5. Conclusion

This exposition is by no means complete, particularly as regards the astronomical part of the procedure. Considerable work remains to be done. Nevertheless, several important conclusions may be drawn.

(1) For many problems requiring the measurement of small differences, the mirror stereoscope and parallax bar is a powerful tool. Photogrammetrists generally agree that the precision of setting a mark stereoscopically is twice that of usual monocular setting.

(2) The application of sound principles to observations made with simple instruments can result in astonishing accuracy. This is particularly important in training and educational programs where expensive instrumentation is generally not available. It will be readily recognized that all of these experiments could be performed with much higher precision on a stereocomparator, but at present no suitable stereocomparators are being manufactured. Even if they were, they would probably cost around $8,000, while the cost of the mirror stereoscope and parallax bar is about $400.

(3) The use of stereoscopic vision for the measurement of small differences greatly reduces the chance of errors due to identification. Such errors are not at all uncommon in problems like the measurement of spectrograms. Also since the observations are much simpler to perform, and two plates are measured simultaneously, a considerable saving in time and strain for the observer may be expected.

(4) It is a common principle in mathematics that when differences are to be determined, differences should be observed. At present, star velocity differences are computed by finding the absolute velocity values separately from each plate. It may be expected that computation from directly measured spectral line shifts will considerably increase the accuracy of the difference determination.

DETERMINATION OF THE ACCURACY OF TERRESTRIAL STEREOPHOTOGRAMMETRIC PROCEDURES*,†

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Introduction

PHOTOGRAMMETRIC measurements are generally founded upon complete information about the inner and outer orientation of the camera in the moments of exposure. In *terrestrial photogrammetry*, all elements of orientation normally are assumed to be known from the photography as results of direct or indirect measurements, but small errors of course are always present. In *aerial photogrammetry*, on the other hand, the elements of the outer orientation are only approximately determined in connection with the photography. Consequently, terrestrial photogrammetry normally means a simpler procedure than

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aerial photogrammetry to the extent that it concerns the principles of the plotting procedure.

But as soon as we start to determine with the theory of errors the real accuracy of terrestrial and aerial photogrammetry, we find that the problem is very similar in both cases. Up to now, however, the theory of errors of terrestrial stereophotogrammetry does not seem to have been treated to the same degree as aerial stereophotogrammetry. In particular the error propagation within the photogrammetric model as a function of the errors of the elements of orientation, before and after an adjustment of discrepancies in control points, has not been completely treated in any one of the existing textbooks. Therefore there will be discussed some of the main principles which are of interest in this case; also the principles will be applied to a practical experiment.

For the introduction to the problem, and with the aid of some illustrations

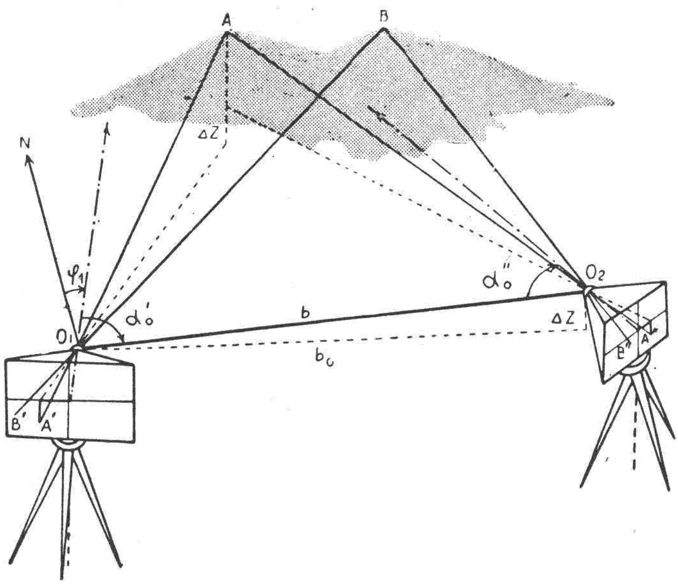

FIG. 1.—The principles of terrestrial photogrammetry. The photography. Some elements of the inner and outer orientation are indicated.

there will be briefly demonstrated the general principles of terrestrial photogrammetry.

In Figure 1 the feature containing the points A and B is photographed from the stations O_1 and O_2.

The inner orientation of the cameras—the principal point, principal distance and the distortion—is known from the camera calibration; the outer orientation—the three coordinates x, y, z of each exposure station, the two angles ϕ and ω which determine the direction of the camera axis and the angle κ which determines the location of the pencils of rays around the camera axis—is determined in connection with the photography.

Figure 2 shows the general principles for the determination of the coordinates of the point P_1.

The rays towards P_1 from the two stations can be reconstructed from the image coordinates of the points P_1' and P_1''. From the left picture we obtain the angles α_1 and β_1 which determine the ray O_1P_1' from the coordinate plane as:

$$\tan \alpha' = \frac{x_1'}{c} \tag{1}$$

$$\tan \beta' = \frac{z_1' \cos \alpha}{c} \tag{2}$$

In the same manner the corresponding ray from the right picture can be determined. Since the direction angles of the camera axis are known from the photography, the direction angles of the rays O_1P_1 and O_2P_1 can be determined.

Finally the coordinates of P_1 can be determined by an intersection from the known points O_1 and O_2. The figure also indicates that two different determinations of the elevations of P_1 are possible. This gives a welcome check of the measurements and computations. A discrepancy in the elevation determination is known as *vertical parallax* and the presence of such a parallax makes evident that something is wrong in the measurements or computations. Normally in *terrestrial photogrammetry*, the accuracy of the measurements is so high that such vertical parallaxes can be neglected. Sometimes, however, they have to be taken

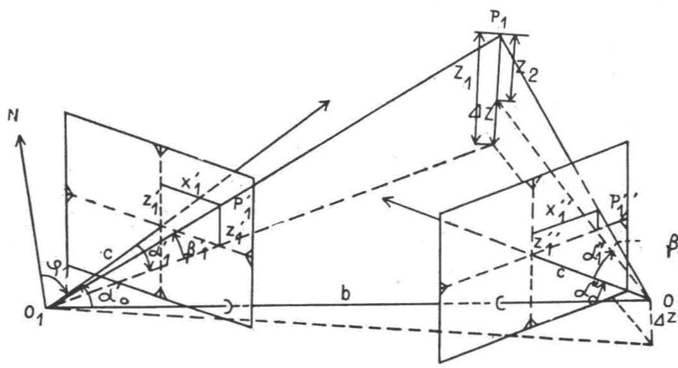

FIG. 2.—The principles of terrestrial photogrammetry. The reconstruction of the rays from image coordinates and the inner orientation. The determination of object coordinates by intersection.

into account. In *aerial photogrammetry*, on the other hand, the vertical parallaxes have to be corrected before other measurements can be performed. The correction of the vertical parallaxes is the well known procedure of relative orientation. In aerial photogrammetry the vertical parallaxes usually are called y-parallaxes due to the orientation of the coordinate systems. The term vertical parallax is still in use and originates obviously in terrestrial photogrammetry.

If there are control points, given with respect to the stations, discrepancies are normally found in these points, between the photogrammetrically determined coordinates and the given coordinates. These discrepancies are primarily due to the small errors in the inner and outer orientation of the cameras and the plotting instrument, and the errors of the measurements of the image coordinates. Normally, the discrepancies of the distance ($y-$) determination are the largest compared with x and z and must obviously depend to a great extent upon the ratio between the distance and the base.

