SOME USES OF THREE-DIMENSIONAL MODELS FOR ILLUSTRATING PHOTOGRAMMETRIC PRINCIPLES*

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Abstract

It is common practice, when illustrating the photogrammetric principles governing relief displacement, tilt displacement, stereoscopy, etc., to employ either line drawings or actual aerial photos. In this paper examples are given of ways in which photos of three-dimensional models can be used advantageously, in conjunction with line drawings and aerial photos, for illustrating such principles. In attempting to indicate with only a few examples how extensively such models can be used, three diverse types of photogrammetric principles are discussed and illustrated: (1) those governing image displacements, (2) those involved in planning and executing a photographic mission and (3) those involved when developing some new photogrammetric technique.

Introduction

In any young and fast-growing profession it is almost inevitable that some positions will be filled by individuals who have had inadequate professional training. For example, many practicing photogrammetrists, who like the speaker have never had the benefit of a formal course in photogrammetry, are employed in positions which presume of the incumbent a thorough knowledge of elementary photogrammetric principles.

There are perhaps both advantages and disadvantages arising from such an incongruity in the photogrammetric profession. On the one hand there is the encouraging prospect that an inadequately trained photogrammetrist will be prompted to develop some rather unconventional and meritorious approaches to photogrammetry, in an effort to convince himself or others, of certain photogrammetric truths. On the other hand there is always the peril that certain of his methods or conclusions will prove to be photogrammetrically unsound.

It is the perilous purpose of this paper to present some of the unconventional methods developed by the speaker for illustrating photogrammetric principles through the use of three-dimensional models. Knowing, as he does, the forthrightness of many of his peers who are present in the audience, he rests assured that any unsound remarks contained in his presentation will be promptly rectified at the conclusion of his paper.

As will presently be shown, *photographs* of a model, rather than the model itself, usually are employed in illustrating the principles discussed in this paper. It might well be asked at the outset, then, why photographs of models should be employed instead of either line drawings or actual aerial photographs, as used in most of our successful photogrammetry texts. While this question might better be answered after a few examples have been presented, at least a tentative answer seems appropriate at this point.

In comparison with line drawings the beginning photogrammetrist usually finds an actual photo stereogram to be more easily understood and somewhat more convincing.—More easily understood because of his inability to grasp the 3-dimensional implications of a rather complicated 2-dimensional drawing.—More convincing because of his inherent skepticism that any line drawing can portray the situation existing on an actual photograph.

In comparison with aerial photographs, 3-dimensional models are frequently superior because (1) models can be ideally constructed so as to illustrate the photogrammetric principles most effectively and (2) the absolute position and orientation of the terrestrial camera can be precisely controlled so as to obtain photos of the model that will best reveal these principles.

It should not be inferred from the foregoing that the speaker believes that 3-dimensional models should be used to the exclusion of line drawings and aerial

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photos. As the examples in this paper will attempt to illustrate, these 3 media should be considered mutually complementary, rather than competitive. Neither does he claim that others before him have not, on occasion, used 3-dimensional models effectively to illustrate certain photogrammetric principles. However, a survey of the rather voluminous literature on photogrammetry, including current textbooks, will show that rarely, indeed, have such models been used.

It is hoped that this paper will serve to highlight the desirability of our making increased use of 3-dimensional models to illustrate photogrammetric principles. In an effort to accomplish this in the allotted time, only three types of principles will be discussed: (1) those governing image displacements, (2) those involved in planning and executing a photographic mission and (3) those involved when developing some new photogrammetric technique.

Image Displacements

Displacement is defined as the departure of photographic images from their true relative positions. On conventional aerial photography the most significant displacements are those caused by relief and tilt. In the ensuing discussion an attempt will be made with the aid of 3-dimensional models to show, first, the nature of relief displacement in the absence of tilt; secondly, the nature of tilt displacement in the absence of relief; and thirdly, the combined effect of both types of displacement.

Relief Displacement in the Absence of Tilt

Relief displacements are those caused by differences in elevation of the corresponding ground objects. The 3-dimensional model shown in Figure 1 was constructed in an effort to illustrate certain consequences of relief displacement. In essence the model consists simply of a uniformly gridded base on which have been placed the frustums of two pyramids (hereafter referred to simply as "pyramids"). The two pyramids differ from each other only in over-all height and steepness of slope.

Figure 2 is a vertical photo stereogram of the same model taken from specific camera stations and with the exercise of much care to ensure true verticality of the optical axis of the camera. The left photo of the stereogram was taken from a point directly above the center of the left pyramid, while the right photo was taken, at the same elevation, from a point directly above the center of the right pyramid.

It is quite instructive for the beginning photogrammetrist to analyze the manner in which areas, distances and directions are presented in this stereogram. While his observations may be somewhat empirical and his conclusions stated in a language that for the moment lacks photogrammetric preciseness, he will clearly

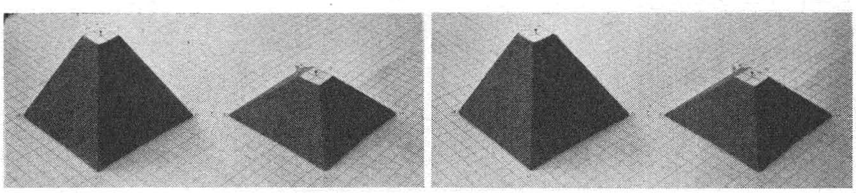

Fig. 1. Oblique stereogram showing the construction of a 3-dimensional model that is useful in illustrating relief displacements. Compare with Fig. 2.

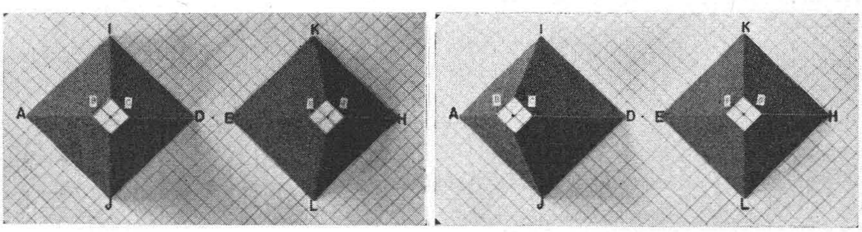

Fig. 2. Vertical stereogram of the model shown in Fig. 1. Note distortions of area, distance and direction, as explained in text.

see the practical importance of correcting for displacements on photos.

Here are the observations and empirical conclusions relative to *area, distance* and *direction* which a beginning photogrammetrist might draw from a study of Figure 2.

1. *Area:* In reality, the area of the shaded half of a pyramid is, of course, equal to the area of the illuminated half. But the left photo of Figure 2 shows unequal areas for the two halves of the right pyramid; and the right photo shows even greater inequality of areas for the two halves of the left pyramid. From these observations it is concluded that when a vertical photo is taken from a point other than directly above a pyramid, *slopes facing toward the camera lens appear too large and slopes facing away from the camera lens appear too small; the steeper the slope, the greater the distortion of area.*
2. *Distance:* In reality the four lines, AB, CD, EF, and GH all have the same horizontal distance. But the left photo of Figure 2 shows EF as somewhat too long and GH as somewhat too short. The right photo of Figure 2 shows AB as much too short and CD as much too long. From these observations it is concluded that *on slopes which face the camera lens, lines running parallel to the slope* (such as EF on the left photo) *are elongated;* conversely *on slopes which face away from the camera lens, lines running parallel to the slope* (such as GH on the left photo) *are foreshortened. The steeper the slope, the greater the distortion of distance.*
3. *Direction.* In reality the lines IJ and KL are straight North-South lines. But in the left photo of Figure 2 KL appears somewhat bent and in the right photo IJ appears even more bent. Here the beginning photogrammetrist's empirical observations may not enable him to draw conclusions more definitive than the following. *The direction of a sloping line may be incorrectly represented on a vertical photo; the steeper its slope the more it is bent.*

Another pertinent conclusion which a beginning photogrammetrist might draw from a study of Figure 2 is the following: *At a given datum* (such as the gridded base on which the pyramids rest) *there is no distortion of area, distance or direction throughout the photo.* Obviously, if this were not true, the individual grid squares of the model's base would not appear, at all points on the photo, to be of uniform size, shape and orientation.

Finally, the beginning photogrammetrist may note that the squares on top of the right pyramid appear somewhat larger than those at the base; and that squares on top of the left, or taller, pyramid appeared still larger. In reality, all of these squares were drafted to the same size on the model. From these observations it is concluded that *the higher the datum, the larger its photographic scale.*

From the foregoing example it is apparent that our hypothetical beginning photogrammetrist was typical of his kind in that he displayed some native intelligence relative to such concepts as datum, scale and distortion. Nevertheless, his observations based on Figure 2 were only empirical because the model was designed merely to show that distortions of area, distance and direction on photographs are very real, may be of considerable magnitude, and therefore cannot be ignored. With this background he can now benefit immensely from studying a series of line drawings of the type portrayed in Figure 3. These drawings permit a more careful analysis of the factors affecting relief displacement.

