PHOTOGRAMMETRIC MEASUREMENT OF SPECTROGRAMS*

Frederick J. Doyle, Research Associate and Assistant Professor of Photogrammetry, Ohio State University, Columbus, Ohio

1. Introduction

IN THE history of instrumental photogrammetry, we have seen a development from elementary stereoscopes, extending through the stereocomparator, simple plotting machines, instruments like the Multiplex and Kelsh which are generally classified as second order, up the the first order machines like the Wild A7 and the Zeiss Stereoplanigraph. This is indeed as it should be, and when I addressed this group a year ago I advocated a further advance to digital rather than analog computers. Today I do not intend to reverse that direction, but rather to take a different one.

In our concentration on the high powered instruments, we are often inclined to neglect the potentialities of the simpler devices. I should like to discuss one of the simplest of all instruments—the mirror stereoscope and the parallax bar—and show how it can be applied to quite complex problems. For a given quality of results, the product of instrumentation and work is approximately constant. Consequently to obtain high grade results with such an elementary instrument, we may expect that a fairly high amount of computation would be required. I shall briefly mention one application to aerial photogrammetry, and one to structural deformations, and then describe in some detail the measurement of astronomical spectrograms. All of the experiments were performed with the Wild mirror stereoscope Model ST-3 and parallax bar, which seems to be a superior instrument for the purpose.

2. Tilt Determination for Aerial Photographs

Every photogrammetrist is familiar with the measurement of elevations by mirror stereoscope and parallax bar, but may not be aware of the fact that y-parallaxes may be easily and accurately measured with the same instrument. The photographs are set up for normal stereoscopic observation and then rotated 90 degrees about the principal point and transferred principal point. Figure 1 shows how this is done. The y-parallaxes now appear as x-parallaxes and may be measured with the parallax bar. Of course the model which is seen has no relief, but the floating mark may still be brought into contact with it. The effect is exactly the same as that obtained in the first order instruments by rotation of the dove prisms.

The y-parallax values for the normal six orientation points are then substituted in simple formulas from which the elements of relative orientation may be determined. Except for differences in the positive directions of the elements, these formulas are identical with those used in the numerical relative orientation of first order plotting instruments.

The photographs may then be rotated back to the normal position, and the elevations of vertical control points are measured in the usual fashion. These measured elevations are then corrected for the elements of relative orientation by means of the well known formula for model deformations. Finally from these corrected elevations the absolute orientation is determined.

Much more significant information may be obtained if the y-parallax obser-

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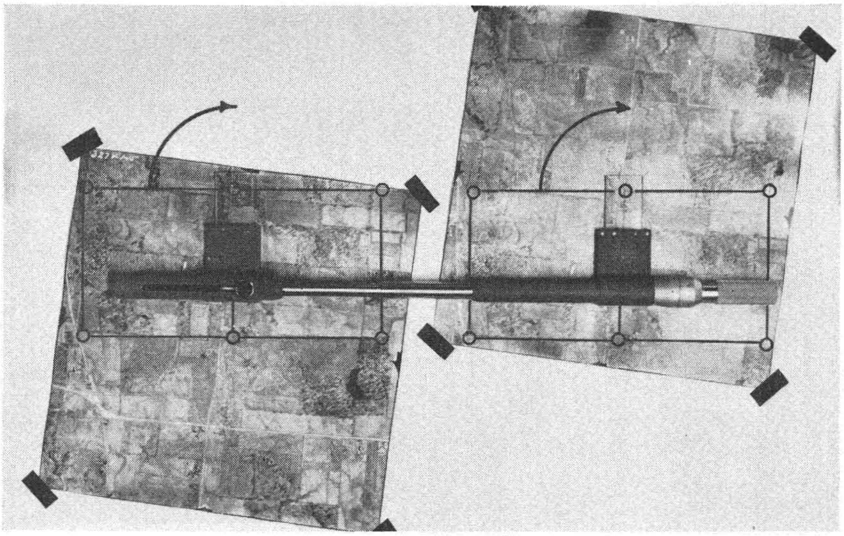

FIG. 1.—Measurement of Y-parallaxes. From the position for normal stereoscopic observation both photographs have been rotated counterclockwise through 90°. The right photograph has been rotated about its own principal point, the left photograph about its conjugate principal point.

vations are made in nine or fifteen points rather than in the normal six. The relative orientation is computed from the six points, after which the measured parallaxes in the remaining points are correlated for the orientation elements. The pattern of the residual y-parallaxes may then be interpreted as the distortion of the aerial camera lens. This technique has been applied by students in the Institute of Geodesy. Photogrammetry and Cartography to plates made with the Wild Aviogon, a normal Metrogon, and a Metrogon with a Tham compensating back on the camera. The results show close agreement with the published distortion characteristics of these lenses.

