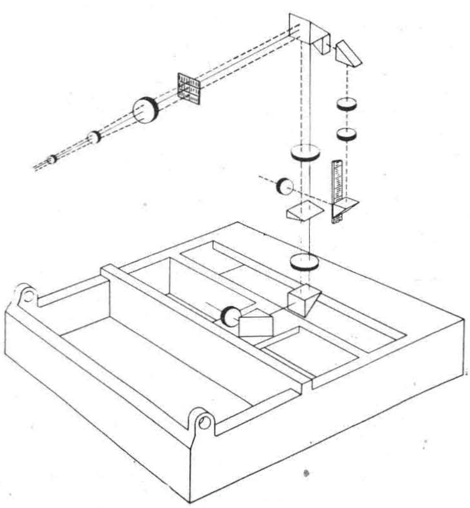

FIG. 6.—Base and optical micrometer.

through use of prisms attached to the x platform, to be collinear with the axis of a second lens attached to the base. The x scale attached to the x platform moves in the focal plane of the second lens, and therefore the image of the y scale is superimposed on the x scale above the x coarse graduations. The x platform is shown in Figure 5. The y scale lens on the x platform and the x scale lens on the base have the same focal length so that the images of the y graduations have a one to one ratio to the x graduations. The light rays between the two lenses are collimated and therefore are independent of the variable distance due to x translation. Thus the two scales are optically linked at a common magnification despite orthogonal translation.

The x scale lies in the focal plane of the reading microscope objective attached to the base. The base platform is shown in Figure 6. The images of the two scales, formed by the microscope objective after passage through the optical wedge, and the image of the micron ladder lie in a common focal plane where they are viewed through the reading microscope eyepiece. The micron ladder is illuminated from the side, and the viewing stage from the bottom.

This concludes a brief description of the design considerations of the MM 100 mm. comparator.

DISCUSSION OF MR. FISCHER'S PAPER: PHOTOGEOLOGIC INSTRUMENTATION IN THE U. S. GEOLOGICAL SURVEY

Bertil Hallert

IN HIS paper, Mr. Fischer mentioned the use of the mirror stereoscope and the parallax bar for the determination of elevation differences on the ground, from approximately vertical aerial photographs. Because, up to now, only the x-parallaxes seem to be used for such determinations, I will briefly mention how the quality of the results can be considerably increased if the y-parallaxes are also taken into account. Some other related questions will be discussed.

In my teaching work at Ohio State University it was necessary that I use the mirror stereoscope and parallax bar as the main stereoscopic instrument because none of the ordinary high precision stereoscopic instruments were available. The German phrase "In der Not frisst der Täufel Flöhen" is a good expression in such situations.

This simple instrument—the mirror stereoscope and parallax bar—has also

NOTE: Following Mr. Fischer's reading of his paper at the Semi-Annual Meeting, a discussion was invited. At that time Dr. Hallert made some remarks. Due to the interest in his statements then and later indicated, Dr. Hallert, after his return to Columbus, embodied the statements in this paper and also included other material—*Editor*.

proved to be of very great value for advanced photogrammetric teaching and training and in some respects can be used as a good substitute for the high-precision, high-cost instruments which are too expensive at least for beginners.

Some of the principles employed seem to be of considerable interest for normal photogrammetric practice, and in some respects an important development of the use of simple stereoscopic instruments can be expected, especially for radial and aerial triangulation.

The fundamental operation is the *measurement* of y-parallaxes, that can be performed very conveniently with the mirror stereoscope and parallax bar in the same manner as the measurements of x-parallaxes—a procedure that is well known to every photogrammetrist.

From the normal set-up of the pictures for x-parallax measurements, one picture (normally the left) is rotated 90 degrees around the principal point, and the other is rotated through the same angle around the transferred principal point. The y-parallaxes from the ordinary set-up now appear as x-parallaxes to the observer, and can be measured with the parallax bar in arbitrary points. This is an old well-known trick that often is used in stereoscopic instruments, early for instance, in the Zeiss radial-triangulator. Colonel Löfström in Finland has reported on the application of the same principle to the mirror stereoscope.