In Figure 3A it is seen that whether the print be truly vertical or tilted, there is no relief displacement for a point directly beneath the camera lens. All other portions of the photograph are subject to relief displacements. This is further indicated by Figure 3B where the points T_1 and B_2 are at the datum plane, with the result that their corresponding photo images, T_1' and B_2' exhibit no relief displacement. Obviously, since B_1 is vertically beneath T_1 it would appear in the same position on a photo as T_1 if it had no relief displacement. Instead it appears at B_1'. Similarly, since T_2 is vertically above B_2 it would appear in the same position on a photo as B_2 if it had no relief displacement. Instead it appears at T_2'. From Figures 3A and 3B, therefore, the following conclusions are indicated:

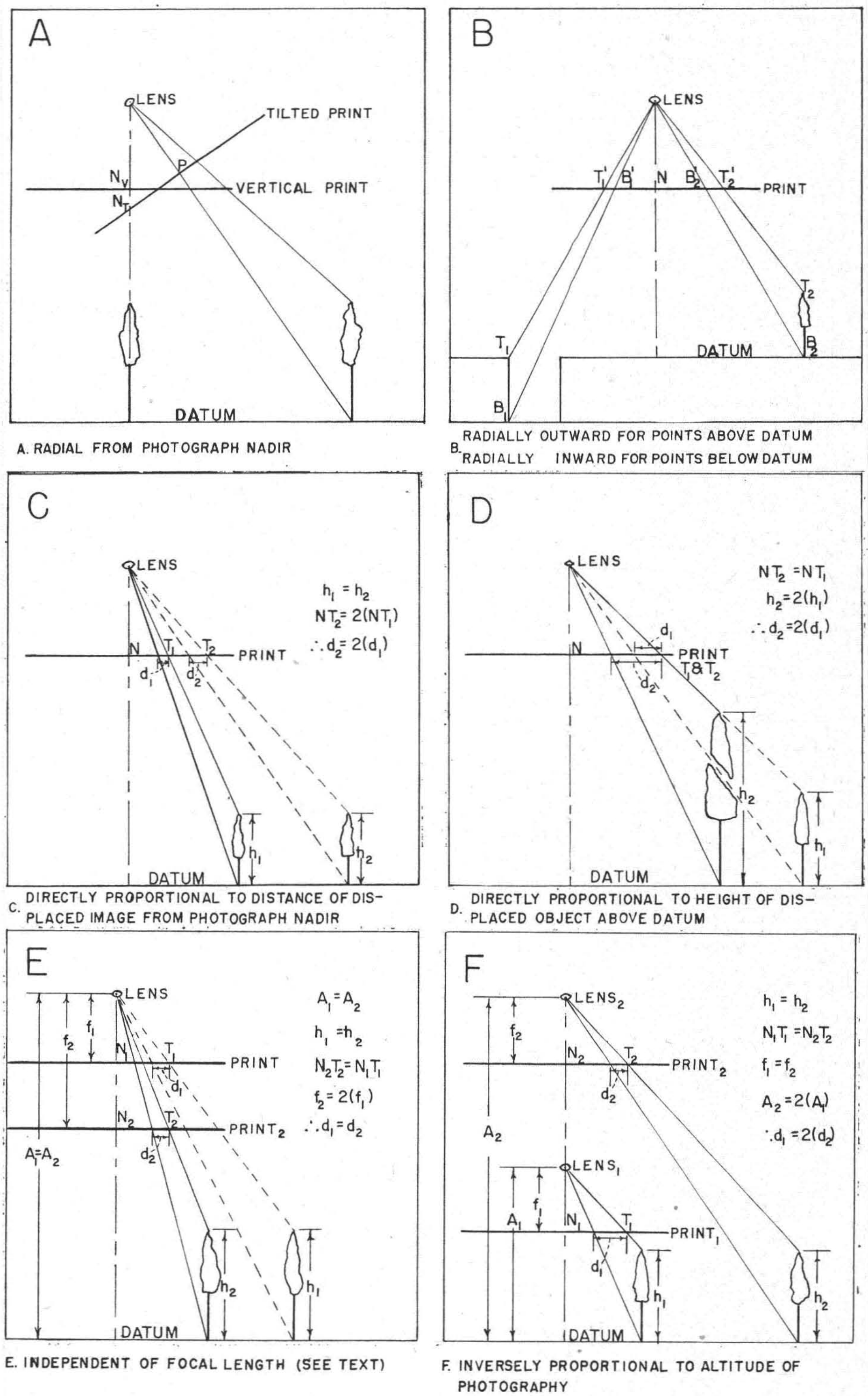

FIG. 3. An example of the effective use of line drawings to illustrate the direction and magnitude of relief displacements.

(1) *there is no relief displacement at the photograph nadir,*
(2) *relief displacement is radial from the photograph nadir,*
(3) *objects above the datum are displaced radially outward,*
(4) *objects below the datum are displaced radially inward.*

In Figure 3C the radial distance, NY_2 is twice NT_1, and the corresponding relief displacement of the top of the tree, d_2, is twice d_1. This indicates that:

(5) *the magnitude of relief displacement is directly proportional to the distance of the displaced image from the photograph nadir.*

Figure 3D shows one tree, h_2, having twice the height of another tree, h_1, but with an identical distance from the photograph nadir to the displaced image, in this case the top of the tree. The corresponding relief displacement of the top of the one tree, d_2, is seen to be twice that of the other, d_1. From this it is concluded that:

(6) *the magnitude of relief displacement is directly proportional to the difference in elevation between the displaced object and the datum.*

In Figure 3E it is assumed that two vertical aerial photos are taken from exactly the same camera station, one with a camera having a focal length, f_2, which is twice that of the other, f_1. However, the heights of the two trees are equal, as are the radial distances from the photo nadir to the displaced images of the tops of the trees. Under these conditions, the tree top images are seen to have equal relief displacements. This diagram serves to illustrate that if altitude of photography, height of object, and distance of displaced image from the photograph nadir are all kept constant, then

(7) *the magnitude of relief displacement is independent of focal length of the camera.**

In the situation diagrammed in Figure 3F it is seen that doubling the altitude of photography reduces to one half the amount of relief displacement. This diagram serves to illustrate that if focal length of camera, height of object, and distance of displaced image from the photo nadir are kept constant, then

(8) *the magnitude of relief displacement is inversely proportional to the altitude of photography.*

At this point the beginning photogrammetrist may profess an understanding of the practical importance of relief displacements as pictured in Figure 2 and of the factors affecting their magnitude and direction, as diagrammed in Figure 3. However, Figure 3 is only a two-dimensional drawing and therefore shows the nature of relief displacement only for the special case in which the displaced image lies in the plane of the page. There are perhaps only two *conventional* ways of bridging the gap between this highly special case and the more general one in which the displaced image may lie anywhere on the photo. The first way is merely to tell the beginning photogrammetrist to visualize the rotation of a two-dimensional drawing (Figure 3) through any desired number of degrees about its central vertical axis. In so doing, he is told, he should see that the situation depicted in the two-dimensional drawing prevails everywhere throughout the photo. The second way is to construct a perspective sketch, or series of sketches, showing the displacement of objects at several positions on the photos even though this may necessitate the drawing of a confusingly large number of rays.

Perhaps these two time-honored methods should not severely tax or surpass the beginner's comprehension of descriptive geometry—but they frequently do. Accordingly a third approach may be worth investigating—one which will make complementary use of actual photographs in conjunction with the simple line drawings just presented. At least some of the photographs used in this complementary fashion

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* This conclusion is surprising to some be ginning photogrammetrists. Nevertheless, it is in keeping with the fact that neither the formula expressing height of an object in terms of its displacement on a single photo

$$\left(h = H \times \frac{d}{r}\right),$$

nor that giving its height from displacement on a stereo pair of photos

$$\left(h = \frac{H \times dP}{P + dP}\right)$$

makes mention of the camera's focal length.

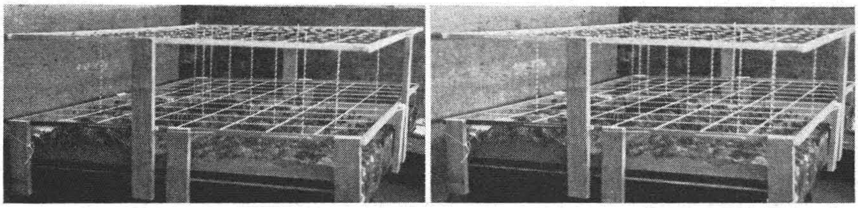


FIG. 4. Oblique stereogram of a 3-dimensional model that was designed to make the line drawings of Fig. 3 more convincing to a beginning photogrammetrist. Compare this stereogram with Figs. 5, 8, and 9.

should be actual aerial photos as will be shown in Figure 6. But others might better be precisely oriented photos of a terrain model (Figures 4 and 5) especially constructed to illustrate the points in question.