The accuracy obtainable by these methods is surprising. Five observations are usually made in each point, and students regularly attain a standard error of eight microns for a single observation and three microns in the mean. This is directly comparable to the results normally obtained with first order instruments. This high accuracy leads one to recommend that the least reading of the parallax bar be decreased from the present one one-hundredth of a millimeter to one micron.

3. Deformation Measurements in Structures

At the Annual Meeting of this Society in Washington last January, Dr. Hallert discussed "Some Experiences from Deformation Measurements by Photogrammetric Methods."* In this technique two photographs of a structure are made from the same camera station, one with the structure unloaded, the other with the load applied and the structure consequently deformed. When these two photographs are rotated 90 degrees, the deformations appear as x-parallaxes which may be measured under the mirror stereoscope with the parallax bar.

Figure 2 shows a pair of plates made by a student investigating the deflection of a centrally loaded simple beam. The right picture was made before loading,

* See page 836 et seq. of the December 1954 issue.

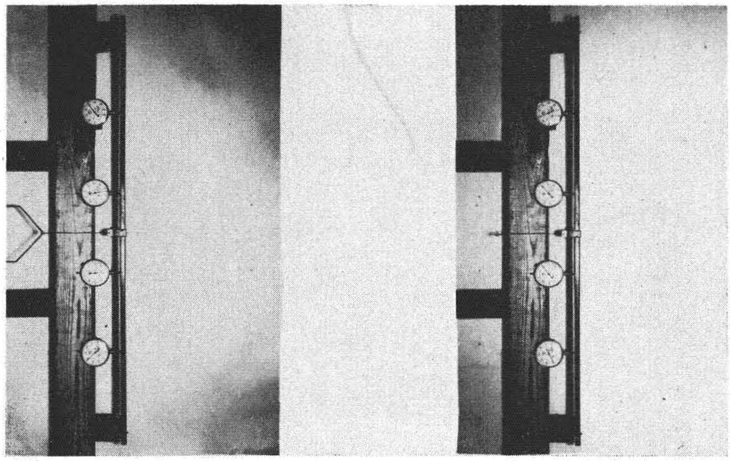

FIG. 2.—Deformation measurement. The beam deformations appear as x-parallaxes which may be measured with the mirror stereoscope and parallax bar.

the left one afterwards. When these plates are viewed under the stereoscope, the center of the beam appears to be bent away from the observer. The observed parallax multiplied by the scale number of the photograph obviously gives the deformation. Small fixed marks on the background serve as a means of checking the orientation of the photographs. In this example dial guages are being used to verify the results. The parallax bar is capable of measuring the deformations with a standard error of five microns at the scale of the photography. This accuracy is sufficient for many engineering purposes.

Another student is applying the same principle to measure the deformations of a reinforced concrete slab. Here the problem is complicated by the fact that the measuring points are distributed over a plane rather than along a line, and by the fact that the camera axis cannot be placed normal to the object.

4. Measurement of Stellar Spectrograms

An extremely interesting problem was presented to us by Dr. Allan Hynek of the astronomy department of The Ohio State University. Astronomers are interested in determining the radial velocity of stars and more particularly, changes in the radial velocity from one year to another.

In order to make this determination they take spectrograms of the star through an astronomical telescope. Our particular spectrograms were made with the 69 inch reflecting telescope of the Perkins Observatory located at Delaware, Ohio. Figure 3 is an enlargement of a section of one of these spectrograms, made from the star 5 Lacertae on September 25, 1949. The actual spectrum band is about 1/32 inch wide.

