A PRACTICAL PROCEDURE TO CARRY OUT SPATIAL STEREOTRIANGULATION

Ing. Umberto Nistri, Ottico Meccanica Italiana, & Rilevamenti Aerofotogrammetrici S.p.A., Via della Vasca Navale 81, Roma

WHEN the first air photograph appeared, technicians and students of this new applied science realized immediately the great help it would render to ground surveys if it would be possible to determine the reciprocal and absolute spatial position of the camera stations at the moment of exposure.

From the very beginning it was realized that the analytical solutions, with their complicated computations arising from instrumental measurements on the photographs, could not satisfy the requirements of practicality and at the same time the economy of use. Although the theoretical fundamentals of the plotting problem, given two perspectives from two points in space, and knowledge of at least three generical points of a detail, had already been studied and analytically solved, it was deemed advisable to resort to automatic optical-mechanical procedures.

Two solutions appeared almost at the same time giving rise to the creation of the modern instrumental equipment suitable for the re-creation and orientation of the model and the successive orthographical plotting. The two methods that have in common the principle of the systematic attempts and of the successive approximations based on theory, differ only in the operational practice followed for the re-creation and absolute orientation of the model.

These methods bear the name of "double survey in space" (Figure 1) or "annulment of the vertical parallax" due to Gruber, and of the "mobile auxiliary screens" (Figure 2) due to the author of this paper.

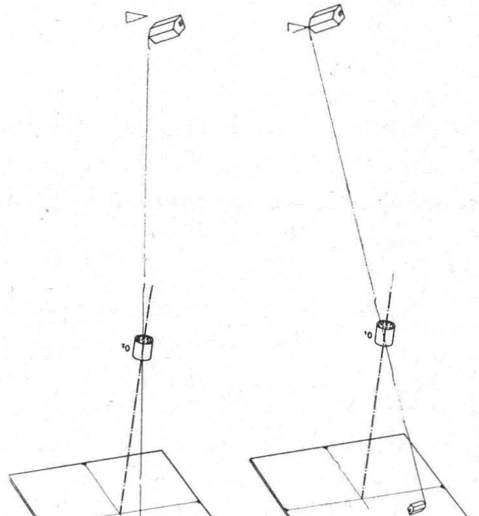

FIG. 1.—Solution through the procedure of the elevation parallax annulment (Gruber).

By means of these screens it is possible to re-create the reciprocal position of the known control points, in order to make the homologous image points of the projection of the two photograms to coincide with it.

Gruber's method is suggested particularly for stereoscopic, binocular vision plotters.

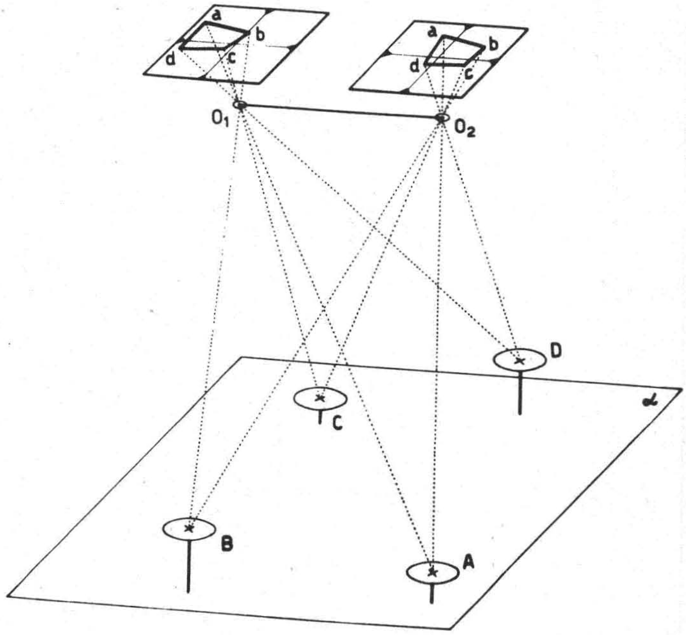

FIG. 2.—Solution through the auxiliary mobile screens (Nistri 1919).

The author's method is recommended for those plotters which are based on the principle of the double optical direct projection. It was conceived as far back as 1919 and applied in all plotters based on the above mentioned principle. With these plotters it is possible to utilize Gruber's method; in addition they have the advantage that known points of the terrain can be determined by means of the auxiliary marks of the mobile screens, in their true reciprocal position.[1]

A problem closely associated with air surveys and which has been under consideration for some time is that of "spatial stereotriangulation." Normally such bridging is done where only a few control points are available on the terrain (generally at either end of the strip), rather than in each photo model.

In any case the problem, although widely dealt with, is still far from its practical solution. The need of a practical and economical solution is particularly felt, when one considers the cost of carrying out field surveys in connection with engineering and cadastral work, or mapping extensive areas on medium or small scales.

In reality the methods which were studied and realized until recently, did not provide the longed-for practical and technical advantages, with a consequent stalemate in the evolution in this field, lasting many years.

The classical, conventional system which is universally followed, and which was anticipated by the author[2] consists of tying the photographs of the strip together, availing oneself only of the geometrical relations prevailing between them.

For the application of this method all modern plotters are particularly adapted. Its purport is limited by the amount of the error propagation in the strip, that invalidates their accuracy and performance.

Other methods that aim at peculiar conditions, such as determining known spatial directions (horizon, sun) in order to restrict error propagation in the strip, proved to be unsatisfactory because of the difficult computations and the necessary

[1] U. Nistri—Patent no. 178590, dated as of August 19th, 1919 (Italy)—"Determination of the elevation of the planimetric aerial negative in the space at the moment of exposure."

[2] U. Nistri—Patent no. 174490 of May 15th, 1919 (Italy)—"An apparatus to obtain the topographical map from aerial stereoscopic photographs."