Figure 4 is an oblique photo stereogram showing the construction of such a model. This model was designed for the purpose of illustrating the direction and magnitude of relief displacements wherever they may be found on an actual photograph. The model consists of (1) a "war surplus" terrain model which serves as a base. (This base was used in order to convey a more realistic impression, on subsequent photography, that an actual portion of the earth's surface was being depicted, as in aerial photography); (2) two horizontal reference planes or "datum planes" formed by stretching grids of string lines above the model. (The lines in the lower reference plane are made of unpainted white string and therefore photograph as solid white lines; the lines in the upper reference plane contain alternate segments of blackened and unblackened string, and therefore photograph as dashed lines); and (3) thirteen vertical rods situated symmetrically at various grid intersections throughout the model. (The rods have been spirally wrapped with white string to give them a dashed line effect also.)

Figure 5 is a truly vertical photo of this same model taken from a point directly above the intersection of string lines labelled "*4*" and "*D*." This single photo illustrates most of the principles conveyed by the six line drawings of Figure 3. For some photogrammetrists it will illustrate the points more convincingly because (1)

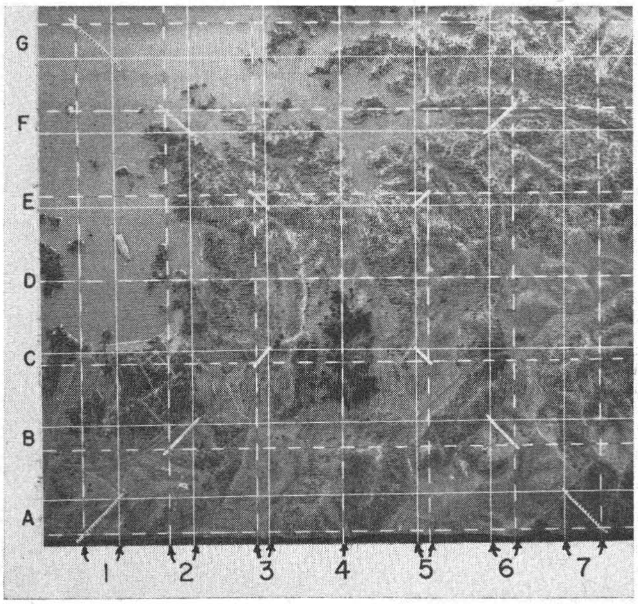


FIG. 5. Vertical photo taken from a point directly above the center of the model shown in Fig. 4. For a discussion of the relief displacements shown here, see text.

the relief displacements shown here are actually photographic rather than diagrammatic and (2) they are not confined to a special portion of the area photographed, as in the previous line drawings.

Specifically, the following conclusions are indicated from a study of the spirally-wrapped poles in Figure 5.

(1) There is no relief displacement at the photo nadir which in this case is the point "*4-D*."
(2) Displacement due to relief throughout the entire photo is radial from the photo nadir.
(3) Objects above a given reference plane (such as the plane represented by the solid white lines) are displaced radially outward.
(4) Objects below a given reference plane (such as the plane represented by the dashed lines) are displaced radially inward.
(5) The magnitude of relief displacement is proportional to the distance of the displaced image from the photo nadir.
(6) The magnitude of relief displacement is proportional to the height of the object being displaced. Thus, each successively higher spiral on a pole is displaced farther outward than the next lower one.

A second vertical photo of this same model was taken from twice the altitude and readily showed half the relief displacement, thereby demonstrating that the magnitude of relief displacement is inversely proportional to the altitude of photography. A third vertical photo taken from the same altitude as Figure 5 but with a lens having only half the focal length showed the same relief displacement as Figure 5 when enlarged to the same scale. This shows clearly that *at a given scale and altitude* focal length has no effect on relief displacement.

By way of illustrating the mutually complementary functions of line drawings, three-dimensional models and aerial photos, it is pertinent to consider next an actual aerial photo stereogram (Figure 6) of an area having an appreciable amount of relief. It is obvious in Figure 6 that despite relief displacements the clearing for the power line appears straight on the right photo, just as it was constructed. By now the beginning photogrammetrist readily appreciates that this is not due to an absence of relief displacements in the photo but rather to their being radial from a point (the photo nadir), through which the power line passes. This is confirmed by his noting the appearance of the power line in the left photo of Figure 6 where the low points, such as A and C, are seen to be displaced in toward the photo nadir while the high points, such as B, are displaced outward. If he had any doubts up to this point as to whether photos of three-dimensional models convey principles which are applicable to actual aerial photos, he should now feel somewhat reassured.

At this point in the discussion of relief displacement, the concept of stereoscopic parallax is logically introduced. Stereoscopic parallax is defined as the apparent displacement of one object with respect to another, caused by a shift in the point of observation. Absolute stereoscopic parallax is the algebraic difference, parallel to the line of flight, of the distances of two images of a given object from their respective principal points, assuming that the two photos are truly vertical and that they are taken from the same height. Y-parallax is displacement at right angles to the line of flight.

In presenting this subject, line drawings and three dimensional models again prove to be mutually complementary. Thus, the derivation of the parallax formula,

$$h = \frac{H \times dP}{P + dP},$$

is more readily seen from a line drawing such as that shown in Figure 7. On the other hand, a carefully positioned vertical photo stereogram of a terrain model, such as is shown in Figure 8, is far more effective in explaining the following important concepts with reference to stereoscopic parallax:

(a) stereoscopic parallax pertains only to the x-component of relief displacement (i.e. the component which is parallel to the stereo base)
(b) the images of all objects having the same elevation have the same absolute stereoscopic parallax on truly vertical photos
(c) the y-parallax of any image is exactly the same on one member of a

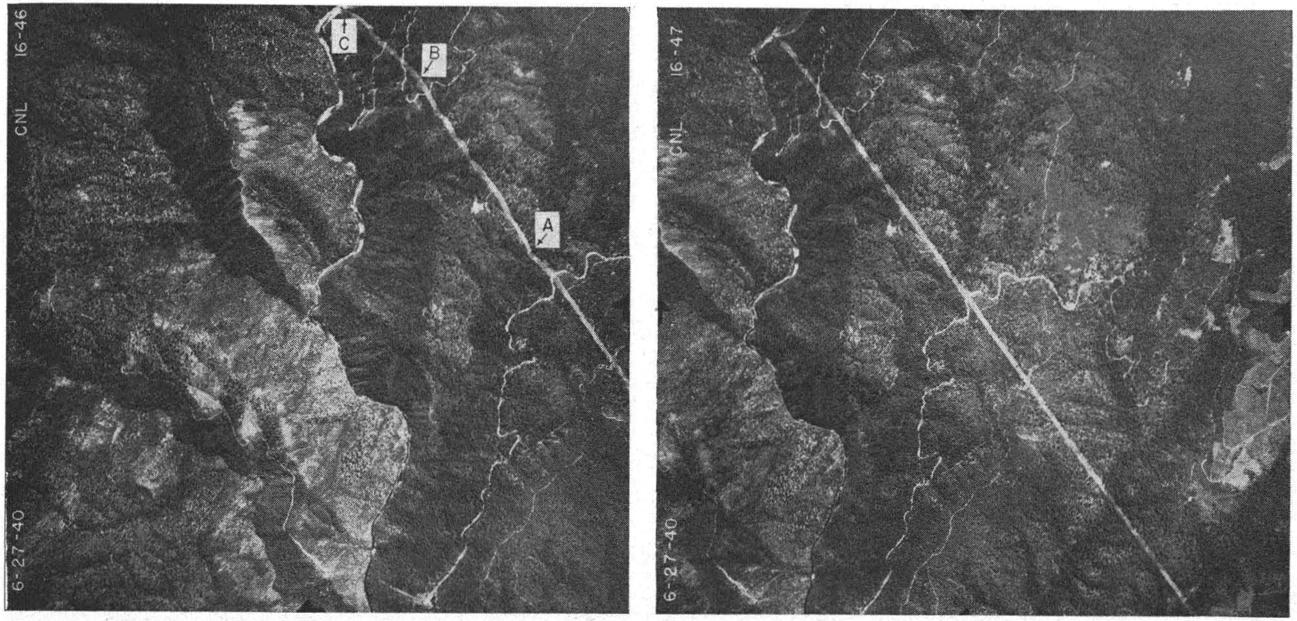
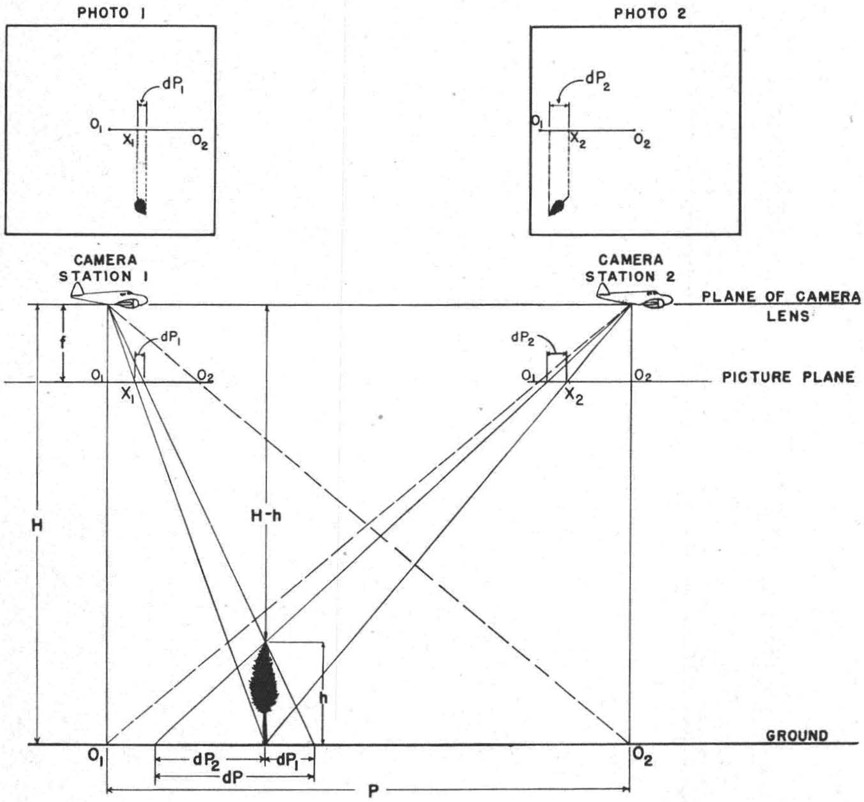


