line segment *DR* internally and externally in the same ratio (harmonic ratio). Thus:

$$\frac{DB}{BR} = \frac{DA}{RA}$$

$$\frac{q}{2f-q} = \frac{p}{p-2f}$$

$$\frac{f+x'}{f-x'} = \frac{x+f}{x-f}$$

This converts to the Newtonian lens equation:

$$xx' = f^2$$

The quadrangle of Figure 1A could be mechanized to produce computations of p and q, but it would not be a practical mechanism because of the required multiplicity of linkages, sliding, pivoting joints, and consequent binding tendencies.

Inversors are, in fact, more practical autofocus mechanisms. Most of these involve linkages also (Pythagorean right-angle, Carpentier, Paucellier), which suffer from looseness and from inaccuracy where indeterminate or dead-center positions are approached, and they too may, under certain conditions, suffer from wrap-up and binding.

The exception to these drawbacks is the band inversor. It, however, is limited in that it is positive in but one direction, the direction of pull on the band. A flexible band also is less dimensionally stable than a rigid member.

The *M.D.C.* inversor described in the next section seeks to circumvent the problems posed by other inversors. It is positive in both directions, and being mechanized through racks and pinions it does not have binding tendencies* nor is it dimensionally unstable.

The following is a description of the *MDC* inversor as designed to be applied to most cameras, projectors, and enlargers.

Description

The simplest form of the inversor is shown in Figure 2. Pinions *S* and *R* are of the same size. The latter does not rotate and is constrained to move parallel to racks *g* and *h* along a straight line at a distance *f* from pinion *S*.

Taking *S* as a center, and striking an arc of radius *SR*, inscribe right angle *TRU*

$$\frac{TY}{YR} = \frac{YR}{YU}$$

* Provided the correct member is used as the driver.

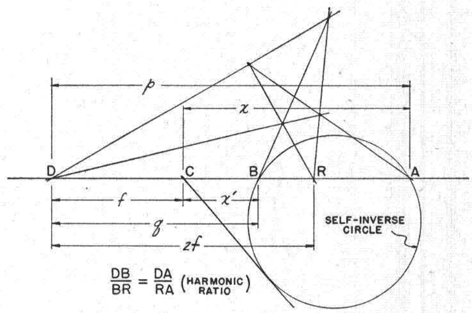

FIG. 1A.—Construction of geometric relationships of harmonic ratio and inversion which exist between the lens formulae:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \text{ and } xx' = f^2$$

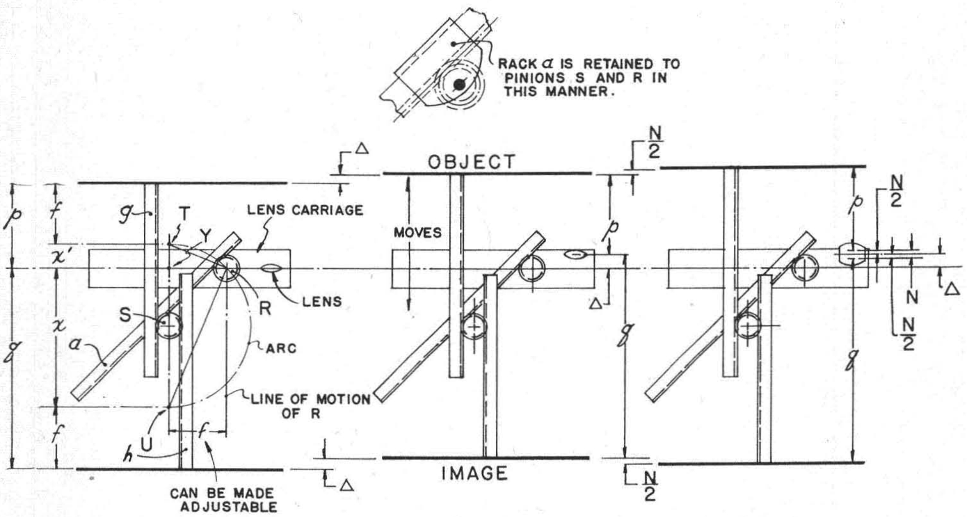

FIG. 2 FIG. 2A FIG. 2B

Simple Rack-and-Pinion Inversor.

or

$$\frac{x'}{f} = \frac{f}{x}$$

$$xx' = f^2, \quad \text{the Newtonian lens equation}$$

As R travels points T and U remain in fixed relationship to their respective racks*—hence, at distances of f from object and image respectively. As a result:

$$x' = p - f$$

and

$$x = q - f$$

for *all* positions of R. Consequently the Newtonian equation is solved for all positions of R. In this simplest form pinion R is fastened to the lens carriage *in line* with the lens. But suppose the lens were offset a fixed distance "Δ" from pinion R as shown in Figure 2A. The object and image would also be offset the same fixed amount in the same direction on their respective racks per Figure 2A.

If, in addition, we account for the nodal separation, the configuration of Figure 2B results, where N is shown as a positive quantity.

* Because rotation of S feeds out equal amounts on racks a, g, and h. *Furthermore SR changes by this same amount, as inspection of the mechanism will show.*

Figure 3 animates the inversor through magnification and reduction ranges. The labels "image-object" are interchangeable. The labels "magnification-reduction" are also interchangeable. Motions are symmetrical about the 1×1 condition.

With S rotating about a fixed center (lens, platen 1, platen 2 all in translation motion) *no binding results if the lens carriage is the driver.*** Usually, however, one platen must be fixed (say platen 2). Then platen 1, lens, and S move. For S to move, its shaft must be mounted on a carriage (not shown) that is free to move in translation parallel to g and h.