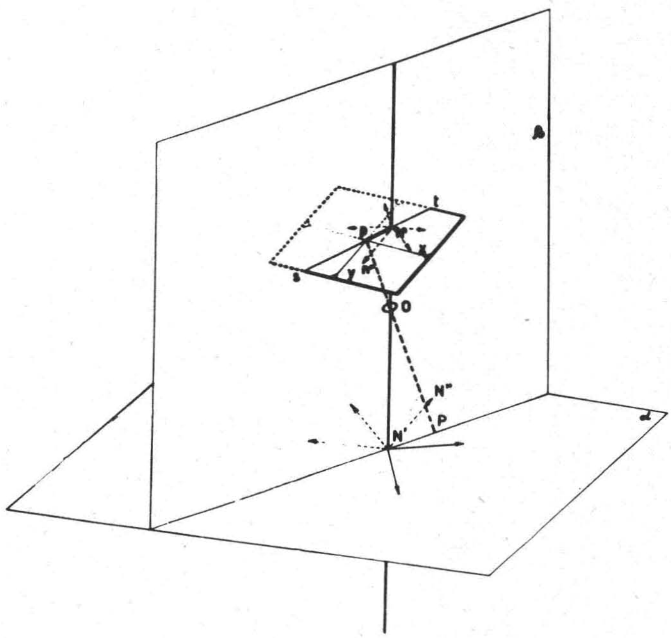

FIG. 3.—Re-creation of the tilt of the photogram with respect to the datum plane through the knowledge of the nadir point.

preliminary instrumental measurements.

For these reasons some are slightly used; others are still in such an experimental stage that one cannot foresee, at least for the time being, that the achievable degree of accuracy is equal to that of their theoretical design.

Hence, it is obvious that the awaited passage from stereotriangulation to the operative stage of wide-scale use will take place only when, as happened in the first stage of air survey, the determination of the exterior orientation of the photograms in the plotters will be directly achieved without intermediate instrumental or computation passages. It is in this direction, therefore, that new researches have been directed.

Consideration has been given to the manner of recording the photograph nadir, with the major possible approximation and with the best conditions for the immediate utilization in the plotters. As is known, the knowledge of the position of this point leaves to the angular attitude of the cameras one freedom degree only, namely the rotation about a vertical axis, of easy instrumental realization.

Before describing this approach it is perhaps fitting to examine how it is possible to re-create and absolutely orientate the model of a stereogram, once the nadir position for each photogram is known.

Figure 3 makes evident that once the nadir point is brought on the vertical line which passes through the air station,[3] the angular attitude of the photogram with respect to the plotter horizon is already re-created; actually the plane containing the air station and the line which connects the principal point with the nadir —this line is that of the maximum slope of the photogram—is vertical.

The vertical line which passes through the air station, that is through the photogram nadir, is called the "nadir axis." By rotating the camera about this axis (this rotation is indicated by ρ), the projection of the photogram on the horizontal plane of the plotter rotates about it without undergoing deformations (Figure 4), provided the exterior orientation of the camera itself remains unaltered, that is its tilt and relief displacements and particularly the angles formed by the directions emerging from the resection of the

[3] For the sake of simplicity, it is considered that the 1st and 2nd nodal points coincide with the air station.

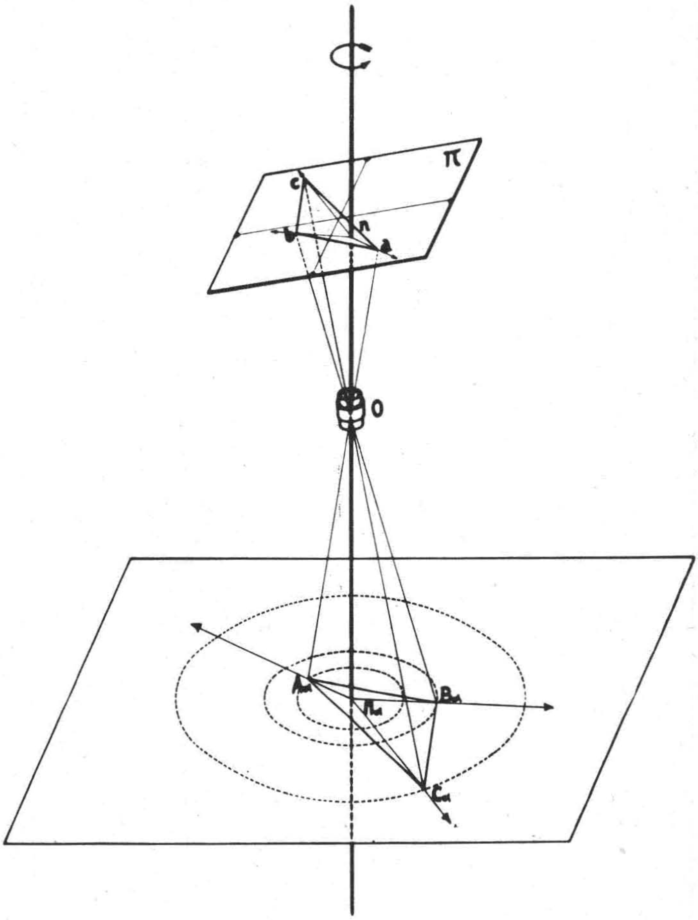

FIG. 4.—Rotation of the photogram about its nadir axis.

nadir axis with said plane, are not changed.

Among these there is one $(N'P)$ which enjoys special and useful properties; this is the direction which passes through that point on the photogram, the homologous point of which is the nadir point of the next photogram of the considered stereogram. The vertical plane which contains this direction and, therefore, the nadir axis is called "nadir plane." It is obvious that when the nadir planes of both photograms are made to coincide by means of the rotations of the two cameras about their nadir axis, there has been re-created the angular attitude (Figure 5) relative to the two photograms, without altering the absolute one for either of them with respect to the plotter horizontal line.

It is also obvious that the same result will be attained if instead of actuating the rotations ρ of both photograms, a translation (b_y) is given to one camera, in order to take its camera station on the nadir plane of the other, and then to the rotation ρ of the camera which has been translated.