FIG. 6. Aerial photos such as these lend credence to conclusions based on line drawings and 3-dimensional models. Note that the power line appears perfectly straight on the right photo, but exhibits obvious relief displacements on the left photo.

truly vertical stereo pair of photos taken from a given altitude as it is on the other.

Although it is not considered within the province of this article to do so, one can more readily explain the logical steps entailed in horizontalizing a stereo model if he understands what is shown in Figure 8. Figure 9 is included here for the purpose of showing how readily one can illustrate with this same three-dimensional model that the magnitude of stereoscopic parallax is (essentially) proportional to the magnitude of the stereo base. Thus in Figure 8, where the two photo nadirs were at 4.0 and 5.0, each grid intersection of the top grid had a dP of x units with respect to the bottom grid. But in Figure 9 the two photo nadirs are at 4.0 and 4.5. Hence the stereo base, P, is only half as great as in Figure 8 and consequently the corresponding dP is only half as great, or $x/2$ units.

Once stereoscopic parallax is understood from a study of Figures 8 and 9 it is quite easy to introduce the concept of contouring from a stereo pair of photos. It has been shown, that all points at a given



h = ht. of tree

H = ht. of camera lens above base of tree

P = Absolute parallax of base of tree (photo equivalent = $O_1 O_2$ = $O_1 X_1 + O_2 X_2$)

dP = Parallax difference of top of tree referred to base plane (photo equivalent = $dP_1 - dP_2$)

From similar triangles: $$\frac{h}{H-h} = \frac{dP}{P}$$

Transposing: $$h = \frac{H \cdot dP}{P + dP}$$

FIG. 7. Line drawing showing derivation of the parallax equation.

datum or reference plane have the same stereoscopic parallax. Accordingly it is obvious why any "floating dot" instrument can be used in contouring from a pair of truly vertical (or horizontalized) photos merely by adjusting the stereoscopic parallax of the instrument's floating dot for the plane of the contour. Then as the operator moves the instrument with respect to the photos, he need only keep

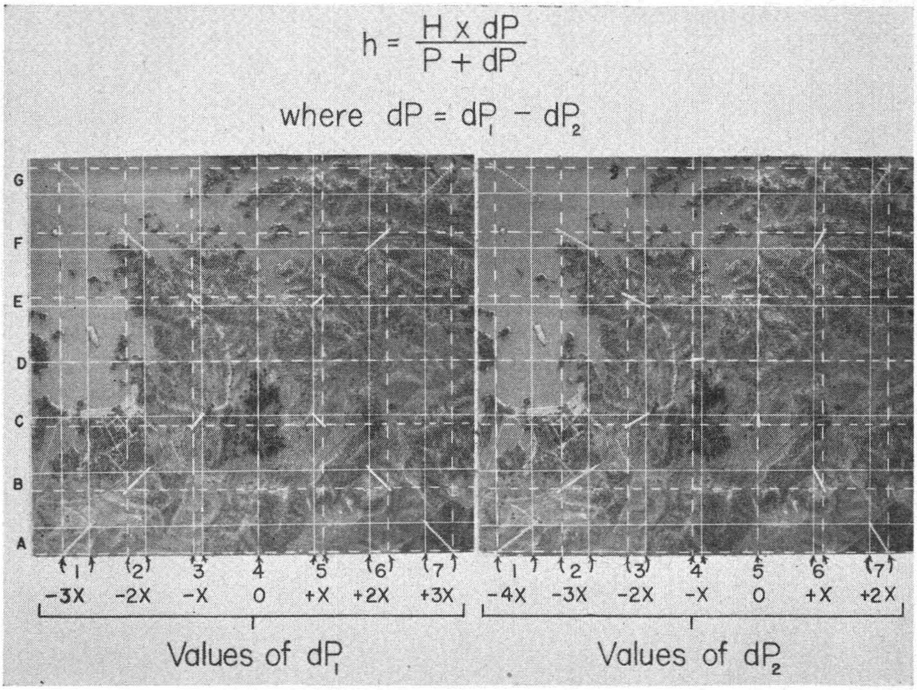


FIG. 8. Vertical stereogram of the terrain model shown in Fig. 4.

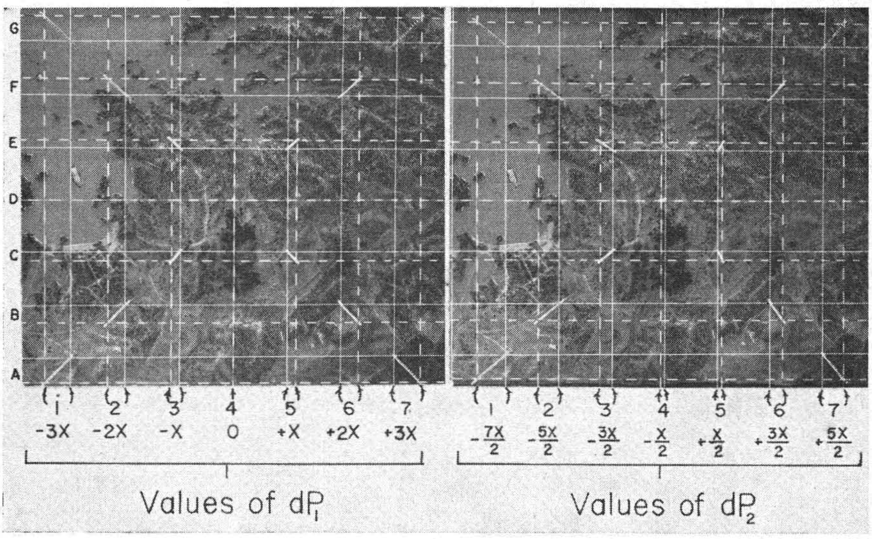


FIG. 9. Vertical stereogram of the same terrain model taken with only half the stereo base used in Fig. 8. Note that when studied stereoscopically, the top grid appears only about half as far above the bottom grid here as in Fig. 8 because of the reduced stereo base.

this floating dot in apparent contact with earth's surface (as seen stereoscopically on the photos) in order to delineate the contour. Indeed, this concept frequently sounds so simple to the beginning photogrammetrist that he may think that the very first contour he delineates from a stereo pair of photos is perfectly placed. It is more than likely however that it contains numerous errors, and should be corrected by a more experienced photogrammetrist. Here some uncertainties may arise, because the beginner's pride in his visual acuity may lead him to say privately, "Well, that's where the boss says the contour should go, but my eyes are as good as his, and as I see it the contour should go back here, just where I had it before."