It is obvious from Figure 2 that also very small errors, for instance in the determination of the azimuth angles ϕ of the camera axes, have a strong influence upon the distance determination, and that the errors of the distances grow rapidly with increasing distances. If the analytical connection between different sources of errors and the resulting errors in the coordinate determination were known, it would be possible to determine numerical corrections to the elements of orientation of the cameras; so that the discrepancies would vanish in the same number of control points as the number of orientation elements which are assumed to have an influence upon the coordinate determination. But if there are more control points than the used elements of orientation, there are superfluous observations, and consequently one has the problem of distributing the discrepancies which cannot be compensated by the available elements of orientation. In such cases the method of least squares is normally used. The corrections to the elements of the orientation are to be determined so that the square sum of the residuals or corrections to the measurements of image coordinates is a minimum. This principle means that one assumes the discrepancies, which cannot be corrected by the elements of orientation, to depend primarily upon the errors of the image coordinate measurements. The principles of the least square method furthermore permits one to determine the standard error of the measurements, to determine the accuracy of the elements, and to study the error propagation to the determination of the coordinates of arbitrary points.

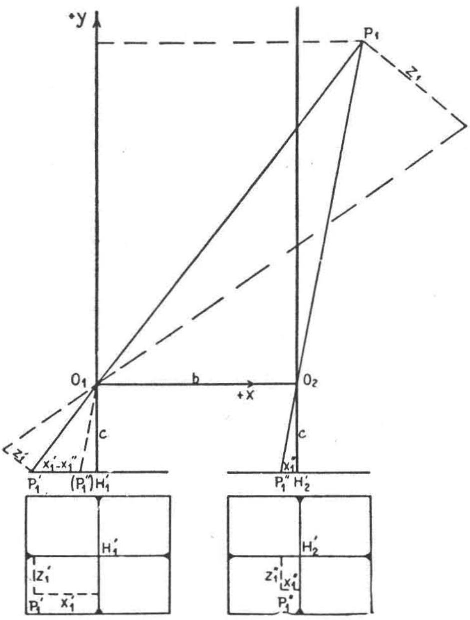

FIG. 3.—The normal case of terrestrial stereophotogrammetry. The principles for the determination of the coordinates x, y and z are indicated.

These general principles will be applied to the most important case of terrestrial photogrammetry, the *stereophotogrammetric normal case*, Figure 3.

The pictures are assumed to have been photographed at the stations O_1 and O_2, with the same camera, or with equal cameras having the same principal distances c and free from distortion. If distortion is present, the measurements must be corrected; this is no real problem. The camera axes are perpendicular to the base and horizontal. A phototheodolite and a stereocamera are shown in Figures 4 and 5.

A coordinate system is assumed to have the origin at the left camera station, the $+x$-axis horizontal towards the right station, the $+y$-axis in the left camera axis and the $+z$ axis upwards. The image coordinates are denoted x', x'', z', z''.

The coordinates of an arbitrary point can be determined from the following expressions which can easily be derived. See Figure 3.

$$y = \frac{bc}{x' - x''} = \frac{bc}{p} \tag{3}$$

$$x = \frac{yx'}{c} = \frac{bx'}{p} \tag{4}$$

$$z_1 = \frac{yz'}{c} = \frac{bz'}{p} \tag{5a}$$

$$z_2 = \frac{yz''}{c} = \frac{bz''}{p} \tag{5b}$$

The difference $x' - x''$ of the image coordinates is thus denoted p and is called the $x-$ or horizontal parallax. Below only the word parallax will be used.

One also sees that two determinations of the z-coordinate are possible, and that any disagreement indicates errors of measurements or computations. Of course, attention has to be paid to the relative elevation of the stations.

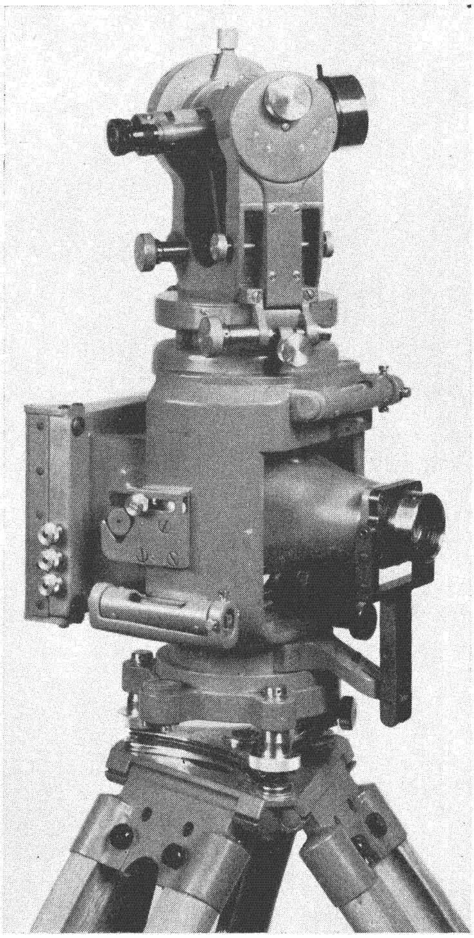

FIG. 4.—Wild phototheodolite.

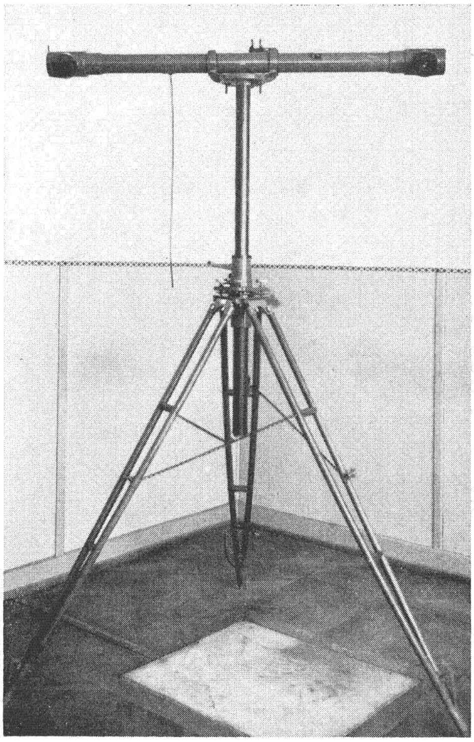

FIG. 5.—Wild stereocamera, b = 120 cm.

The given formulas are called the *fundamental formulas of stereo-photogrammetry*.

For other cases, for instance the vergent, parallel averted, etc. cases, similar formulas can easily be derived. See for instance 5.

1. Theory of Errors for the Normal Case

1.1 THE FUNDAMENTAL ERROR EXPRESSIONS.

The accuracy of the coordinate determination according to the formulas (3) – (5) is of course an important question. Small errors in the determination of the quantities b, c, images coordinates and parallaxes are assumed.

$$dy = \frac{y}{b}\,db + \frac{y}{c}\,dc - \frac{y^2}{bc}\,dp \tag{6}$$

$$dx = \frac{x}{y}\,dy + \frac{x}{x'}\,dx' - \frac{x}{c}\,dc \tag{7}$$

$$dz = \frac{z}{y}\,dy + \frac{z}{z'}\,dz' - \frac{z}{c}\,dc$$

$$= \frac{z}{y}\,dy + \frac{z}{z''}\,dz'' - \frac{z}{c}\,dc \tag{8}$$

These equations can be changed into different forms, with the aid of (3)–(5). For instance, equations (7) and (8) can be converted into:

$$dx = \frac{x}{b}\,db + \frac{y}{c}\,dx' - \frac{xy}{bc}\,dp \tag{7a}$$

$$dz = \frac{z}{b}\,db + \frac{y}{c}\,dz' - \frac{yz}{bc}\,dp$$

$$= \frac{z}{b}\,db + \frac{y}{c}\,dz'' - \frac{yz}{bc}\,dp \tag{8a}$$

From (6) (7) and (8) it is immediately found:

1. Because y generally is much larger than x and z, the influence of the errors, db, dc, and dp is largest upon y.

2. The most important source of error is dp, since the influence is proportional to y^2. The errors db and dc can be assumed to be very small if the measurements of the base and the determination of the inner orientation of the camera are performed carefully.