However, the model is still not re-created, as the reciprocal position of the two camera stations with respect to the horizontalizing plane has not yet been restored. To accomplish this it is necessary to give to one camera a translation (b_z) along its nadir axis (Figure 6), so as to cause any two homologous rays to lie in the same plane; thereby all pairs of homologous rays will be coplanar, and the model re-created. It is obvious that in order to achieve major possible accuracy the two homologous rays, on which the operation is to be carried out, must be chosen among

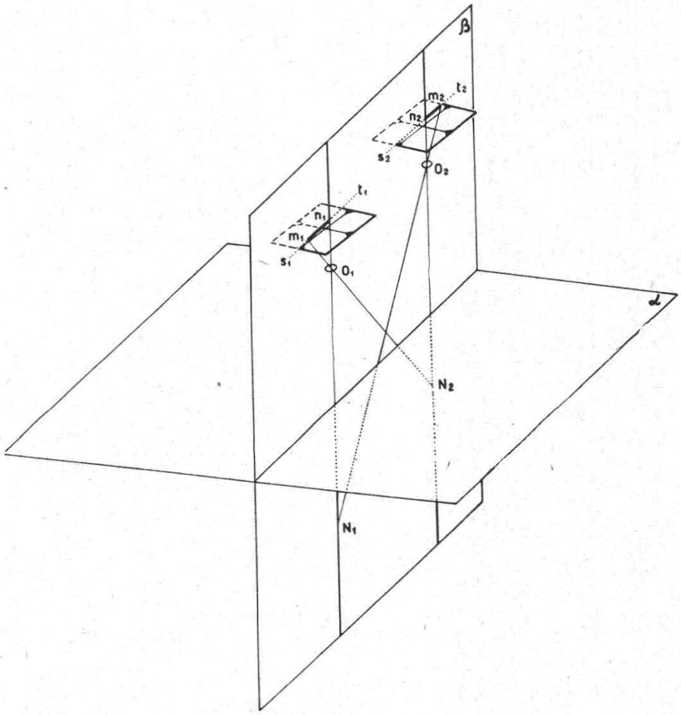

FIG. 5.—Re-creation of the angular attitude of two photograms through their nadiral planes.

those having maximum angular distances away from the joint nadir plane.

With the model thus re-created, it is assumed that its absolute orientation has been obtained as well, because it is known that the three elements—scale, azimuth and elevation of the model—are purely conventional. In order to complete the absolute attitude and orientation of the re-created model the following known elements will therefore be sufficient: distance and azimuth of two points and the absolute height of an object.

In summation therefore, it may be stated that the knowledge of the nadir point for each photogram is reduced to only three movements (ρ', ρ'', b_z; or b_y, ρ, b_z) which are necessary for the absolute orientation of any stereogram as against the five which are necessary for the relative orientation only with the normal double pyramid vertex; and to only one, the known absolute heights of the terrain, as against the three which are necessary for the absolute horizontality of the model in the latter case.

Hence, by interlinking each photogram of a strip to its precedent by means of its movements b_y, ρ, b_z, one takes advantage of an abundance of control elements for bridging—and thereby checks the propagation of the errors.

Since the orientation ρ of any photogram with respect to its preceding one can be determined with a very high degree of accuracy, in the same way as for the rotation K, the absolute angular attitude of each photogram depends almost entirely on the accuracy of locating the nadir point.

It follows also that if, in the taking camera, and in the plotters, a device suitable for recording this point directly on the photograms is provided, there are both a means suitable to exploit such recording with an optical-mechanical direct procedure, not influenced by a residual error of instrumental adjustment and the rotation of the camera about the Z axis, and one will have realized a unit of a very practical application for spatial stereotriangulation. As a matter of fact, the re-creation and absolute orientation of the models for every pair will be carried out in such unit by means of the simple instrumental

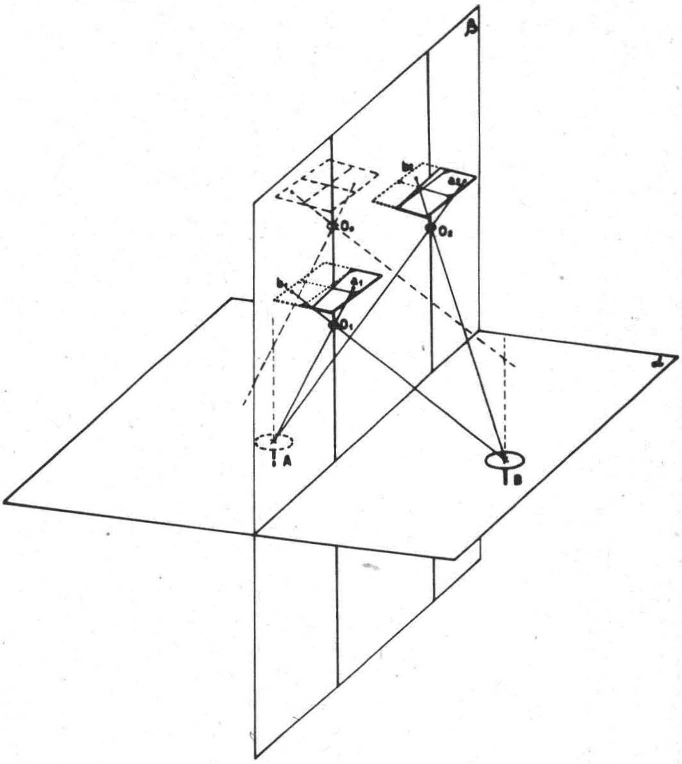

FIG. 6.—Re-creation of the model through the nucleal planes.

operation of coincidence of image points of the photograms, referred to said *Z* direction, realized in any plotter.

The following describes in short the devices used by the author to record the nadir point and to produce plotters suitable to easily carry out the indicated operations; all of these instruments are connected with each other by the leading idea expressed in the premise hereto.

It is known that recording the nadir by means of gyroscopic instruments was studied in the past and tried with varying success and inadequate results, mainly because the accuracy of the gyroscopes used proved insufficient to the purpose. Likewise, in some instances, the recordings were carried out by means of secondary cameras synchronized with the taking camera, so that the determination of the reciprocal position of the cameras required such computations and measurements as to render the project very difficult, and the attainable accuracy very uncertain. In other cases the recording took place on the photogram itself, but, because the indication of the nadir point did not occur in its actual position and with the same measurement unit as that of the axis system of the interior orientation of the photogram, preliminary operations were needed with the consequences above indicated.