Three dimensional models can do much to prevent such controversy and doubt. If the beginning photogrammetrist really wants to master the science and art of contour delineation, why not let him try his skill on a model such as that shown in Figure 10? This model has been carefully devised to include most of the terrain features encountered in contouring the earth's surface—from vertical cliffs to flat surfaces; from sink holes to hoary peaks. At the same time long uniform slopes are included which can be used to illustrate the principles of interpolation, and concave and convex slopes are provided to show how contour spacing varies with steepness of slope. All the beginner need be given is a few spot elevations and the information that this is a truly vertical stereogram of a model for which the contours fall in truly horizontal planes. Then, when he has finished delineating "contours" on this stereogram and wishes to assess the accuracy of his contouring, he need only refer to Figure 11. The thin film which obscured the edges of the successive plywood "lifts" has now been scratched away just enough to reveal the top outer edge of each lift. Each such edge accurately defines a contour in the position where it should have been delineated on Figure 10.

TILT DISPLACEMENT IN THE ABSENCE OF RELIEF

The nature of tilt displacement can best be demonstrated by comparing the positions of photographic images on a tilted photograph with their positions on a vertical photograph taken from the same camera station. Such a situation is represented diagrammatically in Figure 12. Since only objects situated on flat and level ground are considered in the diagram, they have neither tilt displacement nor relief displacement on the truly vertical photograph and therefore appear in their true plane positions. On the tilted photograph

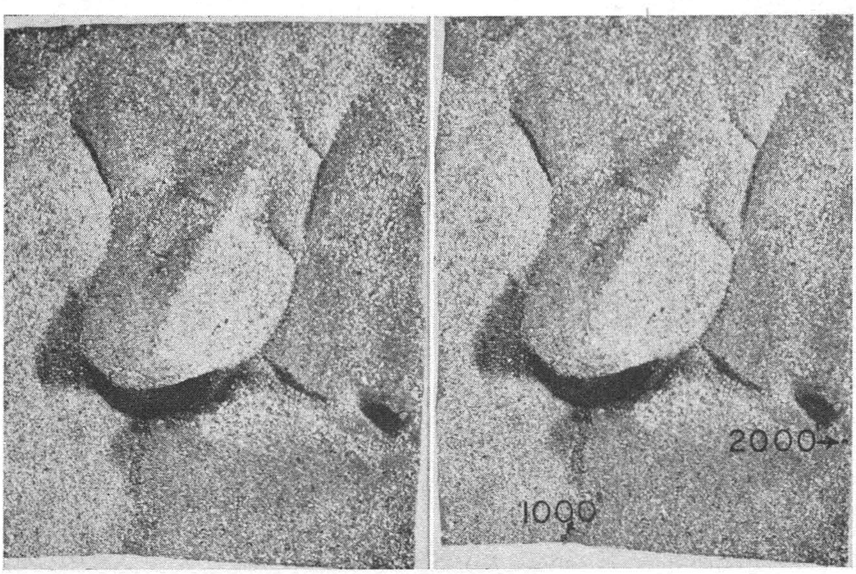


FIG. 10. Stereogram of a 3-dimensional model which illustrates the photogrammetric principles of contour delineation.

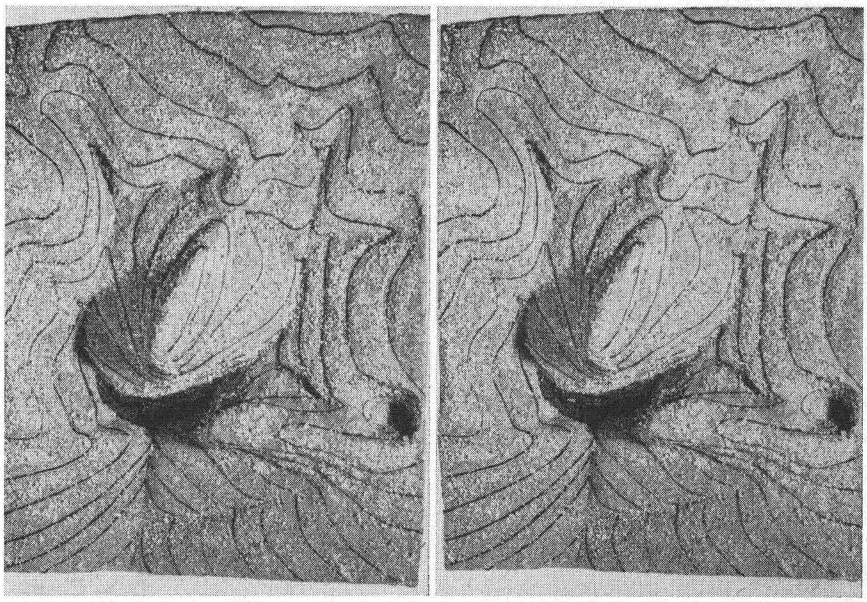


FIG. 11. Stereogram of the same 3-dimensional model, photographed after removing the thin surface which previously had obscured the true positions of the contours.

any departure from these positions of the corresponding photographic images will be solely due to tilt displacement.

In Figure 12 it is seen that the point I_G, on the ground has no photographic tilt displacement since it is imaged in exactly the same position, I, whether the photograph is tilted or truly vertical. All tilt displacement of other objects is radial from the point I, which therefore is known as the isocenter.

Thinking of Figure 12, three-dimensionally, it is apparent that the plane of the tilted photograph intersects the plane of the vertical photograph along a line which is perpendicular to the page and which passes through the point I. Since all objects imaged on the line would have no tilt displacement it is known as the *isoline.*

With respect to the isoline, all objects *in the foreground are displaced radially outward from the isocenter due to tilt, and all objects in the background are displaced radially inward toward the isocenter due to tilt.* Furthermore, as seen in Figure 12, *the amount of tilt displacement of an object is directly proportional to its distance from the isocenter.*

As will presently be shown, an actual photo of a model (Figure 13) lends credence to the foregoing analysis.

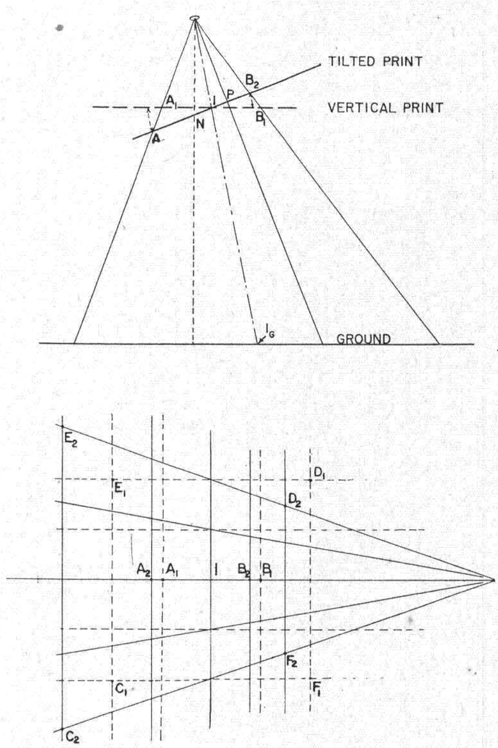


FIG. 12. Line drawing showing the direction and magnitude of tilt displacements.