The relation between the measured y-parallaxes and the elements of the relative orientation of the pictures is well-known and is normally given in the form of differential formulas for small quantities. For dependent pairs of pictures we have:

$$p_y = dby_2 + (x - b)d\kappa_2 + \frac{(x - b)}{h} d\phi_2 - \frac{y}{h} dbz_2 - \left(1 + \frac{y^2}{h^2}\right) hd\omega_2. \quad (1)$$

The signs are chosen in accordance with the stereoplanigraph and can easily be converted to any other instrument with given scales. p_y is defined as $y_2 - y_1$ (the y-coordinate of points from the right picture minus the y-coordinate of the corresponding points from the left picture, if the subscripts of equation (1) refer to the right picture).

From at least five measured y-parallaxes, the corrections of the elements of the relative orientation can be computed from (1). If the measurements have been performed in 6, 9 or 15 symmetrically located points, the computations can be performed in connection with an adjustment of the measurements, according to the method of the least squares, from convenient formulas that also allow a determination of the obtained accuracy. See references [1] [2] and [3]. The formulas have certain limitations and well defined assumptions which must be noted.

In this connection it should be mentioned that the formulas for dbz_2 of scheme 5 and $[vv]$ of scheme 4 and 5 in [2] need corrections. The expression for dbz_2 should be:

$$dbz_2 = \frac{h}{12d}(-p_3 + 5p_4 + p_5 - 5p_6 + 2p_7 - 2p_8).$$

In the expressions for $[vv]$ the two first terms should be read $-2p_1+2p_2$. It should also be emphasized, even though it was mentioned in the publication, that the formulas for the error propagation of the absolute orientation in reference in [2] have an approximate character, since the correlation with the relative orientation is neglected in the formulas. But the influence is taken into account in the expressions for $[vv]$ in a generalized manner.

From the determined corrections of the elements of the relative orientation, corrections of the elevations of the model points can be determined from the well known formula:

$$dh = \frac{hy}{b} d\kappa_2 - \frac{h^2 + (x - b)^2}{b} d\phi_2 - \left(1 - \frac{x}{b}\right) dbz_2 - \frac{(x - b)y}{b} d\omega_2. \tag{2}$$

Of particular interest are primarily only the terms which have a non-linear influence upon the elevations. The linear influences will be taken into account in the next step—the translation and rotations of the entire model. This is performed with the aid of the simple formula

$$dh = xd\eta - yd\xi + dh_0 \tag{3}$$

which expresses the relation between the elevation discrepancies in control points and the rotations and translation of the entire model. Of course, the x and y in this and other formulas are determined from the left picture.

From at least three elevation control points the rotations and translation of the model can be determined, and then applied to the correction of arbitrary points. If more than three elevation control points are given, the adjustment formulas from reference [2] can be applied.

Under the normal assumptions concerning the pictures and the ground an astonishing accuracy can be obtained with this simple procedure. The results, obtained by students at Ohio State University from contact diapositives of aerial pictures, are in many cases better than can normally be expected from high precision plotters. With a limited accuracy also ordinary contact prints can be used.

The advantages of the described method are obvious. The number of theoretically necessary control points can be decreased from five to three although of course additional points always are wanted. The measurements can be performed under simple field conditions. The measurements of the y-parallaxes check themselves and give good information on the accuracy that can be expected. A complete theory of errors can easily be established for the procedure in which the accuracy of the measurements of the y- and x-parallaxes is of fundamental importance. In reference [4] a rather complete investigation of the method is given.

The same principles can be applied to some other problems.