When undertaking the study for recording the nadir point, the author was faced of course with the following two possibilities: either to control the camera by means of a gyroscope, in order to keep its axis vertical; or to record on the photogram the direction of the vertical line furnished by a gyroscope[4] by means of optical and mechanical media.

Although offering the practical advantage of the coincidence of the nadir point with the principal point of the photogram, the first instance was accompanied by the

[4] U. Nistri—"New orientations in the photogrammetric apparatus and procedures to compile aerial photogrammetrical maps." Paper to the 41st Congress of the Italian Society for the Progress of Science—Rome—September 1941.

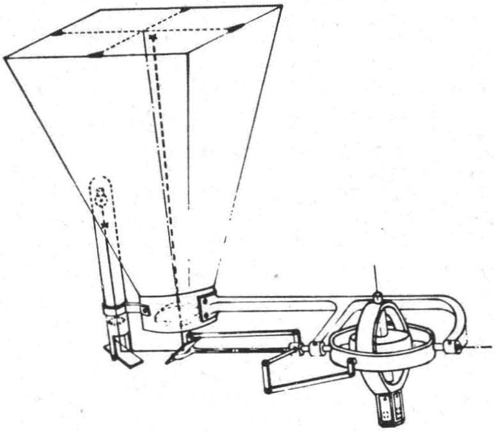

FIG. 7.—Diagram of the recording of the nadir point with the aid of the gyroscope. (Nistri 1949).

difficulties of the complexity of those electro-mechanical media suitable to insuring the verticality, and of its inadequate approximation, because of the inertia to the gyroscope stresses offered by the mass of the camera.

It was considered therefore that the second case offered major successful possibilities; while difficulties similar to the ones above take place, especially if the gyroscope is required to make an effort which upsets its stability (owing to the very delicate reaction of its axes to any effort to which they are subjected), they are present in a much smaller amount.

In 1949 a unit was designed whereby the direction, emerging from a collimator and made vertical by means of a gyroscope controlled mirror (Figure 7), was recorded through the very taking optics; this allowed one to achieve the desired object with remarkable practicality, because a small cross projected by the collimator would directly indicate on the photogram the true position of the nadir point. Almost insurmountable obstacles were occasioned by the unfavorable condition that the secondary axis of the gyroscope—this is very delicate to any stress—should assist in furnishing the requested indication, as well as the loss in luminosity due to the size of the mirror placed before the taking lens—size that is remarkable specially for wide-angle lenses.

These difficulties were known by the author. Also experience derived from numerous tests carried out on gyroscopes, in connection with their application to non-photogrammetrical purposes, made evident that the primary gyroscope axis is more stable than the secondary axis. The author accordingly turned his attention to a recording system which would ex-

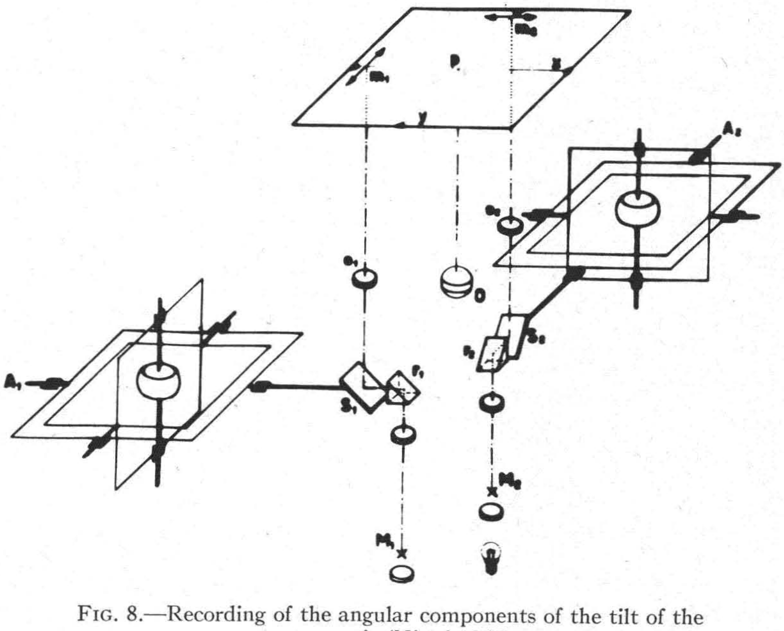

FIG. 8.—Recording of the angular components of the tilt of the camera axis (Nistri 1953).

ploit only the indication of the primary axis of the gyroscopes, avoiding at the same time submitting them to such a stress as to influence their freedom.

The unit which has thus been realized (Figure 8) is represented by two gyroscopes, the primary axes of which are at 90 degrees to each other. These axes are coplanar and parallel to those determined by the reference marks on the photogram.

At one end of the primary axis of each gyroscope there is a mirror placed 45 degrees to the direction of the axis itself; on the photogram plane, through a secondary lens which is fixed to the camera and has the same principal distance as the taking optics, this mirror reflects the image of a luminous point emerging in parallel rays from a collimator which also is fixed to the camera and parallel to the primary axis of the gyroscope.

When the camera axis is vertical, the image of each luminous point is formed in the center of the corresponding photogram axis. When the gyroscopes are spinning, and the camera undergoes a rotation about an axis parallel to the primary one of either gyroscope, it will happen that, because of the fixed position of the mirror attached to the primary axis of the gyroscope, the image of the corresponding luminous point will move along a line common to the photogram plane and to that normal to said axis; at each instant its displacement with respect to the original position, namely, the correspondent axis of the photogram, will represent the product of the value of the principal distance by the tangent of the camera rotation; that is, it will give exactly, in amount, direction and sense, the departure of the nadir point from the principal point of the photogram.

Obviously for any tilt and tip of the camera, the displacements of the two crosses from their original position will represent the components on the photogram axis of the vector, the origin of which is the principal point and its end is the nadir point, and therefore, finally the latter's position.

It is to be noted that the only work of the primary axis of the gyroscope is that of controlling the mirror fixed to it, and this gives the system the maximum stability.