For an accurate method for the determination of the inner orientation of the camera, see reference [3]. A regular grid is photographed perpendicular to the grid from two stations on the normal from the center of the grid. The corrections to the preliminary determined data of the inner orientation can conveniently be determined, and also the distortion. Also the standard error of the observations can be found in an easy way, and furthermore the standard errors of the elements of the inner orientation and the error propagation in functions of the elements.

The next step is to investigate the sources of the error dp. Since p is defined as $p=x'-x''$ and consequently $dp=dx'-dx''$, it is noted that any error of the inner or outer orientation that has influence upon the image coordinates x' and x'' causes the error dp. Consequently there must be found the connection between small errors of the inner and outer orientation on one hand and the errors dx' and dx''. Then formula (6) gives the relation between the y-errors in control points and the errors of the inner and outer orientation. Knowing this connection there can be found the corrections to the elements of the inner and outer orientation from the discrepancies in a sufficient number of suitably located control points.

From the literature, for instance [1], [2], one finds the differential formulas for the influence of the errors of the inner and outer orientation upon the image coordinates.

The errors of the inner and outer orientation of the left camera (See Figure 6) are defined as follows:

dc_1 is the error of the principal distance c_1 at the photography.

dx_t' and dz_t' are the errors in the positive x'- and z'-directions of the determination of the position of the principal point from the fiducial marks in the image.

dx_r' and dz_r' are the translations of the principal point in the positive x'- and z'-directions due to rotations of the negative around the z'- and x'-axis in connection with the photography. Compare Figure 7.

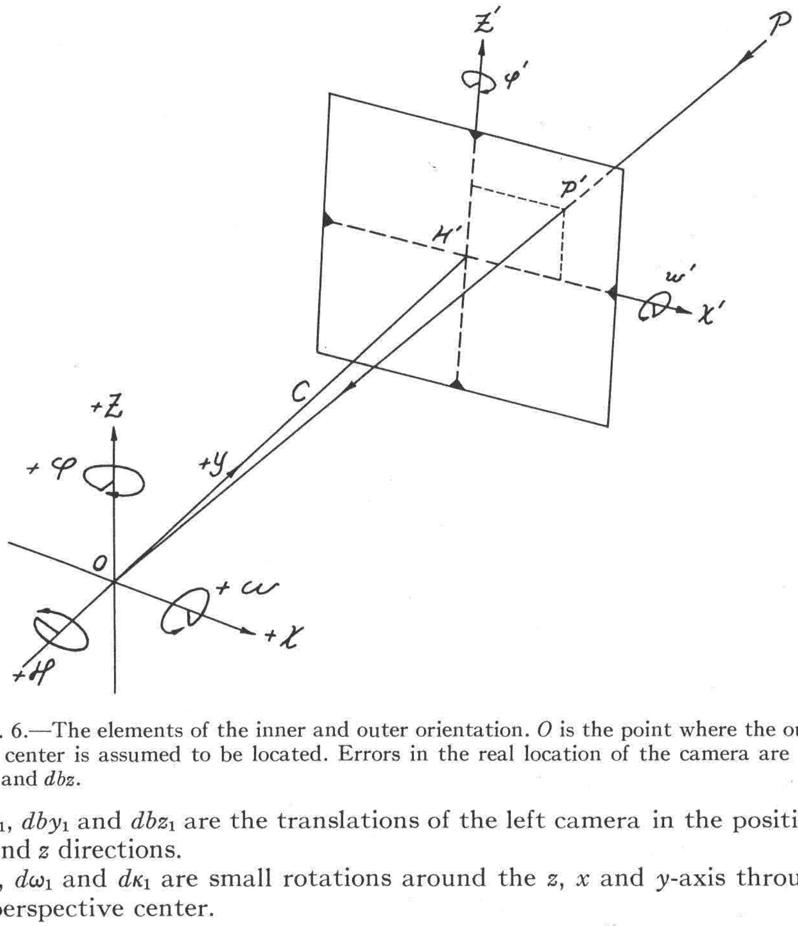

FIG. 6.—The elements of the inner and outer orientation. O is the point where the outer perspective center is assumed to be located. Errors in the real location of the camera are denoted dbx, dby and dbz.

dbx_1, dby_1 and dbz_1 are the translations of the left camera in the positive x, y and z directions.

$d\phi_1$, $d\omega_1$ and $d\kappa_1$ are small rotations around the z, x and y-axis through the perspective center.

The directions of the rotations in the formulas below are shown in Figure 6. Since the rotations always can be assumed to be very small, all the approximations in the definitions of the rotation axis are allowable.

The complete differential formulas for the influence upon the x'- and z'-coordinates of the assumed errors of inner and outer orientation are:

$$dx' = -dx_t' - \frac{x'^2}{c_1^2}dx_r' - \frac{x'z'}{c_1^2}dz_r' + \frac{x'}{c_1}dc_1 - \frac{c_1}{y}dbx_1 + \frac{x'}{y}dby_1$$
$$- \left(1 + \frac{x'^2}{c_1^2}\right)c_1 d\phi_1 - \frac{x'z'}{c_1}d\omega_1 + z'd\kappa_1 \qquad (9)$$

$$dz' = -dz_t' - \frac{z'^2}{c_1^2}dz_r' - \frac{x'z'}{c_1^2}dx_r' + \frac{z'}{c_1}dc_1 - \frac{c_1}{y}dbz_1 + \frac{z'}{y}dby_1$$

$$-\frac{x'z'}{c_1^2}d\phi_1 - \left(1 + \frac{z'^2}{c_1^2}\right)c_1 d\omega_1 - x'd\kappa_1 \tag{10}$$

For the x'' and z'' coordinates of the right picture the corresponding expressions are obtained:

$$dx'' = -dx_t'' - \frac{x''^2}{c_2^2}dx_r'' - \frac{x''z''}{c_2^2}dz_r'' + \frac{x''}{c_2}dc_2 - \frac{c_2}{y}dbx_2 + \frac{x''}{y}dby_2$$

$$-\left(1 + \frac{x''^2}{c_2^2}\right)c_2 d\phi_2 - \frac{x''z''}{c_2}d\omega_2 + z''d\kappa_2 \tag{11}$$

$$dz'' = -dz_t'' - \frac{z''^2}{c_2^2}dz_r'' - \frac{x''z''}{c_2^2}dx_r'' + \frac{z''}{c_2}dc_2 - \frac{c_2}{y}dbz_2 + \frac{z''}{y}dby_2$$

$$-\frac{x''z''}{c_2}d\phi_2 - \left(1 + \frac{z''^2}{c_2^2}\right)c_2 d\omega_2 - x''d\kappa_2 \tag{12}$$

A verification and also a derivation of these formulas can easily be made by studying the effect upon the image coordinates of the varying individual errors sources geometrically. Take a ray from a point in the feature through the origin as fixed, and investigate step by step the effect upon its image coordinates of small individual translations and rotations of the image plane and the entire camera. See Figure 7.

The effect of the errors of the image coordinates, as expressed by (9)–(12) can be found from the expressions (6)–(8).