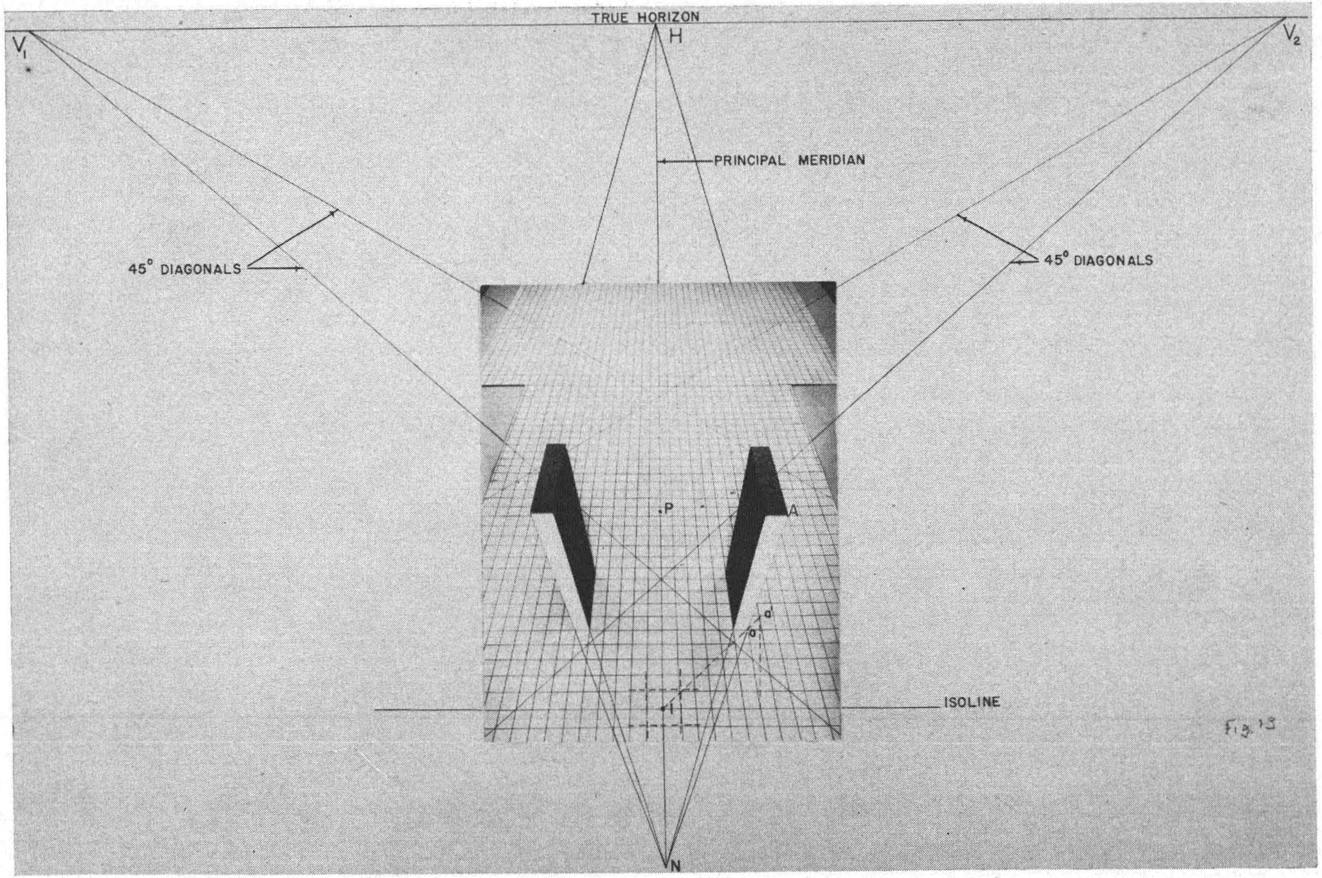


FIG. 13. Oblique photo of a 3-dimensional model illustrating the combined effect of displacements due to relief and tilt. For explanation see text.

COMBINED EFFECT OF TILT AND RELIEF DISPLACEMENT

In concluding this discussion of displacements it is helpful to consider the three dimensional model shown in Figure 13. This model illustrates the combined effects of both tilt and relief displacement. The gridded base of the model is truly horizontal and the two upright blocks are truly vertical. In taking the tilted (or oblique) photo shown in Figure 13, the optical axis of the camera was elevated to an angle of approximately 45 degrees with respect to the vertical position.

The apparent convergence of the vertical sides of the blocks shows clearly that relief displacement is radial from the photo nadir, *N*. Comparison of the dashed and solid lines for squares immediately surrounding *I* shows as in Figure 12 that tilt displacement is radial from the isocenter, *I*. But what is the true plan position, relative to the gridded base, of a point such as *A*, which has both tilt and relief displacement? The answer to this question is readily obtained by first correcting for relief displacement, which places *A* at the point, *a*, on the reference plane, and then correcting for tilt displacement which finally places this point in its true plane position at *a'*.

Of incidental interest, the following points are also well illustrated by Figure 13:

(1) the principal point (*P*) isocenter (*I*) and photo nadir (*N*) all fall on the principal meridian.
(2) the vanishing point (*H*) for the meridianal system of lines also falls on the principal meridian, at the point where it intersects the true horizon.
(3) the vanishing points V_1 and V_2 for the 45 degree, diagonal lines also fall on the true horizon and are equidistant to left and right of *H*.

PLANNING AND EXECUTING A PHOTOGRAPHIC MISSION

Three dimensional models can be quite effective in illustrating the photogrammetric principles involved in some closely related series of events. Such a series is exemplified in the planning and execution of a photographic mission for some specific purpose. Among the interrelated events in this series are the following: (1) planning the mission, (2) flying the mission, (3) inspecting the results, (4) indexing the photos, and (5) interpreting the photos. As will presently be shown, a single three-dimensional model serves effectively in illustrating most of the photogrammetric principles involved in this series. The use of a single model permits all steps in the series to be illustrated in terms of the same piece of terrain. A greater continuity is achieved thereby than might otherwise be possible. As in the preceding treatment of image displacements, however, the model should be used only in conjunction with appropriate drawings and actual aerial photographs.

PLANNING THE PHOTO MISSION

Figure 14 shows a typically crude map of the area to be photographed in our example. The map was made in this case merely by tracing pertinent detail from a vertical photo of the model shown in Figure 15. The actual area for which photo coverage is desired has been designated in solid black on the map. The project

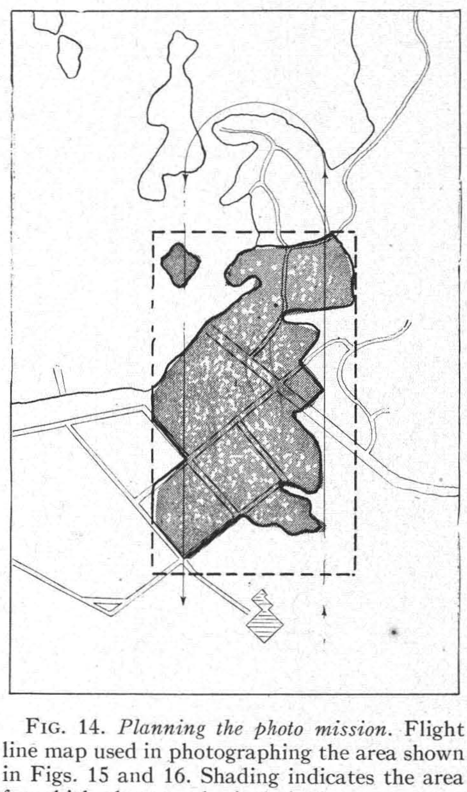


FIG. 14. *Planning the photo mission.* Flight line map used in photographing the area shown in Figs. 15 and 16. Shading indicates the area for which photography is desired.

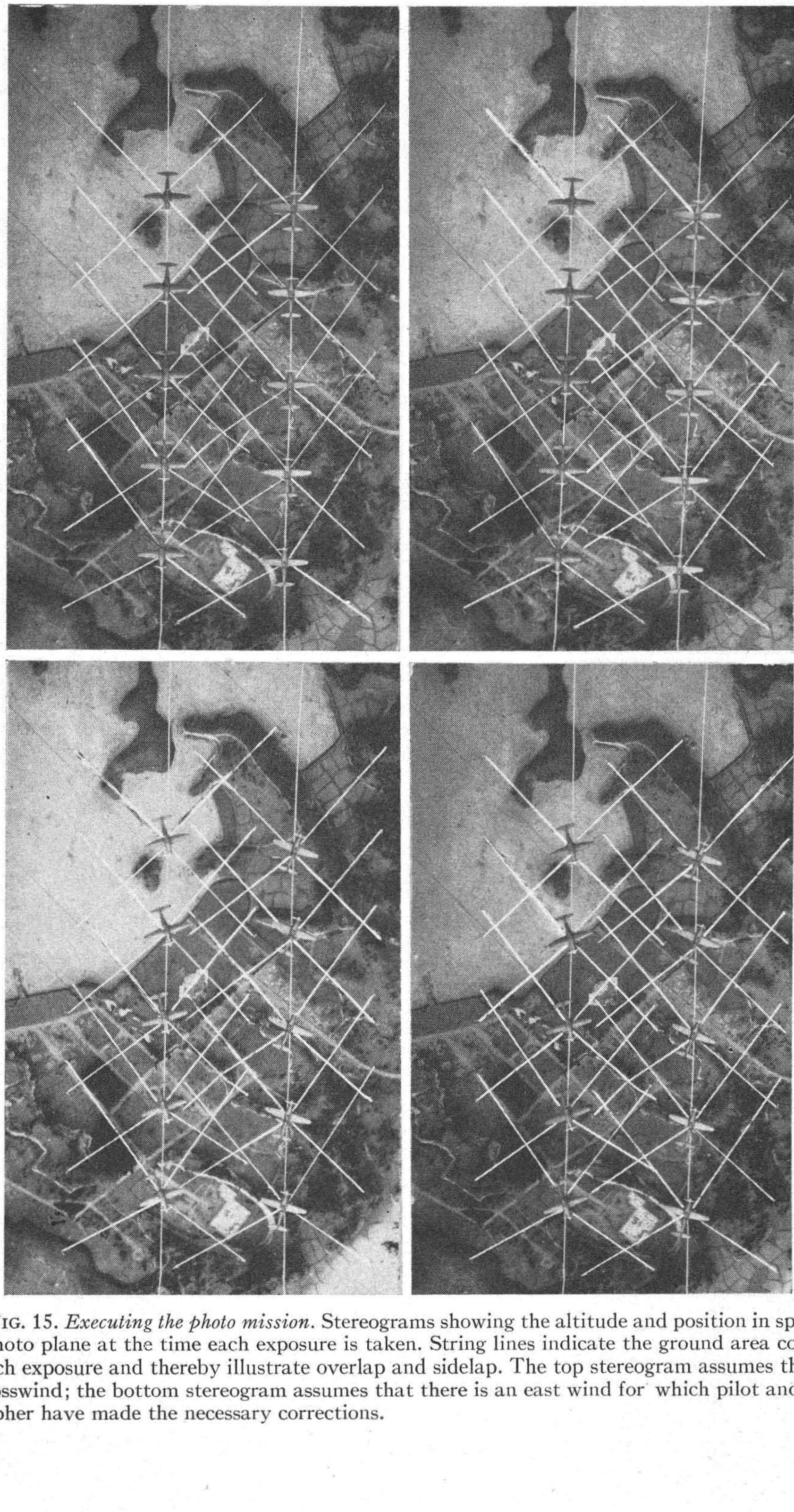


FIG. 15. *Executing the photo mission.* Stereograms showing the altitude and position in space of the photo plane at the time each exposure is taken. String lines indicate the ground area covered by each exposure and thereby illustrate overlap and sidelap. The top stereogram assumes there is no crosswind; the bottom stereogram assumes that there is an east wind for which pilot and photographer have made the necessary corrections.