Figure 9 shows the position on the photogram of the two luminous points for a certain tilt and tip of the principal camera axis. In order to restore the latter's position with respect to the vertical line in whichever plotter (furnished with the normal rotations ω, ϕ) is used it will be sufficient at the very beginning to set the axes of the camera focal plane parallel to the corresponding plotter axes. The cameras can then be swung by means of the rotations ω, ϕ, in order to take the images of the two luminous points respectively on the planes XZ and YZ, which pass through the camera station. Once the attitude for either photogram of a stereogram with respect to the vertical line is restored, it will be necessary to make the nadir planes coincide, photogram by photogram, as previously illustrated, without varying their individual attitudes, in order to create the model.

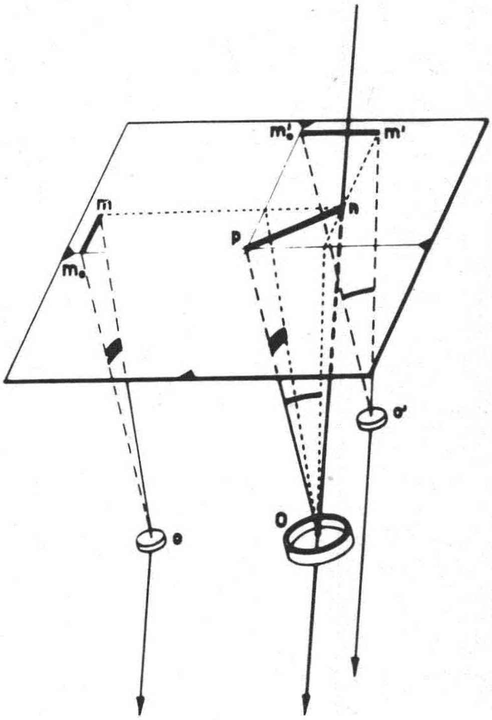

FIG. 9.—Recording of the components of the nadiral distance (Nistri).

To obtain this result in all plotters presently in use, it is necessary to utilize the computation which gives the variations to be given to the ω, ϕ rotations of the photogram, in order to keep the same tilt along the vertical line, in function of the

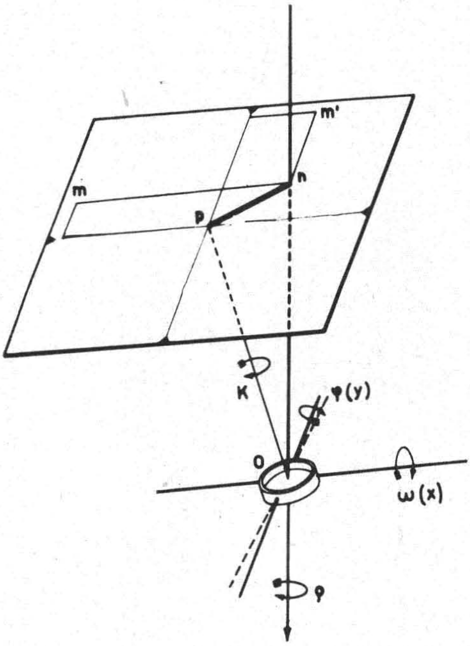

FIG. 10.—Rotations of the camera: $\omega\phi$ tilt components; K about the principal axis; P about the nadir axis.

rotation K to be imposed so as to obtain said coincidence of the nucleal planes.

This procedure can be avoided, if the support of each camera allows it being rotated about its nadir axis Z, namely about the axis which is perpendicular to the instrument horizon, which passes through the camera station.

This possibility has been realized in the new camera supports of the author's recent direct optical projection plotters, such as the Photocartograph Mod. V, and the Photomultiplex D/III; it represents an advantage that all plotters should utilize.

Figures 10 and 11 indicate that the camera support makes use of three rotations, ω, ϕ and K, completely similar in shape and succession of controls to those realized in modern conventional plotters; however, the axis of the ω rotation which bears the other two—in conventional plotters this is in a fixed position and parallel to instrumental X direction—is not a primary axis, but is supported in turn by another axis of rotation, that is the primary rotation, and namely the nadir Z axis, corresponding to the vertical direction. Thus the rotation axis, although

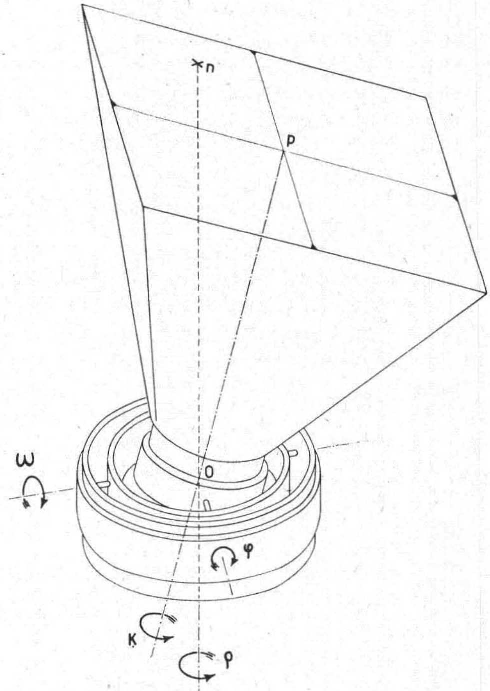

FIG. 11.—Camera support to assure the rotations ω, ϕ K and P about the nadir axis (Nistri).

always remaining parallel to the horizon XY of the instrument, can rotate about the instrumental direction Z.

In the new supports (Figure 12) the

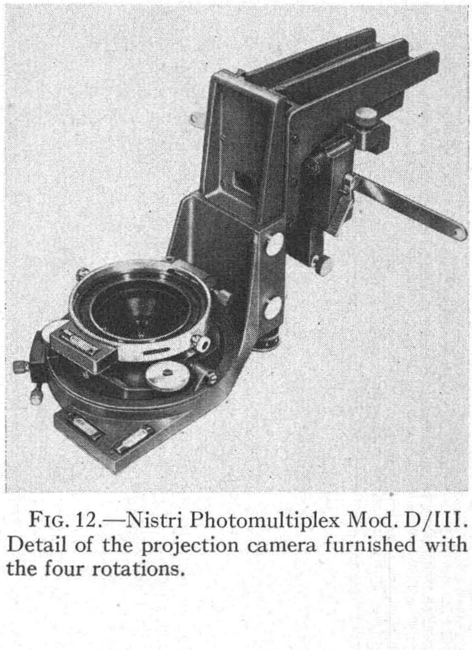

FIG. 12.—Nistri Photomultiplex Mod. D/III. Detail of the projection camera furnished with the four rotations.