If it is assumed that $c_1 = c_2 = c$ and there is used the relation $dp = dx' - dx''$, there is obtained, for instance for the influence of dp upon y:

$dy = -\dfrac{y^2}{bc}(dx' - dx'')$, and after inserting the expressions (9) and (11):

$$dy = \frac{y^2}{bc}\left\{dx_t' - dx_t'' + \frac{x'^2}{c^2}dx_r' - \frac{x''^2}{c^2}dx_r'' + \frac{x'z'}{c^2}dz_r' - \frac{x''z''}{c_2}dz_r''\right.$$

$$-\frac{x'}{c}dc_1 + \frac{x''}{c}dc_2 + \frac{c}{y}dbx_1 - \frac{c}{y}dbx_2 - \frac{x'}{y}dby_1 + \frac{x''}{y}dby_2 + \left(1 + \frac{x'^2}{c^2}\right)cd\phi_1$$

$$\left.-\left(1 + \frac{x''^2}{c^2}\right)cd\phi_2 + \frac{x'z'}{c}d\omega_1 - \frac{x''z''}{c}d\omega_2 - z'd\kappa_1 + z''d\kappa_2\right\} \tag{13}$$

The formula using the expressions (4), (5a) and (5b) can be rewritten. From this there is first obtained:

$$x' = \frac{xc}{y} \tag{14}$$

and

$$x'' = \frac{c}{y}(x - b) \tag{15}$$

$$z' = \frac{cz}{y} \tag{16}$$

$$z'' = \frac{cz}{y} \cdot \tag{17}$$

Obviously it is assumed that, $z_1 = z_2 = z$.

Through inserting the expressions (14)–(17) into (13) there is obtained, after some rearrangement:

$$\begin{aligned} dy = {} & \frac{y^2}{bc}(dx_t' - dx_t'') + \frac{1}{bc}\left\{x^2 dx_r' - (x-b)^2 dx_r''\right\} + \frac{z}{bc}\left\{x dz_r' - (x-b)dz_r''\right\} \\ & + \frac{y}{bc}\left\{-x dc_1 + (x-b)dc_2\right\} + \frac{y}{b}(dbx_1 - dbx_2) \\ & + \frac{1}{b}\left\{-x dby_1 + (x-b)dby_2\right\} + \frac{1}{b}\left[(y^2+x^2)d\phi_1 - \left\{y^2 + (x-b)^2\right\}d\phi_2\right] \\ & + \frac{z}{b}\left\{x d\omega_1 - (x-b)d\omega_2\right\} + \frac{yz}{b}(-d\kappa_1 + d\kappa_2). \end{aligned} \tag{18}$$

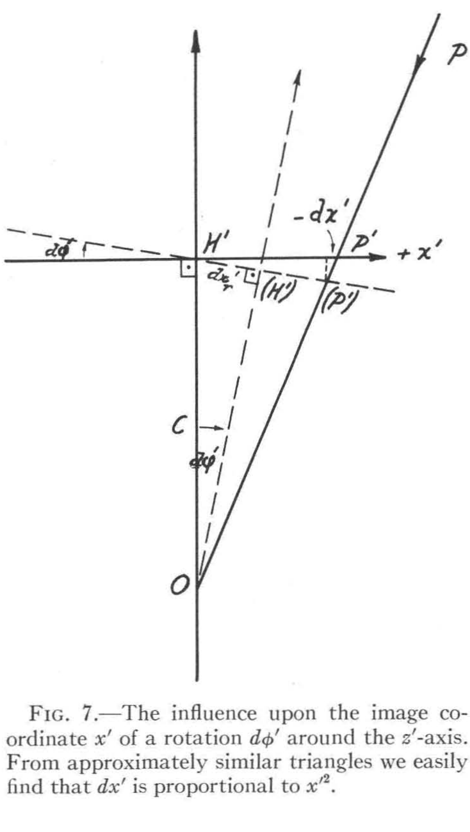

FIG. 7.—The influence upon the image coordinate x' of a rotation $d\phi'$ around the z'-axis. From approximately similar triangles we easily find that dx' is proportional to x'^2.

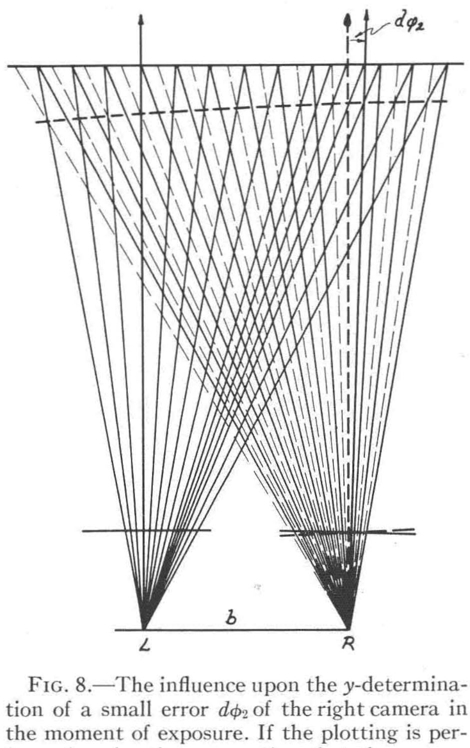

FIG. 8.—The influence upon the y-determination of a small error $d\phi_2$ of the right camera in the moment of exposure. If the plotting is performed under the assumption that the camera axis had the dashed position at the exposure, the straight full line at the top of the figure will get translated and deformed into the dashed curve, a parabola. A vertical plane would have been deformed into a parabolic cylinder.

Formula (18) expresses the influence upon the y-determination, of the errors of the elements of orientation in the moments of exposure and the error of the determination of the principal point from the fiducial marks in the pictures (dx_l', dx_l'').

The expression for the total error dy must also include the influence of the errors of the settings of the elements of orientation in the plotting instrument and the errors of the measurements of the parallaxes. The influence of the errors of the settings of the elements of orientation in the instrument is, of course, identical to the corresponding terms of (18) but with opposite signs. The influence of the errors of the parallax measurements is given by the last term of (6).

There follows an investigation of the influence of the errors of the elements of orientation of the photography; for this will be used the expression (18). In connection with the adjustment of the orientation there will also be included the errors of the orientation elements in the plotting instrument and the errors of the measurements of the parallaxes.

Formula (18) provides many interesting relations between the different errors.

The relative importance of the different sources of errors can, for instance, be found. Of course, those errors which propagate with the square of x or y are particularly dangerous. The most critical errors are obviously $d\phi_1$ and $d\phi_2$, but also dx_l' and dx_l'' cause errors dy proportional to the y^2. The determination of ϕ_1 and ϕ_2 in connection with the photography is also normally rather weak since the angle ϕ means the angle between the relatively short base and the camera axis. Therefore, great care has to be used for the determination of this angle. Compare Figure 8.

An example will demonstrate the error propagation.

If it is assumed that

$$x = b/2$$

$$y = 20b$$

$$d\phi_1 = 1'$$

then the influence upon the y from (18) is

$$dy = 1/b(400b^2 + b^2/4)1/3438 = 0.11b.$$

The error of the y is consequently rather large also for the small error of the angle ϕ. The ratio b/y is of course of importance. The influence of assumed errors in the other elements of orientation can be investigated in a similar way.

Since $dbx_1 - dbx_2$ represents the error of the base, projected upon the x-axis, then

$$dbx_1 - dbx_2 = -dbx$$

and it is found that the relative base error propagates linearly with the distance y. The influence of dbx upon y is of course the same as the influence of db in the first term of formula (6).

If, furthermore, $dc_1 = dc_2 = dc$, the second term of formula (6) is obtained. The signs are different because formula (18) refers to the errors of the elements of orientation at the photography, and formula (6) with the corresponding errors at the plotting.

From (18) it is further found that several errors of the elements of orientation can substitute for each other more or less completely. For instance, $d\phi_1$ and $d\phi_2$ can partly compensate the influence of dx_l', dx_l'', dx_r' and dx_r''. $d\omega_1$ and

$d\omega_2$ can compensate for the influence of dz_r' and $dz_r''\Delta$. $d\kappa_1$ and $d\kappa_2$ can compensate for each other, dby_1 and dby_2 can compensate the influence of dc_1 and dc_2 but only for a certain distance y.

It is also found that equal errors of some elements can be compensated by another element. For instance, the influence of $d\phi_1=d\phi_2=d\phi$ can partly be compensated by dby_1, dby_2 and for a certain distance y by dc_1 or dc_2.