At this point it is necessary to mention briefly the significance of these lines and the technique of determining the star's velocity from them. If an element is heated to incandescence, it emits light composed of a number of different wave lengths. When this light passes through the prism of a spectrograph, it is dispersed and the various wave lengths are recorded as distinct lines. The pattern of these lines is unique for each element. This is seen at the top and bottom of Figure 3. These lines are made by an iron spark which is exposed at the beginning and end of each star exposure. The spectrum of a body, such as a star, which is composed of many elements, contains the characteristic lines of all of these elements. Many of the elemental lines are closely adjacent or even superimposed.

This is observed in the center band of Figure 3. The pattern of these lines, and the wave lengths which they represent, are well-known to physicists.

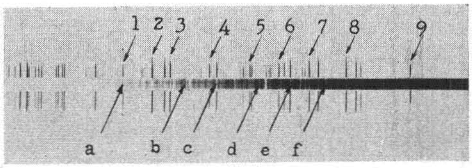

FIG. 3.—Stellar spectrum plate. The numbered lines are emission lines from the standard iron spark. The lettered lines are absorption lines from the star.

If the star has a velocity the apparent wave lengths are shortened if the star is approaching the observer, and lengthened if the star is receding, according to the well known Doppler principle. This causes the position of a given line in the star spectrum to shift with respect to the corresponding line in the fixed iron comparison spectrum. The amount of the shift is dependent upon the velocity of the star and the wave length of the line. This condition may perhaps be observed at the left end of Figure 3 where the first iron line in the star is shifted to the left when compared with the fixed iron line above. This shift towards the blue, or shorter, end of the spectrum indicates that the star is coming nearer.

Quite obviously, if two spectrograms are made on different occasions and the star's velocity has changed in the interim, the amount of shift will be different on the two plates. Measurement of differences of this type is a problem ideally suited to instruments like the parallax bar.

Nine lines, indicated by numbers, were selected in the iron spectrum, and six lines, indicated by letters, were chosen in the star spectrum. The lines selected are not those that an astronomer would normally use, but they serve very well to illustrate the technique.

One spectrogram, that of June 21, 1951, was chosen as a base, and the x coordinate of each of the selected lines was measured. This was done by laying a Wild glass scale graduated in tenths of a millimeter over the plate and estimating the reading to the nearest hundredth of a millimeter. Though such a high accuracy was by no means required, the readings were repeated five times with different positions of the scale, and the average was taken to microns. The coordinates were then reduced to zero at the left end. The average standard error for one measurement was six microns, and the average standard error in the mean was only three microns.

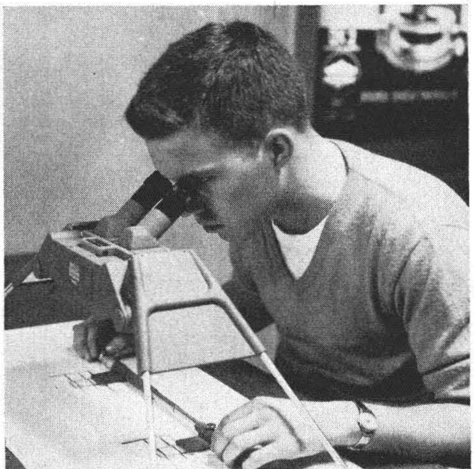

FIG. 4.—Measurement of spectrum line parallaxes.

Next the plates were aligned under the mirror stereoscope (Figure 4) and the parallax of each line was measured five times. The Wild parallax bar has two types of marks—one a circle, and the other a cross. Test measurements were made with both types on both the star lines and the iron lines. No significant difference was found in the accuracy of setting, and it was decided to use the cross. The readings were estimated to microns and the mean taken to microns, after which the parallaxes were also reduced to zero at the left end. The average standard error in one

TABLE 1

OBSERVED DATA

Point	x mm	m_x microns	μ_x microns	Parallax microns	m_p microns	μ_p microns
			Iron Lines			
1	0.024	5	2	11	8	2
2	2.106	5	2	19	4	2
3	3.386	5	2	25	4	2
4	6.202	8	4	41	3	1
5	9.078	9	4	42	5	2
6	11.158	8	4	53	4	2
7	13.327	5	3	64	5	2
8	15.980	8	4	70	7	3
9	20.520	0	0	80	5	2
Averages		6	3		5	2
			Star Lines			
a	0.000	0	0	0	7	3
b	4.362	4	2	14	4	2
c	6.720	12	5	23	2	1
d	9.760	7	3	33	2	1
e	12.077	8	4	43	7	3
f	14.487	5	3	55	3	1
Averages		6	3		4	2

observation was five microns and in the mean two microns. This completed the observations. The results of the two series of measurements are given in Table 1.