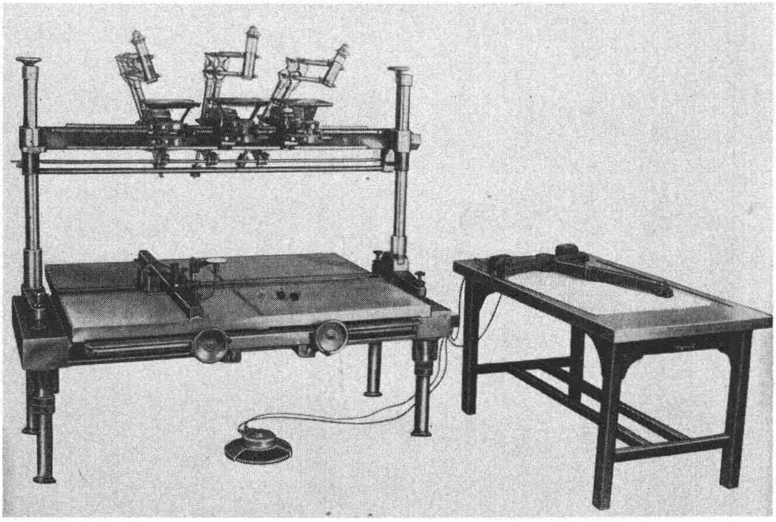

FIG. 13.—Nistri Photocartograph Mod. V.

primary rotation—indicated by ρ and called "orientation"—takes place, therefore, about a fixed axis parallel to the instrumental direction Z; the axis of the secondary rotation ω is a straight line at right angles to the former direction and therefore always parallel to the horizon XY of the instrument; the tertiary rotation ϕ takes place about an axis that is constantly at right angles to the axis of the previous rotation ω, whereas the axis of the fourth rotation K is that of the camera always at right angles to the axis of the rotation ϕ.

If the camera tilt with respect to the vertical line be zero ($\omega=0$, $\phi=0$), the axis of the rotations K and ρ would obviously coincide, and therefore the two rotations would be equal; in all other cases the rotation ρ causes the camera axis to describe a conical surface that has for its aperture the tilt of the axis itself.

The only freedom degree which remains to the camera in consequence of the knowledge of the position of the nadir point on the photogram, leaves undetermined the position of the camera axis on this cone.

In the Photocartograph Mod. V (Figure

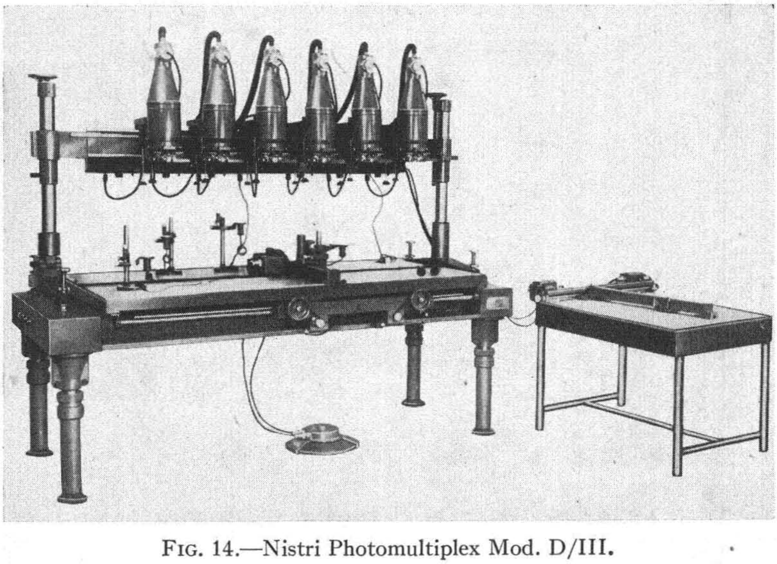

FIG. 14.—Nistri Photomultiplex Mod. D/III.

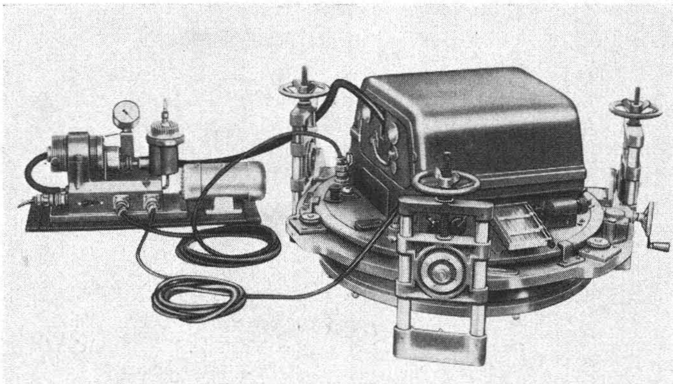

FIG. 15.—Nistri Camera Mod. FOMA 53/B, with wideangle lens OMI-Rigel, and recording of the components of the nadir distance through gyroscopes.

13) and in the Photomultiplex Mod. D/III (Figure 14), the orientation axis can be taken to coincide with the instrumental Z axis by means of two bubbles at 90 degrees, and foot screws. Another set of two bubbles is fixed parallel to the axis of the ω, ϕ rotations of the camera, in order to make the principal axis of the latter to coincide once again with the orientation axis and, hence, to coincide with the Z axis, which in said instruments is vertical.

Furthermore, the camera supports can be rigidly connected with each other, and can slide along the supporting beam without changing their recpirocal and absolute orientation, that is without disturbing the model. Thus the model formed by two or more cameras can be moved within the instrumental limits in order to facilitate aerial stereotriangulations, introducing the successive cameras for the bridging operation.