Some errors of the elements of orientation are harmless for points in certain locations. For instance dz_r', dz_r'', $d\kappa_1$, $d\kappa_2$, $d\omega_1$ and $d\omega_2$ have no influence upon the y-coordinates of points which have $z=0$ or are located in a plane through the camera axes.

Several more interesting conclusions can be found from formula (18). It is of great value to investigate how vertical planes, located at different distances from the base and perpendicular to the ideal camera axes, in the moments of exposure, get deformed at the plotting if errors of the elements of orientation are assumed.

The results are very similar to corresponding deformations in aerial photogrammetry.

1.11 ERROR PROPAGATION BEFORE ADJUSTMENT

The *true* errors of the elements of orientation are never known but normally an estimate of the *standard* errors of the elements can be made by statistical methods. If for instance, from repeated measurements of the elements of orientation, the standard error of the measurements has been determined, then the standard error of the determination of y can be found by the special law of error propagation, applied to (18).

The measurements of the different elements of orientation are assumed to be free from mutual correlation, and the standard errors are denoted by m and a subscript indicating the element.

From the special law of error propagation, and assuming the standard errors of the elements of orientation at the photography to be equal in both stations, one obtains the standard error m_y of y as:

$$\begin{aligned} m_y{}^2 = {} & \frac{2y^4}{b^2c^2} m_{x_l}{}^2 + \frac{1}{b^2c^2}\{x^4+(x-b)^4\} m_{x_r}{}^2 + \frac{z^2}{b^2c^2}\{x^2+(x-b)^2\} m_{z_r}{}^2 \\ & + \frac{y^2}{b^2c^2}\{x^2+(x-b)^2\} m_c{}^2 + \frac{y^2}{b^2} m_{bx}{}^2 + \frac{1}{b^2}\{x^2+(x-b)^2\} m_{by}{}^2 \\ & + \frac{1}{b^2}[(y^2+x^2)^2 + \{y^2+(x-b^2)\}^2] m_\phi{}^2 + \frac{z^2}{b^2}\{x^2+(x-b)^2\} m_\omega{}^2 \\ & + \frac{2y^2z^2}{b^2} m_\kappa{}^2 \end{aligned} \tag{19}$$

The influence of the standard errors of the settings of the elements of orientation in the plotting instrument and the standard error for the parallax measurements have to be added to (19) for the determination of the total standard error m_y.

The expression for the total standard error $m_y{}^2$ will consequently consist of (19) and another set of terms with the same coefficients as (19), but with special subscripts in the standard errors of the elements of orientation in order to indicate that these standard errors belong to the plotting procedure.

The term which expresses the influence of the standard error of the parallax measurements is $(y^4/b^2c^2)m_p^2$ in accordance with (6). m_p is the standard error of the parallax measurement. Similar expressions can easily be established for the standard errors of the x- and z-coordinates.

After a determination of the standard errors of the measurements of the elements of orientation of the cameras, of the setting of the elements of orientation in the plotting instrument, and of the parallax measurements, the standard error of the coordinates x, y, and z can be determined in the indicated way.

It is obvious that a good determination of the coordinates x, y, and z requires that only very small errors in the elements of orientation from the photography, from the reconstruction of the orientation in the instrument and from the parallax measurement procedure, are present. Also the ratio y/b plays an important role for the error propagation.

If terrestrial photogrammetry is applied to measuring problems where no control data are available, for instance in meteorology, and a good accuracy is wanted in the determination of the coordinates x, y, and z, there must be assumed:

1. A good distance/base ratio.
2. Stabile cameras with an accurately determined inner orientation.
3. Accurate measurements of the outer orientation in connection with the photography.
4. A high accuracy in the reconstruction of the orientation in the plotting instrument.
5. A high accuracy in the measurements of the parallaxes.

Here are the reasons why amateur cameras normally cannot be used for reliable photogrammetric determinations of the coordinates x, y, and z.

For the determination of coordinate *differences* the conditions are easier.

One can easily find the influence upon a coordinate difference y_1-y_2 of the errors of the elements of orientation from the photography in applying (18) to the coordinates of the two points $(x_1y_1z_1)$ and $(x_2y_2z_2)$ and determining the difference dy_1-dy_2.

From the corresponding expressions for dx and dz the errors of coordinate differences in x and z can be determined.

1.2 Correction and Adjustment of the Elements of Orientation with the Aid of Control Points

1.21 The General Procedure

Since obviously considerable errors in the coordinate determination must be assumed, control points normally must be available for a check of the results.

The photogrammetrically determined measured coordinates are denoted with the subscript m and the corresponding given data with a subscript g. The error of the photogrammetrically obtained coordinates is defined by:

$$dy = y_m - y_g \tag{20a}$$

$$dx = x_m - x_g \tag{20b}$$

$$dz = z_m - z_g. \tag{20c}$$

Since the y-determination normally is the most critical, one concentrates upon the treatment of the errors of y. The treatment of the errors of the other coordinates can be performed in a similar way.

The dy is assumed to be caused by:

1. The errors of the elements of the inner and outer orientation of the cameras
2. The errors of the settings of the elements of orientation in the instrument
3. The errors of the measurements of the parallaxes.

Since the two first sources of errors have similar formulas for the error propagation (18) it is assumed that the differentials are combinations of the sources of errors under 1 and 2.

It is also seen that the influence of the errors of the parallax-measurements can be included in the first term of (18).

Formula (18) can therefore be regarded as the error—or correction equation between the elements of orientation and parallax measurements on one hand and the errors dy on the other. Since the parallaxes are assumed to be measured with equal weight, it is suitable to convert the distance error dy into parallaxes. This is performed by dividing both sides of equation (18) by y^2/bc.

The correction formula for the determination of corrections to the approximate elements of orientation from the photography which were used at the plotting for the determination of y_m, is according to (18):

$$\begin{aligned}\frac{dybc}{y^2} &= dx_t' - dx_t'' + \frac{1}{y^2}\{x^2dx_r' - (x-b)^2dx_r''\} + \frac{z}{y^2}\{xdz_r' - (x-b)dz_r''\}\\ &\quad + \frac{1}{y}\{-xdc_1 + (x-b)dc_2\} - \frac{c}{y}dbx + \frac{c}{y^2}\{-xdby_1 + (x-b)dby_2\}\\ &\quad + \frac{c}{y^2}[(y^2-x^2)d\phi_1 - \{y^2+(x-b)^2d\phi_2\}]\\ &\quad + \frac{cz}{y^2}\{xd\omega_1 - (x-b)d\omega_2\} + \frac{cz}{y}(-d\kappa_1 + d\kappa_2).\end{aligned} \tag{21}$$

A check of the signs of this formula can easily be performed from a simple geometrical discussion.

If the corrections of the elements are to be introduced into a plotter, the positive directions of the scales of course at first must be taken into account.

In such cases in terrestrial photogrammetry, when the feature is close to the cameras, the determination of the control distances y_g from the base to the control points can be affected by a constant error due to the difficulty of locating the outer perspective centers of the cameras.

In such cases it is suitable to assume a constant correction to all photogrammetrically determined y-coordinates. If this correction is called dy_0 the corresponding parallax correction is found to be $-(bcdy_0/y^2)$.

If a determination is desired of the corrections to the elements of the orientation completely from the discrepancies in control points, obviously a great number of control points must be assumed.

However it is important to remember that some of the elements have identical or similar influence upon the dy. Such elements, the coefficients of which are identical or proportional, therefore can replace each other completely, and it is sufficient to determine corrections to one of the elements only. An example is $d\kappa_1$ and $d\kappa_2$ which can be substituted for each other completely.

Other elements can substitute for each other partly, as for instance, $d\phi_1$ and dx_r'. In such cases new variables can be introduced. In other cases the sum of equal variables as, for instance, $d\phi_1 = d\phi_2 = d\phi$ can be substituted by another variable, in this example dby_1 or dby_2.