Because the parallax bar readings increase as the marks move towards one

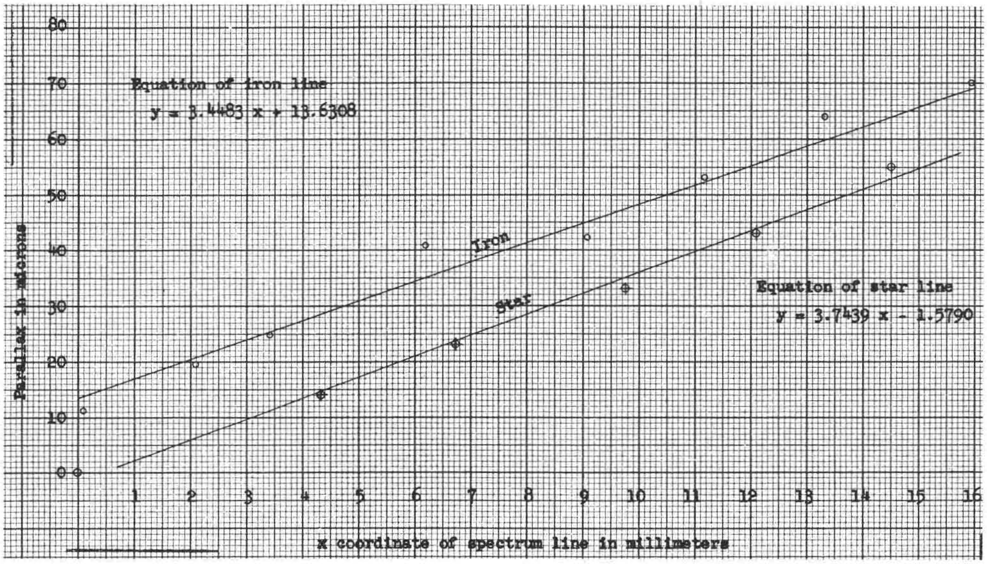

FIG. 5.—Graph of iron and star line parallaxes.

TABLE 2

COMPUTATION OF SCALE CHANGE FROM IRON LINES

OBSERVATION EQUATIONS

$$p_i = ax_i + b$$

1 $11 = 0.024\,a + b$
2 $19 = 2.106\,a + b$
3 $25 = 3.386\,a + b$
4 $41 = 6.202\,a + b$
5 $42 = 9.078\,a + b$
6 $53 = 11.158\,a + b$
7 $64 = 13.327\,a + b$
8 $70 = 15.980\,a + b$
9 $80 = 20.520\,a + b$

NORMAL EQUATIONS

a	b	p	Q_a	Q_b
1115.316	81.781	4964.988	1	0
	9.000	405.000	0	1
		22757.000		
33.3963	2.4488	148.6688	0.0299	0.0
	1.7330	23.6237	−0.0422	0.5770

RESULTS

$a = 3.4483$ microns/mm. $[vv] = 96.51$
$b = 13.6308$ microns $\mu_f = 3.7$ microns

$Q_{aa} = 0.002675$ $m_a = 0.1912$ microns/mm.
$Q_{bb} = 0.332929$ $m_b = 2.13$ microns
$Q_{ab} = 0.024349$

EQUATION OF IRON LINE

$$p_f = 3.4483x + 13.6308$$

another, the increasing parallaxes in the supposedly constant iron lines indicate that the right plate is smaller than the left. This scale variation, which should be linear, is caused by different temperatures of the spectrograph at the times the two plates were made. The determination of the scale change is the first computational step. The observed coordinates and parallaxes (actually differences between the x coordinates on the left and right plate) for the iron lines are used to set up observation equations and normal equations. These are solved by the method of Cholesky-Rubin for (a) the change in coordinate per millimeter along the base spectrum, and (b) the change in coordinate at the zero point. The standard error μ_f of one observation, the weight and correlation numbers of a and b, and the standard errors of a and b, are also found for later use in accuracy determination. These computations are given in Table 2, and the results are indicated by the upper graph line in Figure 5.