boundary as indicated by the surrounding dashed lines is blocked off in conventional rectangular fashion. Photo specifications, including photo scale, are arrived at after considering the uses to be made of the photography. (They are omitted here in the interest of brevity.) Assuming that these specifications can be met by the flying of only two flight lines, the project boundaries are blocked off in conventional rectangular fashion and the two flight lines, properly spaced (calculations here omitted), are drawn parallel to the major axis. Directions of flight and turn around between flights are indicated on the map. Such additional factors as number of exposures required, approximate time interval between exposures, can be introduced also with reference to this model.

EXECUTING THE PHOTO MISSION

Figure 15 indicates the attitude of the plane at the time each exposure is taken. The top illustration assumes that there is no crosswind while the bottom illustration assumes an east wind. These two illustrations should be studied stereoscopically in order to perceive more clearly the area covered by each photograph and the consequent forward lap and sidelap.

INSPECTING THE RESULTS

Figure 16 indicates the photo coverage that is obtained in the presence of a crosswind,

(a) if the pilot and photographer both ignore the wind and thereby suffer drift (both strips flown from S to N in this example),
(b) if the pilot crabs into the wind but the photographer instead of rotating the aerial camera a comparable amount, leaves its sides parallel to the sides of the fuselage, and
(c) if the pilot and photographer both make the appropriate corrections.

From an inspection of such photography the contracting officer determines whether or not it is acceptable.

INDEXING THE PHOTOS

Figure 17 shows one form of photo index, known as the index mosaic, which results if the mission has been properly flown. Note that the number of each exposure and the portion of the terrain which it covers are clearly indicated.

Figure 18 illustrates a second commonly used form of photo index, in which only the principal point of each photo is indicated on a base map, by means of a dot.

INTERPRETING THE PHOTOS

Figure 19 is an attempt to illustrate how any two successive photos from this project can be studied stereoscopically to provide a three dimensional view, at reduced scale, of the actual terrain photographed. It is believed that this type of illustration compares favorably with the "giant eye base" concept which is commonly drawn to illustrate the same point. As in previous illustrations, space limitations prevent a fuller treatment of the opportunities which this stereogram affords for developing such concepts as height measurement, area estimation, and technique of interpretation.

DEVELOPING NEW PHOTOGRAMMETRIC TECHNIQUES

Only one example can be included here to illustrate this important use of three-dimensional models. The photogrammetric technique to be developed is one permitting the *use of terrestrial stereo photos taken vertically upward* through a stand of forest trees as shown in Figure 20. The use of such photography would be in the estimation of timber volumes in dense stands of tall trees having high commercial value, such as the redwood stand here shown.* With such high values at stake more accurate means of estimating timber volumes are being sought. But the density of the stand makes both conventional ground survey techniques and aerial survey techniques difficult.

The photogrammetric principle involved here is quite simply explained by analogy to vertical aerial photos. Because it is possible to measure tree heights and crown diameters quite accurately even on rather small-scale aerial photos, when the camera is pointed vertically downward, might it not be possible to measure merchantable

* Although this is only second growth redwood, approximately 90 years old, the trees are upwards of 200 feet tall, have a volume in excess of 250 thousand board feet per acre, and a value "on the stump" of about $2,500/acre.

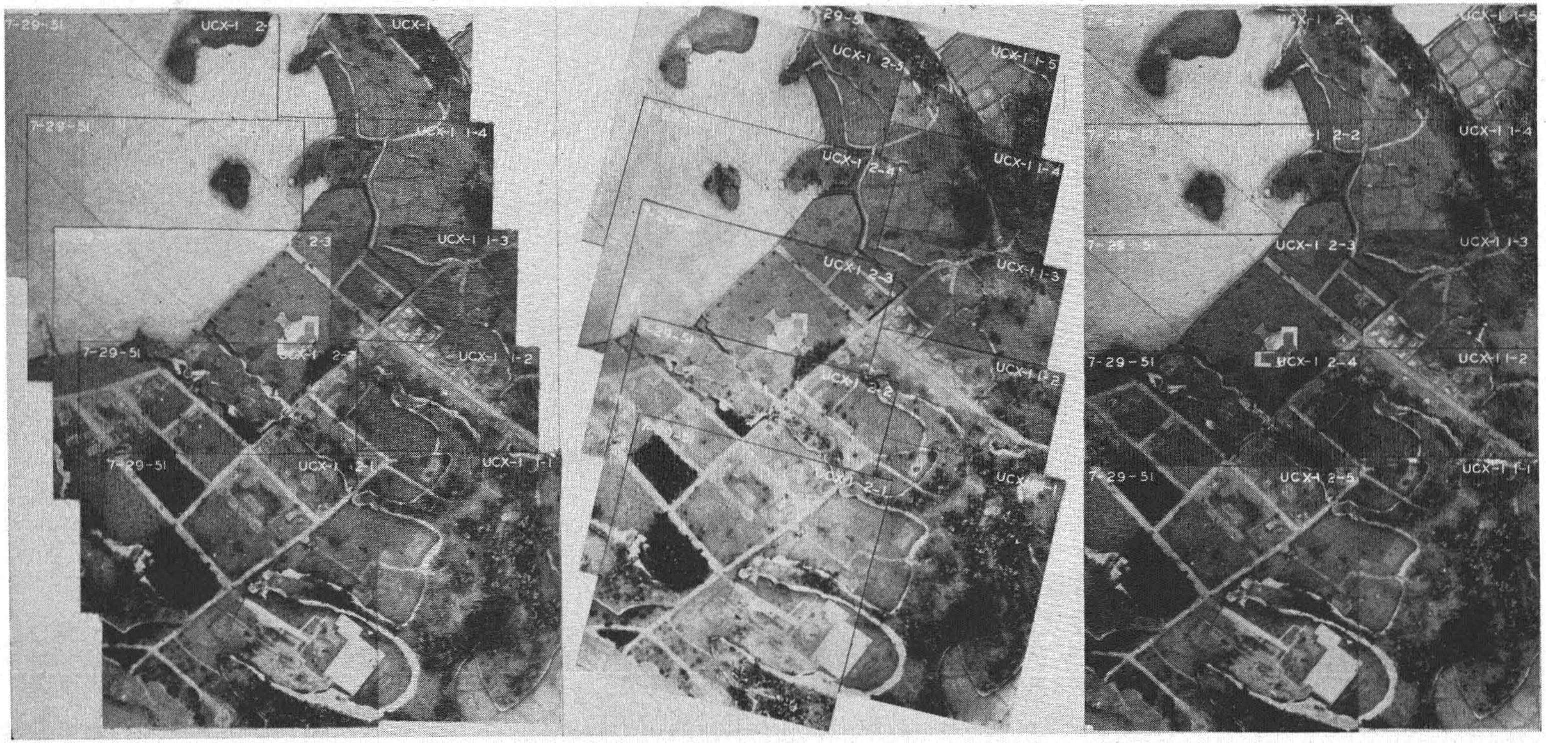


FIG. 16. *Inspecting results of the photo mission* (assuming an easterly crosswind). Left photo: Pilot failed to make drift correction. Middle photo: Photographer failed to make crab corrections. Right photo: Both pilot and photographer made the necessary corrections.