After considerations of these and similar relations one finds that different combinations of elements can be used for the corrections. The choice should also be made with a certain reference to the used instruments and the circumstances at the photography and the measurements of the elements of orientation.

For most cases in close-up terrestrial photogrammetry, the following combination seems to be suitable:

$$dbx,\ dc_2,\ dby_2,\ d\phi_2,\ d\omega_2,\ d\kappa_2,\ dy_0. \tag{22}$$

The corresponding elements of the left picture could of course also be used.

If the y-differences of the feature are comparatively small, and the principal distances of the cameras are determined with good accuracy, the dc_2 can be left out since dby_2 under such circumstances can be a sufficient substitution.

Furthermore, in practice, the ω_1, ω_2, κ_1 and κ_2 normally are very accurately determined with the aid of spirit levels. If in addition the elevation differences within the feature are relatively small and the dz_r' and dz_r'' can be neglected, the $d\omega_2$ and $d\kappa_2$ in (22) can be omitted.

After these simplifications, which for normal work with reliable instruments are allowable, one can concentrate upon the determination of the corrections:

$$dbx,\ dc_2,\ dby_2\ d\phi_2 \text{ and } dy_0. \tag{23}$$

Obviously there is a need for at least 5 control points, well distributed in the feature.

According to (21) and introducing the expression for dy_0 there is secured 5 equations of the type:

$$\begin{aligned} \frac{dybc}{y^2} = &- \frac{c}{y}\,dbx + \frac{x-b}{y}\,dc_2 + \frac{(x-b)c}{y^2}\,dby_2 \\ &- \left\{1 + \frac{(x-b)^2}{y^2}\right\} cd\phi_2 - \frac{bc}{y^2}\,dy_0 \end{aligned} \tag{24}$$

In this equation dy is the difference $y_m - y_g$ in each of the control points with the coordinates x and y.

From the five equations, obviously the corrections of the elements can be solved, and then applied to compute corrections to the photogrammetrically determined distances y to other points than the control points with the aid of (24).

ADJUSTMENT AND ERROR PROPAGATION BY THE LEAST SQUARE METHOD

If there are more than five control points more variables can be introduced or only the five can be used but the discrepancies are treated by adjustment procedures.

For the latter case the expression (24) is then written as a working correction equation:

$$\begin{aligned} v = &- \frac{c}{y}\,dbx + \frac{(x-b)}{y}\,dc_2 + \frac{(x-b)c}{y^2}\,dby_2 - \left\{1 + \frac{(x-b)^2}{y^2}\right\} cd\phi_2 \\ &- \frac{bc}{y^2}\,dy_0 - \frac{dybc}{y^2} \end{aligned} \tag{25}$$

TABLE 1

Point	x_m	y_m	dym_m	Point	x_m	y_m	dym_m
1	0.00	12.00	22	8	1.25	30.81	40
2	0.00	36.00	64	9	0.00	37.54	49
3	−10.00	36.00	83	10	−11.52	37.37	99
4	10.00	36.00	24	11	− 7.08	37.43	87
5	0.00	24.00	39	12	6.86	37.59	62
6	− 5.00	24.00	57	13	11.76	37.65	47
7	5.00	24.00	26				

If one wishes to determine the corrections so that the square sum of the residuals v becomes a minimum, the common principles of the least square method are used. This means the solution of a system of normal equations formed from (25). *Note that v refers to parallaxes.*

The square sum $[vv]$ from the solution of the normal equations will show how well the discrepancies have been corrected by the used elements.

Under the assumption that the residuals are caused by accidental errors of the observations of the parallaxes, there can be determined the standard error of one observation and also the error propagation with the principles of the least square method can be studied. Although this procedure means a certain approximation because the residuals may be caused by the uncorrected elements of orientation, which act as sources of systematic errors, a practical example will show the principles and particularly determine the standard error of the observations.

1.211 PRACTICAL EXAMPLE

A phototheodolite Zeiss, $c=192.09$ mm. image size 13×18 cm. was used for stereoscopic photography of a test field. The base was 4.024 m. and 13 control points were located in the field, see Figure 9.

The photography was performed according to the normal case.

The measurements of the negatives were performed in a Zeiss stereocomparator and the x-, y-, and z-coordinates were computed according to formulas (3)–(5).

The computed y-coordinates were compared with the given data. The

TABLE 2

Point	db'	dc_2	dby_2	$d\phi_2$	dy_0'	e_y
1	−1.601	−0.334	−0.536	−0.214	−5.365	−1.178
2	−0.534	−0.111	−0.060	−0.195	−0.596	−0.383
3	−0.534	−0.390	−0.208	−0.221	−0.596	−0.495
4	−0.534	+0.166	+0.089	−0.198	−0.596	−0.143
5	−0.800	−0.167	−0.134	−0.198	−1.342	−0.523
6	−0.800	−0.376	−0.302	−0.219	−1.342	−0.765
7	−0.800	+0.041	+0.033	−0.192	−1.342	−0.349
8	−0.650	−0.090	−0.053	−0.194	−0.815	−0.326
9	−0.513	−0.107	−0.055	−0.194	−0.549	−0.269
10	−0.515	−0.416	−0.214	−0.225	−0.554	−0.547
11	−0.514	−0.296	−0.152	−0.208	−0.551	−0.479
12	−0.512	+0.076	+0.039	−0.193	−0.546	−0.338
13	−0.510	+0.206	+0.105	−0.200	−0.545	−0.256

From the coefficients of Table 2 the normal equations, Table 3, are formed and solved.

errors $dy = y_m - y_g$ are shown in Table 1, together with the location of the points.

The computation of the coefficients of formula (25) is shown in Table 2. To avoid too large values of the coefficients the following substitutions are used:

$$\frac{dbx}{100} = db' \tag{26}$$

$$\frac{dby_2}{100} = dby_2' \tag{27}$$

$$\frac{dy_0}{1000} = dy_0' \tag{28}$$

Furthermore the dy of Table 1 are expressed in units of 0.1 mm.

The term $dybc/y^2$ is called e_y.

From the normal equations is obtained the square sum of the residuals as $[vv] = 0.0292$. As usual the standard error μ of the measurements is obtained as

$$\mu = \sqrt{\frac{[vv]}{n-u}}\,. \tag{29}$$

Since the number n of control points is 13 and the number of u of used elements is 5, the $[vv]$ corresponds to a standard error of one observations as 0.06 units of 0.1 mm. or 0.006 mm.

The corrections to the elements of orientation are obtained as:

$$\begin{aligned} db_x &= -\ 3.7 \text{ mm.} \\ dc_2 &= +\ 0.04 \text{ mm.} \\ dby_2 &= -\ 17.5 \text{ mm.} \\ d\phi_2 &= -\ 0.000077 \text{ radians} = -\ 2' \\ dy_0 &= +\ 6.9 \text{ mm.} \end{aligned}$$

The standard errors of the elements can be obtained in the usual way from their weight numbers and the standard error of the observations. The weight numbers are determined from the normal equations in the usual way.

The weight number of, for instance, db' is $Q_{b'b'} = 33.5168$ and the standard error of db', consequently

$$m_{b'} = \mu\sqrt{Q_{b'b'}} = 0.035 \text{ mm.}$$

From the relation

$$dbx = 100db'$$

the standard error of bx is found as

$$m_{b_x} = 3.5 \text{ mm.}$$

The standard errors of the other elements are in this case:

$$m_{c_2} = 0.05 \text{ mm.}; \qquad m_{by_2} = 7.9 \text{ mm.}$$

$$m_{\phi_2} = 0.00070 \text{ radians} = 2.'4$$

$$m_{y_0} = 11.5 \text{ mm.}$$

1.22. The Error Propagation after the Adjustment

The y-coordinates or primarily the parallaxes of arbitrary points can be corrected numerically with the aid of the corrections to the elements of orientation. For this purpose the formula (24) is used.