If the y-parallaxes are measured in a great number of points of near vertical pictures in the indicated way, and the relative orientation is computed from six symmetrical points, corrections to the measured y-parallaxes in all points can be computed from formula (1). The residual y-parallaxes are then of great interest since large and systematic errors can be detected. Consequently the possibility of the relative orientation of the pictures in a stereoscopic plotter can be determined from the simple y-parallax measurements. The presence of grave film disturbances and distortions can in this way easily be detected under field conditions. Of course, the limitations concerning the allowable deviations from the vertical must be noted, and also other assumptions, concerning for instance the elevation differences on the ground. Furthermore, the computed angles $d\phi_2$ $d\omega_2$ $d\eta$ and $d\xi$ can be used for a comparatively good determination of the nadir point and the isocenters of the pictures.

The absolute values of the ϕ and ω of the two pictures can obviously be determined as:

$$\phi_1 = d\eta; \qquad \omega_1 = d\xi; \qquad \phi_2 = d\phi_2 + d\eta; \qquad \omega_2 = d\omega_2 + d\xi.$$

From these data and the principal distance of the camera, the location of the nadir point and of the isocenter can be determined. It is of course of great importance that the rotations of the pictures for this purpose be accurately 90 degrees because of the linear influence of errors of the rotations upon the elevations (see formula 2).

In many cases the angles ϕ and ω are of great direct importance. Especially for radial triangulation this simple method for the determination of the nadir point and of the isocenter seems to be of interest, because one of the most important systematic errors of the triangulation can be at least approximately eliminated. The angles ϕ and ω can of course be transformed from model to model so that only a few elevation control points along the strip are necessary for the determination of the absolute angles and for checks.

Finally, if the pictures have been taken with sufficiently small deviations from the normal case, a kind of numerical aerial triangulation may be possible. The formulas for the numerical corrections of the preliminary data of the elements of orientation and the model coordinates from measured y- and x-parallaxes can be derived from such as Bachman's well known work on aerial triangulation. Reference [5]. The results of the experiments now being carried on with automatically stabilized cameras are of the greatest interest for this purpose.

Also ordinary aerial triangulations in stereo instruments can of course be treated with the same principles. In other words, residual y-parallaxes of the individual models, measured with by can in the indicated way be transformed into corrections of the elements of orientation and the preliminary model coordinates along the strip. Each individual model can thus be regarded as numerically adjusted up to the same standard according to the least square method, which is an important condition for the application of a strict adjustment of the entire strip. The relative orientation can thus be performed more approximately than usual which means considerable saving of time and work. The influence of systematic errors can also be corrected numerically to the extent that the sources of the systematic errors are known. In reference [3], some new suggestions for the determination of systematic errors in connection with the photography and in the instruments have been presented. Of especially great importance as systematic errors are the radial distortion in the aerial camera, including the atmospheric refraction and earth's curvature, and the corresponding disturbances in the instrument at the reproduction of the pencils of rays, including the base-in and base-out positions. After the corrections of the strip with respect to the relative orientation and the correction of known systematic errors, an adjustment of the entire strip with respect to the least square method is justified. Practical experiments have indicated that the normal performance of the relative orientation needs considerable corrections that can be determined numerically in the indicated way and then be used for correction of functions of the relative orientation.

Investigations of these questions are being carried on, and also methods for the determination of the weights of the individual models with respect to each other for the strict adjustment.

As a follow-up to these deviations from my original theme, I will mention that the mirror stereoscope and parallax bar can be used for a great number of measurements in non-topographic photogrammetry.

As soon as *differences* between two pictures are determined, the stereoscopic measurements of x-parallaxes are of great value. The pictures of course must be arranged so that the actual differences appear as x-parallaxes. An application is described in the paper by Professor Doyle which was read at this meeting.

An astonishing accuracy can be obtained if the stereoscopic vision and the parallax screw are in good condition. There are a great number of measuring problems in many sciences which can be treated in the same way, for instance deformation measurements etc. Of course, a stereocomparator is the best instrument for such measurements, but the mirror stereoscope and the parallax bar can in many cases serve as a good substitute.

Only rather simple arrangements are necessary to facilitate the measuring procedure.

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