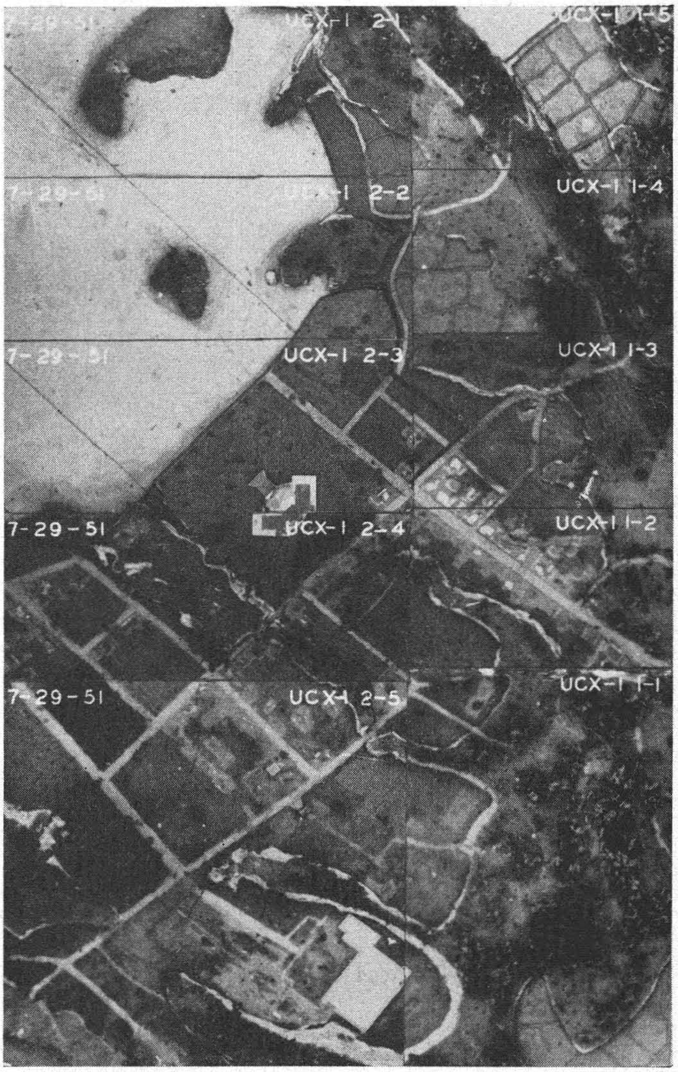


FIG. 17. *Indexing the photos*. This is an index mosaic of the successful mission showing the ground area covered by each exposure.

heights and stem diameters on terrestrial photos when the camera is pointed vertically upward and the stereo base is accurately measured? Ideally this technique might permit a forest photogrammetrist to locate the top of each log in a tree and to measure the diameter of the stem at that point. Such measurements might lead to highly accurate volume determinations.

The possible use of three dimensional models in relation to this problem is indicated in Figure 21. In this case the model is a smokestack, complete with ladder. By way of ascertaining the order of accuracy attainable by this photogrammetric method when applied to forest trees, a check is first made on a comparable object, the smokestack. In this case the smokestack is more easily climbed than the tree and therefore the correct data needed for the check are more easily obtained.

Specifically on the photos here shown of the smokestack a photogrammetrist attempted to locate the top of each of the first five 16 foot "logs" and to measure their "stem diameters." When checked against actual measurements obtained with calipers and tape, at the time the

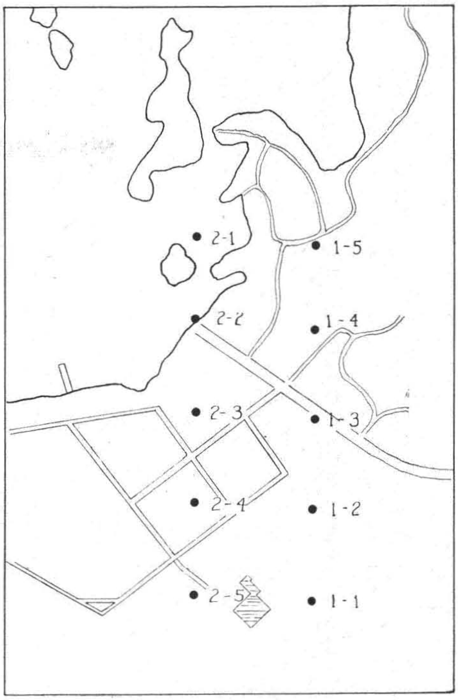


FIG. 18. *Indexing the photos.* This is one of the simplest forms of photo plot. The map position of the center of each photograph is indicated by a black dot, followed by the flight line number and exposure number. Compare with Fig. 17.

smokestack was climbed, the heights were found to be remarkably accurate but the diameters badly in error. Further consideration showed an obvious error entailed in the original photogrammetric theory as applied to diameter measurement. When this error was rectified the results were much more accurate.

CONCLUSION

The examples presented in this paper are by no means the only ones for which three-dimensional models can be used advantageously by the photogrammetrist. Other instances in which such models have been used to advantage by the speaker include the following:

(1) for illustrating the relationship between coordinates of the terrestrial

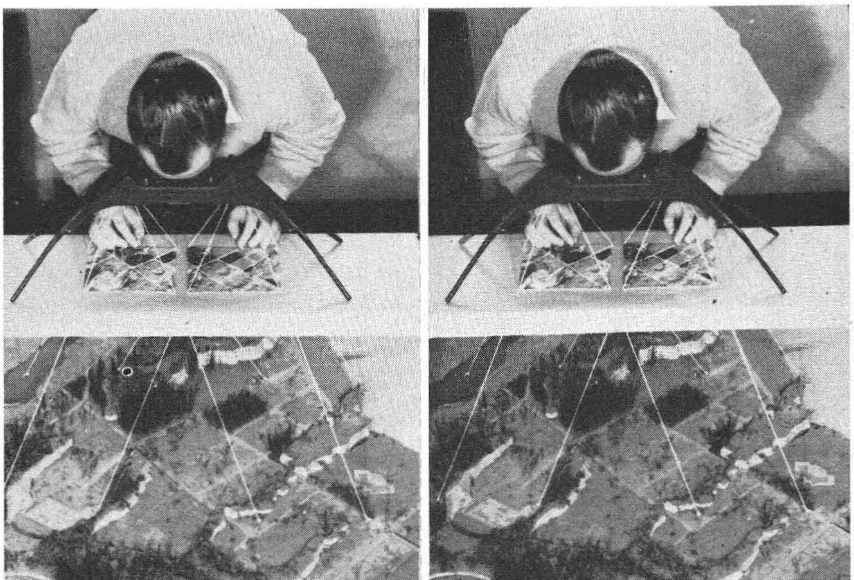


FIG. 19. *Interpreting the photos.* By taking advantage of the "giant eye base" offered by aerial photography the photo user is able to perceive a 3-dimensional model of the actual area photographed. In this figure exposures 2–4 and 2–5 of the previous illustrations are being studied stereoscopically.

FIG. 20. Stereogram looking vertically upward through a stand of timber. For a discussion of the use to be made of such photos, see text.

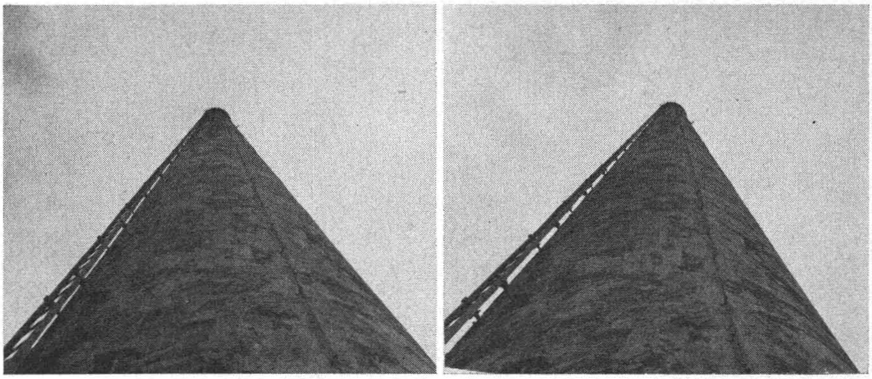


FIG. 21. Stereogram looking vertically upward at a 3-dimensional "model," in this case a smokestack. Such a model, being easily climbed and measured, facilitates determination of the photogrammetric reliability of tree measurements made on Fig. 20.

sphere and those of the celestial sphere,

(2) for illustrating the extent to which light rays are refracted when passing through a camera lens, depending on their wave lengths and the angles at which they enter the lens, and

(3) for illustrating some common conditions under which the shadows of objects, as imaged on photographs, are elongated, foreshortened, bent or otherwise distorted, thereby posing special problems in their interpretation or measurement.

As this paper has attempted to demonstrate, most photogrammetric models are effective only when their three-dimensional aspects can be readily perceived. Fortunately, this poses no great problem in our particular profession, because we are justified in assuming that our reading audience is able to grasp the third dimension from a study of stereograms of the type just presented. Such an assumption would, of course, be invalid in almost any other profession. It is the present speaker's sincere belief that we should exploit this unique advantage much more fully in future photogrammetric literature than we have in the past.