To these features and to the high accuracy of the optical and mechanical manufacturer are added the known mark and pencil controls by means of synchronized electromagnetic devices, which give to these instruments the same possibilities of performance of the larger and more complicated orthographic plotters and with the advantage of a greater practicality of use and lower cost.[5]

To obtain on the photograms the nadir point recorded according to the stated principle, the taking camera FOMA Mod. 53/B has been formulated. It is furnished with all requirements in order to assure a normal use, namely, the taking of aerial photographs for photogrammetric use, as well as the device for recording the nadir point by gyroscopes. This instrument deserves some elaboration. As one can see from Figure 15 all cinematic mechanisms necessary for its functioning and the two gyroscopes are contained within a large drum (Figure 16) which is supported by a simple antivibration base. The drum plane serves as a support to the true and proper camera (Figure 17) represented by the focal plane of the photogram and the lens with shutter.

The magazine rests against the camera

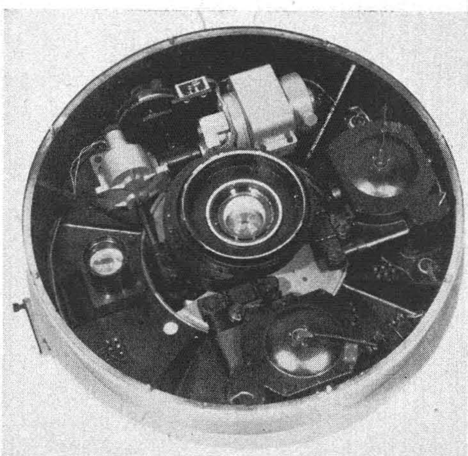

FIG. 16.—Camera FOMA 53/B. View of the interior of the supporting drum wherein are installed the gyroscopes.

[5] U. Nistri—"Double optical direct projection and a new address in the photogrammetric instrumentation"—Revue of the Cadaster and S.S. TT. EE. Finance Ministry, Rome, 1948.

plane and is supplied with means to insure the change of the film and its flattening at the time of exposure. The design is such as to make possible taking out the camera with ease, both as regards the optical and geometrical controls and the eventual substitution of another camera, without loss of adjustment of the nadir recording device, or replacement of operative parts.

The characteristics of the FOMA Mod. 53/B are as follows:

format 23×23 cm.[2] (9″×9″)
photographic material: film
lens: OMI Rigel, 152 mm. focal distance (6″), aperture: 1:6.3, with yellow photometric levelling filter
useful field: 90°
central shutter: max. speed 1/240 sec.
geometrical features: (distortion and resolution power as per graphs in Figure 18)

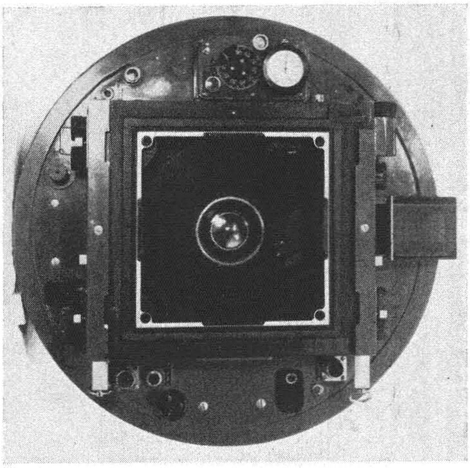

FIG. 17.—Camera FOMA 53/B with the film magazine removed.

flattening and depression with a patented system which insures a sudden

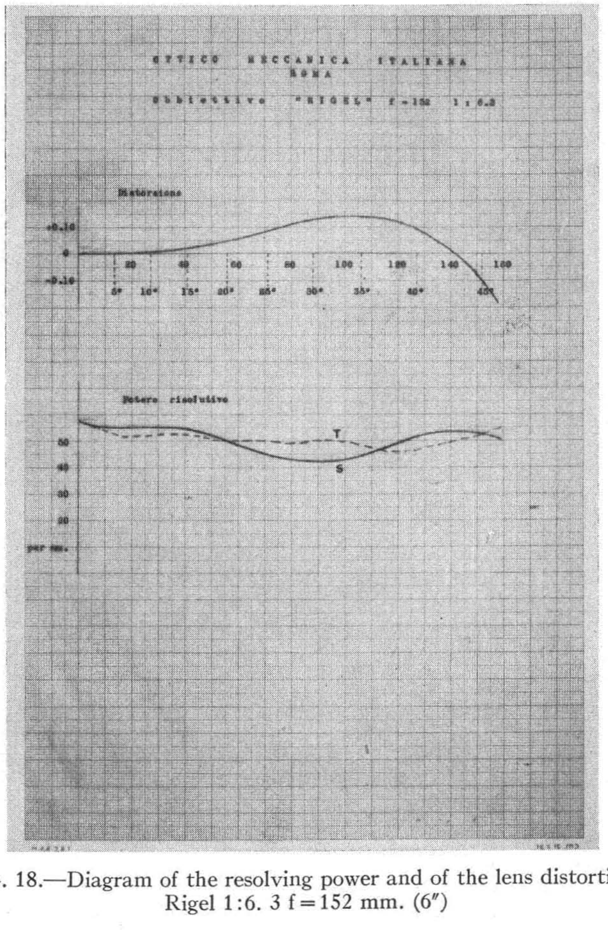

FIG. 18.—Diagram of the resolving power and of the lens distortion.
Rigel 1:6. 3 f=152 mm. (6″)