With platen 2 fixed† the lens carriage (and R) can move upward from 1×1 without bind. A little downward motion from 1×1 is also possible. But soon a condition as in Figure 3A or 3B is reached where rack a pulls on the same side of pinion S as rack h and more nearly in line with h. Binding results because the teeth of S act as common locking pins between a and h.

This binding can be overcome if at about the position of Figure 3B the lens carriage

** In parts of this motion (sufficiently remote from the 1×1 dead center) the individual platens or the rotation of S can be used as driver if desired.

† It is not shown fixed in Figure 3. Rather, the center of S is shown fixed.

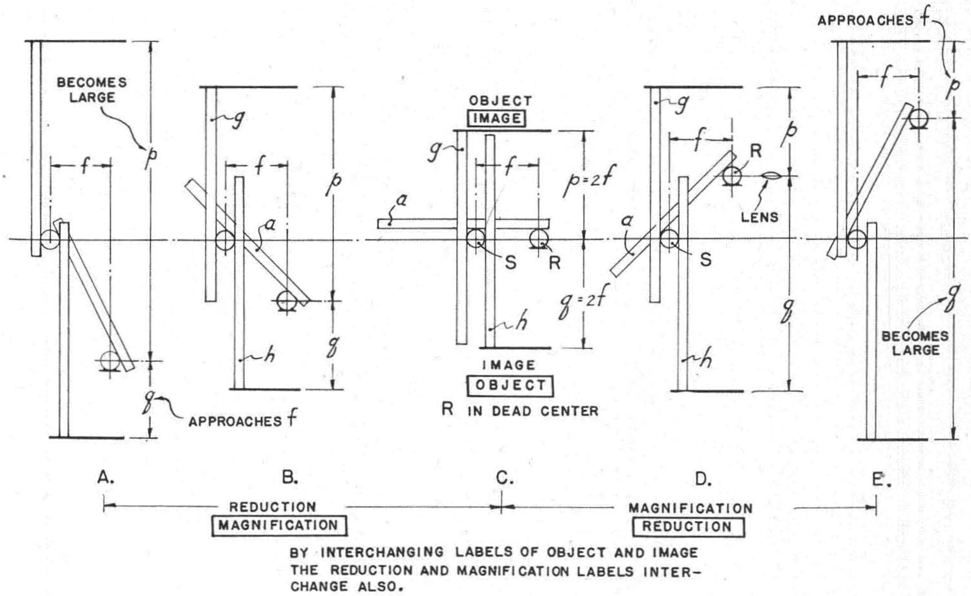

FIG. 3.—Animation of inversor over magnification and reduction ranges.

is no longer used as a driver. Instead use one of the following:

1. the motion of platen 1;
2. the translation motion of the carriage for S;
3. the rotation of S.

Thus, by having two alternative drivers (one of the above three and the lens carriage) *the entire range can be spanned without bind.*

Another possibility is that the lens carriage might be held still and the platens and the center of S (on a carriage) would move. In this event the carriage of S should be the driver. No binding results in either direction.

Description of Alternative Inversor

One difficulty with the inversor just described is the room required laterally for the sweep of rack a. To minimize this clearance problem a half-size similitude can be made per Figure 4.

Racks g and h are related through pinion W which turns with pinion S and is twice the pitch diameter of S.

Rack a is about half the length it was in the previous inversor. It is pulled or pushed by pinion R which moves in a straight line parallel to g and h and at $f/2$ from S. Pinion R is the same size as S and is kept from turning by a "key" that rides between racks i and j.

When R is moved away from the 1×1 condition to produce an angle θ, it pulls out only half as much of rack a as it did in the previous inversor. This is compen-

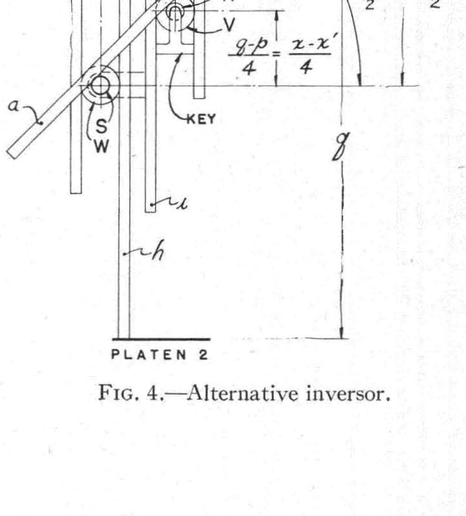

FIG. 4.—Alternative inversor.

sated by W being twice the size of S, thereby feeding out the correct amounts of racks g and h.

Pinion R is no longer attached to the lens carriage, because it travels only half as far as it "should" due to the half-size similitude resulting from its line of action being at only $f/2$ from S.

To produce the correct lens motion pinion V is added. It turns on the same center as R. But, since R doesn't turn, V idles freely relative to it.* Pinion V could be of any convenient size, but is shown (unnecessarily) equal to W.

Rack i carries the bearing for pinion S. The motion of R along its line of action (and relative to S) is doubled from i through V to rack j. The lens is carried on j and moves with the correct total motion.

Thus at the expense of two more racks and two more pinions, rack a is shortened to about half its former length.

The same general considerations apply as for the previous inversor insofar as binding is concerned. The same members in general must be used as drivers to avoid binding under any particular set of conditions.

The incorporation of mirrors in conjunction with the mechanism in Figure 4 suggests itself to double the motion and possibly eliminate some of the mechanical components. This possibility will not be investigated here.

Either inversor could be made applicable to rectifiers by providing an additional mechanism.

* Unlike W and S which turn together.