The standard errors of the corrections can also be found by the same formula, to which is applied the general law of error propagation. (See reference 4.) The elements of orientation are now correlated since their corrections have been determined from the same observations. The correlation is expressed by the correlation numbers, which can be obtained from the normal equations. The weight number of the y-correction of an arbitrary point can also be obtained directly from the normal equations after inserting an extra column containing the differentials of the expression (24). See reference 3.

If here there is used the general law of error propagation and also the weight number 1 for the observed parallax introduced, there is obtained from (25), the weight number of the parallax in an arbitrary point after the adjustment:

$$
\begin{aligned}
Q_{pp} = {} & \frac{c^2}{y^2} Q_{bxbx} + \frac{(x-b)^2}{y^2} Q_{cc} + \frac{(x-b)^2c^2}{y^4} Q_{by_2by_2} + \left\{1 + \frac{(x-b)^2}{y^2}\right\}^2 c^2 Q_{\phi\phi} \\
& + \frac{b^2c^2}{y^4} Q_{y_0y_0} + 1 - \frac{2(x-b)c}{y^2} Q_{bxc_2} - \frac{2c^2(x-b)}{y^3} Q_{bxby_2} \\
& + \frac{2c^2}{y}\left\{1 + \frac{(x-b)^2}{y^2}\right\} Q_{bx\phi_2} + \frac{2bc^2}{y^3} Q_{bxy_0} + \frac{2c(x-b)^2}{y^3} Q_{c_2by_2} \\
& - \frac{2(x-b)c}{y}\left\{1 + \frac{(x-b)^2}{y^2}\right\} Q_{c_2\phi_2} - \frac{2bc(x-b)}{y^3} Q_{c_2y_0} \\
& - \frac{2c^2(x-b)}{y^2}\left\{1 + \frac{(x-b)^2}{y^2}\right\} Q_{by_2\phi_2} - \frac{2bc^2(x-b)}{y^4} Q_{by_2y_0} \\
& + \frac{2bc^2}{y^2}\left\{1 + \frac{(x-b)^2}{y^2}\right\} Q_{\phi_2y_0}.
\end{aligned}
\tag{30}
$$

The weight number of the distances y is then obtained from the weight number of the parallaxes (30) and using the relation $dy = -(y^2/bc)dp$:

$$
Q_{yy} = \frac{y^4}{b^2c^2} Q_{pp} \tag{31}
$$

The standard errors of the distances y are finally found as

$$
m_y = \mu\sqrt{Q_{yy}} \tag{32}
$$

where μ is the standard error of the parallax measurements.

Formula (32) is the theoretically correct expression for the standard error of arbitrary y-coordinates under the assumed conditions and does not seem to have been derived earlier. The distribution of the standard error can be shown graphically after plotting formula (32) for varying x and y. It must be emphasized, however, that the neglected influence of some elements of orientation means a certain approximation, particularly in the method used for the determination of the standard error of the observations. But the complete and theoretically fully correct procedure is still more complicated and less suitable for practical application. The mean value of the y-errors within a certain area can be found after a double integration of (32).

It is, of course, also possible to include the errors of the x- and z-determinations in the adjustment procedure, since partially the same elements have caused the discrepancies of x and z as of y. See formulas (4) and (5).

For the theoretically correct determination of the standard errors of x and z of arbitrary points after the adjustment of the discrepancies of all three coordinates in the control points, a procedure similar to that demonstrated above can be used. The influence of small vertical parallaxes can also be taken into account.

If the discrepancies of y only have been adjusted as demonstrated above, the standard errors of x- and z- can approximately be determined from the

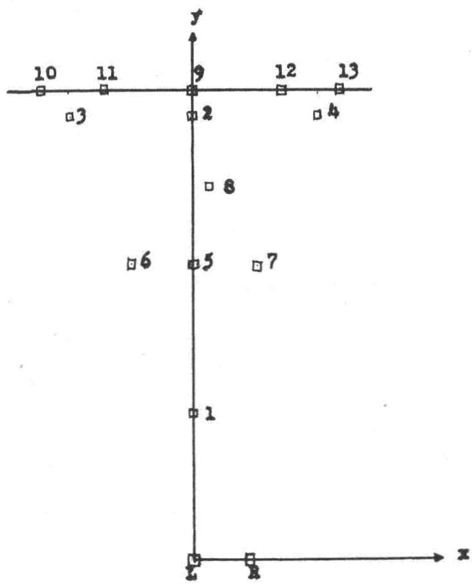

FIG. 9.—The location of the control points in the test field.

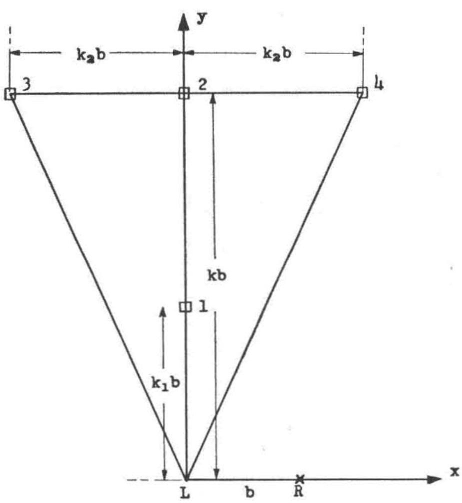

FIG. 10.—The location of control points for the simplified correction procedure.

first terms of formulas (7) and (8). In addition the standard errors of the measurements of the $x'z'$ or $x''z''$ coordinates have to be taken into account. These measurements are normally monocular in the stereocomparator.

Of course, the whole adjustment and error propagation procedure can be varied with respect to the conditions of the photography and plotting. Such elements as are accurately determined, can be neglected regarding their influence upon the coordinate errors, but the selection of such elements can be made from case to case only.

Test measurements are consequently necessary for each combination of cameras, instruments and also operators.

1.23. A Simplified Procedure for the Determination of the Corrections of the Elements of Orientation from Discrepancies of the Distances

The procedure for correcting the elements of orientation can be considerably simplified if the control points are located in certain positions. One possible and

TABLE 3

db'	dc_2	dby_2	$d\phi_2$	dy_0'	$-ey$	$Q_{b'}$	Q_{c_2}	Q_{by_2}	Q_{ϕ_2}	$Q_{y_0'}$	$S+1$	Σ
7.0760	1.4510	1.4536	1.8069	14.7024	−4.9222	1	0	0	0	0	22.5677	22.5677
	0.8029	0.5881	0.3915	3.0371	−1.3150	0	1	0	0	0	5.9557	5.9556
		0.5396	0.3133	3.7203	−1.2161	0	0	1	0	0	6.3989	6.3988
			0.5423	3.0496	−1.2572	0	0	0	1	0	5.8464	5.8464
				37.4230	−10.4297	0	0	0	0	1	52.5027	52.5027
					3.6675	0	0	0	0	0	15.4727	15.4727
2.6601	0.5455	0.5464	0.6793	5.5270	−1.8504	0.3759	0	0	0	0	8.4838	8.4838
	0.7109	0.4079	0.0294	0.0311	−0.4299	−0.2885	1.4067	0	0	0	1.8678	1.8676
		0.2732	−0.2559	2.5172	−0.1087	−0.3210	−2.1003	3.6603	0	0	3.6658	3.6648
			0.1204	−0.5125	−0.1279	−2.7326	−4.8073	7.7799	8.3056	0	8.0274	8.0256
				0.5247	0.0360	−5.0717	5.2971	−9.9609	8.1124	1.9059	0.8407	0.8435

$[vv]=0.0292$

$$\mu=\sqrt{\frac{0.0292}{8}}=0.006 \text{ mm.}$$

$db'=-0.037$ mm.