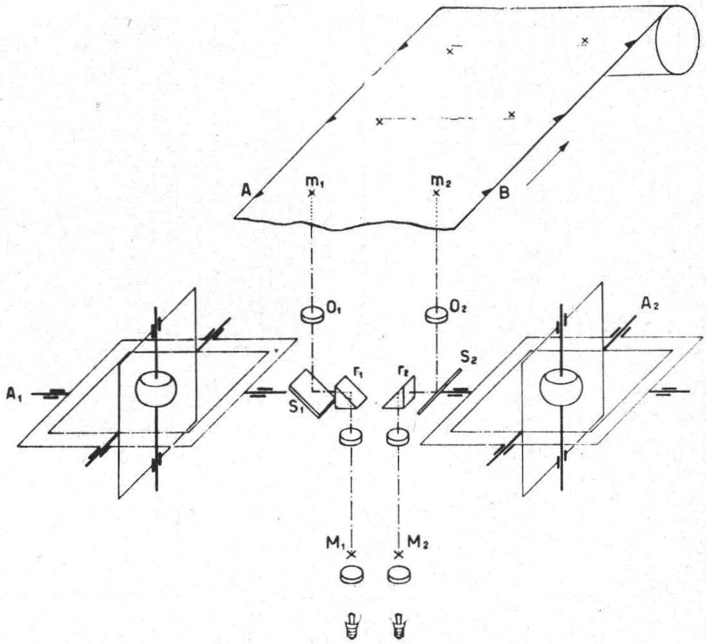

FIG. 19.—Device for the experimental control of the two gyroscopes.

differential pressure at the instant of exposure.

The antivibration mount provides for rotation for drift correction. In addition it is supplied with a target sight—drift indicator-intervolometer and with devices to adjust for the horizontality on board the aircraft, by means of two levels at 90 degrees.

The antivibration system is absolutely safe and the camera is very stable; its center of gravity is at about the same height as the suspension.

The magazine is easily interchangeable and bears either 60 meter film of standard 23 cm. (9″) for 240 photograms 23×23 cm. (9″×9″), or 120 meter film.

Although it is realized that the indicated instruments have been in use only a short time, and that the generical purpose of the taking and plotting instruments has been completely reached[6] as far as stereotriangulation is concerned, definitive experiments must still be carried on.

Recently, a series of preliminary tests have been made, the results of which are very interesting and optimistic. Some of these have already been reported by Dr. Gino Parenti.[7]

A brief summary of these two papers is presented herewith but I suggest the original texts be used by those who desire to delve further into this subject.

The tests were carried out allowing the primary axis of two gyroscopes to face each other, on the same alignment, in order that the stresses might operate in the same direction, but in a contrary sense for each gyroscope (so that their influence could take place with a contrary sign with respect to the two vertical lines which they must determine). The two crosses which indicate the vertical line for either units were projected on the emulsion side of a film in continuous motion, so that the re-

[6] As proved by the exhaustive tests by Prof. Luigi Solaini of the Polytechnic of Milan. "Report on the test carried out on one Photomultiplex D/III." Paper presented to the 2nd National Congress of the Italian Society of Topography and Photogrammetry—Padua, Oct. 1953.

[7] Gino Parenti: "Device to record directly on the photograph at the instant of exposure the nadir point." Paper presented to the VIIth International Congress of Photogrammetry. Washington, Sept. 1952.

Gino Parenti: "The new photogrammetric film camera." Paper presented to the 2nd National Congress of the Society of Topography and Photogrammetry, Padua, Oct. 1953.

cordings did not superimpose each other (Figure 19). The two units were adjusted and the reciprocal positions of the two crosses on the film were measured under static conditions of the gyroscope support.

Their behavior was then examined after submitting the whole system to stresses both under laboratory conditions and under actual flight conditions. The photograph of the position of the two crosses has been recorded periodically on said film in continuous motion. It is obvious that the accuracy of the gyroscope indications in the described system, would be evidenced by the constant reciprocal position of the two crosses on the film, because the system is correct when they keep the same distance after repeated tests. The displacement from such condition, translated in minutes, results therefore in the value of the achieved accuracy.

Laboratory tests were carried out by placing the device on an oscillating bench (of the same type as that used to check the gyroscope used for aerial navigation), and given some acceleration to said plane. The average displacement for some 100 readings was found to be $\pm 2'$.

For the tests carried out in flight (Figure 20) with a light tourist airplane, at the altitude of about 1,000 meters (3,281 feet), under not excessively favorable atmospheric conditions, and on different days, the average displacement for more than 500 readings was $\pm 5'$.

The results are therefore perfectly satisfactory, if one considers that to approach the results achieved in the laboratory tests, better conditions can be realized by using airplanes with a major stability, as generally is the case with those used for the photographic work.

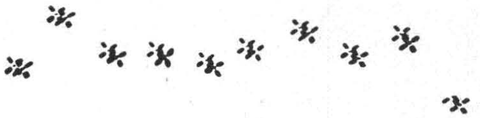

FIG. 20.—Example of recording for the reciprocal control of the two gyroscopes.

In conclusion, it is opportune to make some observation as to the limits to the possibilities of use of the described system.

It is easily understandable that if the tests underway confirm the results achieved in a preliminary stage, the method, besides being practically economical, will show sufficient accuracy for all the mapping necessities at large and medium scale. If not and if the achievable accuracy is of an inferior order, the method nonetheless will still find a very large application in the vast survey fields; in any case its approximation is sufficient to replace the principal point radial triangulation with that utilizing the nadir point; in fact it has a major approximation and permits an economy in determining control points on the ground, the density of which will be set by the degree of accuracy achieved by the method, and by that determined by the scale and scope of the survey.

With the available data, one already can say that the method possesses sufficient approximation to rectify the photograms without the support of ground control points, as the residual deformations of the photogram with respect to its true vertical projection, keep within the graphical tolerance up to at least three enlargements of the photogram.

NEWS NOTE

OTHERS ARE LEARNING ABOUT PHOTOGRAMMETRY

Much interest has been evidenced in two recent newspaper stories about photogrammetric activities. The first appeared in the *Wall Street Journal* of December 17, 1954, was entitled "Number of Firms Expands as Suppliers Develop Faster and Cheaper Equipment—A Time Saver for Business"; and was written by Ellis Haller, Staff Reporter for the *Wall Street Journal*. The *Washington Sunday Star* of January 9, 1955 carried the second story. It described the beginning, growth and present activities of Alster & Associates with headquarters at 6135 Kansas Ave., N. E., Washington, D. C. The story was written by Tom Burke and was captioned "D. C. Photogrammetry Firm Wins Wide Repute."