Before computing the star observations let us analyze the situation a moment. If there were no velocity change between the plates, the scale change for the star lines would be the same as for the iron lines, and, except for observational errors, a similar computation based on the star lines should give values of a' and b' identical with those found from the iron lines. If, however, there was a velocity change, and this were the sole cause of a shift in the lines, we should expect an equal shift in all star lines. This would result in identical values for a and a', and $(b-b')$ would be the amount of the shift. In other words the star

TABLE 3

COMPUTATION OF STAR LINES

OBSERVATION EQUATIONS

$$p_i = a'x_i + b'$$

a	$0 = 0a' + b'$
b	$14 = 4.362\,a' + b'$
c	$23 = 6.720\,a' + b'$
d	$33 = 9.760\,a' + b'$
e	$43 = 12.077\,a' + b'$
f	$55 = 14.487\,a' + b'$

NORMAL EQUATIONS

a'	b'	p	Q_a'	Q_b'
515.1701	45.406	1853.804	1	0
	6.000	168.000	0	1
		6688.000		
22.6974	2.0886	81.6747	0.04406	0.0
	1.2797	−2.0206	−0.07191	0.78143

RESULTS

$a' = 3.7439$ microns/mm.	$[vv] = 13.16$
$b' = 1.5790$ microns	$\mu_s = 1.8$ microns
$Q_{a'a'} = 0.007112$	$m_{a'} = 0.1517$ microns/mm.
$Q_{b'b'} = 0.610633$	$m_{b'} = 1.41$ microns
$Q_{a'b'} = -0.056193$	

EQUATION OF STAR LINE

$$p_s = 3.7439x - 1.5790$$

graph would be displaced parallel to the iron graph. But we know that the shift in any one line depends not only on the velocity change but also on the wave length of that line.

The apparent change in wave length is, as a matter of fact, directly proportional to the actual wave length. Thus a line of longer wave length towards the red end of the spectrum would have a larger change in apparent wave length than a line at the blue end. This does not mean, however, that the actual shift of line on the plate would be larger, because the wave length does not vary linearly along the plate. One millimeter at the red end represents considerably more angstroms than a millimeter at the blue end. But since the actual wave length and the apparent change in wave length both vary at the same rate along the spectrum, we may assume that the star line observations should also fall along a straight line. We proceed to determine this line in the same manner employed for the iron lines. The computations are given in Table 3. The resulting line is the lower one on Figure 5. The difference in the slope is the effect of varying wave length. The shift in microns for a line at any x coordinate along the spectrum is obtained as the difference between the two ordinates at that point. By formulas well known to astronomers, this shift may be converted to an apparent change in wave length, and from the change in wave length, the difference in velocity may be found. The velocity difference, of course, should be constant regardless of the lines from which it is determined. The formulas and techniques for this conversion are beyond the scope of this paper. Suffice it to say that on June 21, 1951, the star 5 Lacertae was approaching the earth faster than it was on September 25, 1949.

It is of interest to determine the accuracy of the difference between two ordinates at any x coordinate. In order to do this we need an expression for the standard error of a point on each of the graphs.

For the iron line the ordinate at any point is given by

$$p_f = ax + b.$$

Since a and b are determined from the same set of observations, they are correlated and the general law of error propagation must be applied.

We have the weight symbol equation

$$Q_{p_f} = xQ_a + Q_b$$

whence by squaring both sides

$$Q_{p_f p_f} = x^2 Q_{aa} + Q_{bb} + 2xQ_{ab}.$$

Finally

$$\begin{aligned} m_{p_f} &= \mu_f \sqrt{Q_{p_f p_f}} \\ &= \mu_f \sqrt{x^2 Q_{aa} + Q_{bb} + 2xQ_{ab}} \end{aligned}$$

in which μ_f, Q_{aa}, Q_{bb}, and Q_{ab} are the quantities determined from the solution of the normal equations.