$dc_2=+0.04$ mm. ± 0.05 mm.

$dby_2=-17.5$ mm. ± 7.9 mm.

$d\phi_2=-0.000077$ radians $=-2'\pm 2.4'$

$dy_0'=+0.0069$ mm.

$db_x=-3.7$ mm. ± 3.5 mm.

$dy_0=+6.9$ mm. ± 11.5 mm.

suitable pattern of a minimum of control points is shown in Figure 10.

It is assumed that there are four control points within the stereoscopic field.

Points 1 and 2 are located in the left camera axis and at the distances k_1b and kb respectively from the left station.

The points 3 and 4 are located on the perpendicular to the left camera axis through point 2 and at the same distances k_2b from point 2.

b is the base of the photography and k, k_1 and k_2 are arbitrary multiples of the base which have to be chosen so that the control points are located within the stereoscopic field, and if possible so that the feature is surrounded by the figure 1-3-2-4.

In principle, the points should be located in the horizontal plane through the camera axes but certain deviations can be allowed, particularly if $\kappa_1\kappa_2$ ω_1 and ω_2 are well determined and furthermore the inner orientation is stable.

It is assumed that the elements db_x, dby_2 and $d\phi_2$ are to be corrected. There is added the term for a constant error dy_0 as introduced above. The correction formula for the connection between the errors of y and the corrections of the elements is, according to (18):

$$dy = -\frac{y}{b}\,dbx + \frac{x-b}{b}\,dby_2 - \left\{1 + \frac{(x-b)^2}{y^2}\right\}\frac{y^2}{b}\,d\phi_2 - dy_0. \tag{33}$$

For each of the control points is obtained an equation of this type in which the coordinates of the coefficients are inserted according to the coordinates of the control points according to Figure 10.

The equations in Table 4 are secured.

TABLE 4

Point	Coordinates		Coefficients of				$y_m - y_g$
	x	y	dbx	dby_2	$d\phi_2$	dy_0	
1	0	k_1b	$-k_1$	-1	$-b(k_1^2+1)$	-1	dy_1
2	0	kb	$-k$	-1	$-b(k^2+1)$	-1	dy_2
3	$-k_2b$	kb	$-k$	$-k_2-1$	$-b\{k^2+(k_2+1)^2\}$	-1	dy_3
4	k_2b	kb	$-k$	$+k_2-1$	$-b\{k^2+(k_2-1)^2\}$	-1	dy_4

By solving the equations of Table 4 is obtained:

$$dbx = \frac{2k_2^2dy_1 - 2dy_2(k^2 - k_1^2 + k_2^2) + (dy_3 + dy_4)(k^2 - k_1^2)}{2(k - k_1)k_2^2} \tag{34}$$

$$dby_2 = \frac{-\,4dy_2 + dy_3(2 - k_2) + dy_4(2 + k_2)}{2k_2^2} \tag{35}$$

$$d\phi_2 = \frac{2dy_2 - dy_3 - dy_4}{2bk_2^2} \tag{36}$$

$$dy_0 = -\,dy_1 - k_1dbx - dby_2 - b(k_1^2 + 1)d\phi_2 \tag{37}$$

In this way the corrections of the elements can be obtained directly from the discrepancies in the control points and the coefficients k, k_1 and k_2. These coefficients can be chosen arbitrarily and with respect to the shape and size of the feature, location of the cameras, etc.

For a study of the error propagation from the measurements of the parallaxes to the elements of orientation and to functions of these elements, there can be used the formulas (34)–(37) directly after having substituted the measurements. From the relation

$$dy = -\frac{y^2}{bc}dp$$

the corrections dbx etc. can be expressed as direct functions of the errors dp of the measurements.

For instance from (36) and Table 4 is obtained:

$$d\phi_2 = \frac{k^2(-2dp_2 + dp_3 + dp_4)}{2ck_2^2} \tag{38}$$

If there is assumed the same standard error μ of the measurements of the parallaxes p and no correlation, the standard error of ϕ_2 can be obtained by applying the special law of error propagation to (38).

The weight number $Q_{\phi_2\phi_2}$ will thus be obtained as the square sum of the coefficients of dp. And from (38):

$$Q_{\phi 2\phi 2} = \frac{6k^4}{4c^2k_2^4} \tag{39}$$

and the standard error m_{ϕ_2} as

$$m_{\phi_2} = \frac{\mu k^2}{2ck_2^2}\sqrt{6} \tag{40}$$

In a similar way the weight numbers and standard errors of the other elements of orientation can be found from (34)–(37).

For the error propagation in functions of the elements, the correlation numbers have to be determined. These can be found from (34)–(37) as the product sum of the coefficients of corresponding dy. After substituting the dy by the corresponding parallaxes one finds for instance:

$$Q_{by_2\phi_2} = -\frac{3k^4b}{k_2^4c^2} \tag{41}$$

By the aid of the general law of error propagation, the standard error of any function of the elements then can be determined.

The standard error of the observations μ is of course of great importance since it appears as a factor in all expressions for error propagation. In principle, the normal procedure for the study of error propagation, as outlined above, can be applied in a theoretically correct way only if μ refers to accidental observational errors. This means in our case that the errors of the neglected elements of orientation must be so small that the accidental observational errors dominate. Of course, systematic errors, as for instance distortion, are also neglected or are assumed to be corrected in all measurements.

Under these assumptions the standard error of the observations can be determined from an adjustment procedure of discrepancies in a great number of control points by a similar procedure as has been demonstrated above under *2.11*.

Great attention must be paid to the qualities of the camera, plotter and operator, in order to find those elements of orientation, the errors of which can be neglected in practical work. Consequently, test measurements are necessary for photogrammetric equipment and procedures in order to determine the standard error of the observations. *The more carefully this work is done the less is the approximate character of the formulas for the error propagation.*

1.231 PRACTICAL APPLICATION

The formulas (34)–(37) are applied to the data in Table 1 and points 1, 2, 3 and 4 are chosen for the determination of corrections.

The obtained corrections are:

$$dbx = -27.3 \text{ mm.}$$

$$dby_2 = -15.3 \text{ mm.}$$

$$d\phi_2 = 0.000422 \text{ radians} = 1.'4$$

$$dy_0 = 58 \text{ mm.}$$

After applying the corrections to the discrepancies of the rest of the control points, there is found the mean square value of the residuals in the remaining control points of 20 mm. A considerable improvement of the y-determination has thus been obtained by the used simplified method for determination of corrections to the elements of orientation. Due to the strong correlation between the elements of orientation, the corrections can vary rather much for different combinations of control points.

In many cases in practice it may of course be difficult to place control points in accordance with Figure 10. But particularly in non-topographic applications a similar arrangement often ought to be possible. An easy check of the accuracy of the elements of orientation is in such cases possible, and corrections can obviously be determined if necessary.

Summary

This paper contains a concentrated demonstration of the theory of errors for the terrestrial stereophotogrammetric normal case. Particularly the connection between the errors of the elements of orientation and the errors of the distances y from the base to arbitrary points has been treated, since the distance determination normally is the most critical. The error propagation from the elements of orientation has been demonstrated before and after an adjustment of discrepancies in control points. In a practical example the theory has been applied to the determination of the corrections of the elements and the error propagation.

Finally, a simplified procedure for the determination of the corrections from discrepancies in suitably located control points has been demonstrated. Such a procedure is of particular interest for many non-topographic applications of photogrammetry, for instance, culture-historical purposes. In such cases the final plotting of the pictures can be expected to be performed several generations after the photography. The operator will surely appreciate an easy way to correct the errors which were made in connection with the photography, particularly if the feature is no longer available for a direct check.

For testing of instruments and operators, the principles, demonstrated above, may have particular use.

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