A similar development for the star line gives

$$m_{p_s} = \mu_s \sqrt{x^2 Q_{a'a'} + Q_{b'b'} + 2xQ_{a'b'}}.$$

The difference in the ordinates is

$$\Delta p = p_f - p_s.$$

Because p_f and p_s are determined from different sets of observations they are uncorrelated, and the special law of error propagation may be applied.

$$\begin{aligned} m_{\Delta p} &= \sqrt{m_{p_f}^2 + m_{p_s}^2} \\ &= \sqrt{\mu_f^2(x^2 Q_{aa} + Q_{bb} + 2xQ_{ab}) + \mu_s^2(x^2 Q_{a'a'} + Q_{b'b'} + 2xQ_{a'b'})}. \\ &= \sqrt{x^2(\mu_f^2 Q_{aa} + \mu_s^2 Q_{a'a'}) + 2x(\mu_f^2 Q_{ab} + \mu_s^2 Q_{a'b'}) + \mu_f^2 Q_{bb} + \mu_s^2 Q_{b'b'}}. \end{aligned}$$

The standard error of the difference is thus dependent upon the x corrdinate. Evaluating this expression for the left and right limits gives

$$\text{for } x = 0 \qquad m_{\Delta p} = 2.56 \text{ microns}$$

$$\text{for } x = 15 \qquad m_{\Delta p} = 2.13 \text{ microns.}$$

These values are important because if the difference in the ordinates approaches them in magnitude, the difference must be ascribed to observational errors rather than to velocity change.

In view of the fact that the spectrum lines are on the order of 50 microns wide, this high accuracy is perhaps surprising. But it must be remembered that the settings are made stereoscopically, so it is unnecessary to attempt to find the midpoint of a line whose boundaries are quite poorly defined. The floating mark is merely brought into the same stereoscopic plane as the spectrum line, and a skilled observer can do this with a high precision.

5. Conclusion

This exposition is by no means complete, particularly as regards the astronomical part of the procedure. Considerable work remains to be done. Nevertheless, several important conclusions may be drawn.

(1) For many problems requiring the measurement of small differences, the mirror stereoscope and parallax bar is a powerful tool. Photogrammetrists generally agree that the precision of setting a mark stereoscopically is twice that of usual monocular setting.

(2) The application of sound principles to observations made with simple instruments can result in astonishing accuracy. This is particularly important in training and educational programs where expensive instrumentation is generally not available. It will be readily recognized that all of these experiments could be performed with much higher precision on a stereocomparator, but at present no suitable stereocomparators are being manufactured. Even if they were, they would probably cost around $8,000, while the cost of the mirror stereoscope and parallax bar is about $400.

(3) The use of stereoscopic vision for the measurement of small differences greatly reduces the chance of errors due to identification. Such errors are not at all uncommon in problems like the measurement of spectrograms. Also since the observations are much simpler to perform, and two plates are measured simultaneously, a considerable saving in time and strain for the observer may be expected.

(4) It is a common principle in mathematics that when differences are to be determined, differences should be observed. At present, star velocity differences are computed by finding the absolute velocity values separately from each plate. It may be expected that computation from directly measured spectral line shifts will considerably increase the accuracy of the difference determination.

DETERMINATION OF THE ACCURACY OF TERRESTRIAL STEREOPHOTOGRAMMETRIC PROCEDURES*,†

Dr. Bertil Hallert, Institute of Geodesy, Photogrammetry and Cartography, Ohio State University, Columbus, Ohio.

Introduction

PHOTOGRAMMETRIC measurements are generally founded upon complete information about the inner and outer orientation of the camera in the moments of exposure. In *terrestrial photogrammetry*, all elements of orientation normally are assumed to be known from the photography as results of direct or indirect measurements, but small errors of course are always present. In *aerial photogrammetry*, on the other hand, the elements of the outer orientation are only approximately determined in connection with the photography. Consequently, terrestrial photogrammetry normally means a simpler procedure than

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† This paper covers a part of a report on a research project concerning the application of terrestrial photogrammetry to different measuring problems within cities. The investigations have been sponsored by the city of Stockholm, Sweden and have been performed at the Division of Photogrammetry of the Royal Institute of Technology, Stockholm, Sweden in cooperation with Mr. A. Jörbeck, City Engineer of Stockholm.