Graphic Determination of the Over-Correction Factor for Use in the Relative Orientation of Vertical Photographs of Any Terrain

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THE relative orientation of aerial photographs, in stereoscopic plotting instruments, is generally accomplished by optical mechanical orientation methods. This is a trial-and-error procedure, which generally must be repeated several times in order to obtain a satisfactory result. Nevertheless, parallax-free stereoscopic models of good theoretical accuracy can be set up quickly after some training. Only a few simple rules concerning the removal of residual y-parallaxes need to be known and observed. The method was developed by O. von Gruber many years ago, and it cannot be discarded as long as plotting instruments are designed in today's conventional form.

Generally, the six standard points shown in Figure 1 are used for the determination and elimination of the y-parallax in a model. In establishing relative orientation by von Gruber's method, image ordinates y' of equal length are usually chosen for points 3, 4, 5 and 6. In its simplest form, von Gruber's method consists of the following procedure: eliminate the y-parallax at points 1 and 2 with the by and swing motions; then, for instance, eliminate y-parallax at points 3 and 4 with bz and tip motions; eliminate the y-parallax p^* at point 5 with the tilt motion—over-correcting this y-parallax $n \cdot p^*$ times; and, finally, remove y-parallax at points 1 and 2. If vertical photographs are used, if the terrain is flat, and if only small adjustments are required to eliminate y-parallaxes at points 3, 4 and 5, then a completely parallax-free model will be obtained by this procedure—provided a suitable over-correction factor be used, with symmetrically chosen standard points. Otherwise, this procedure must be repeated until the y-parallaxes throughout the model are reduced to insignificant values.

The evaluation of the magnitude of the over-correction factor n, is the only difficult part of the above procedure. The following preliminary value of n is used in removing y-parallax at point 5 for the first time:

$$n = \tfrac{1}{2}(1 + f^2/y^2) \tag{1}$$

where f is the calibrated focal length, and y is the ordinate of point 5. (These symbols are illustrated on Figure 2.) In actual practice, however, significant residual y-parallaxes Δp^* will generally remain after the above procedure has been performed once. As a result, the entire procedure must be repeated a sufficient number of times to reduce the residual y-parallaxes to insignificant values—using the following modified over-correction factor:

$$n' = n \cdot \frac{p^*}{p^* - \Delta p^*} \tag{2}$$

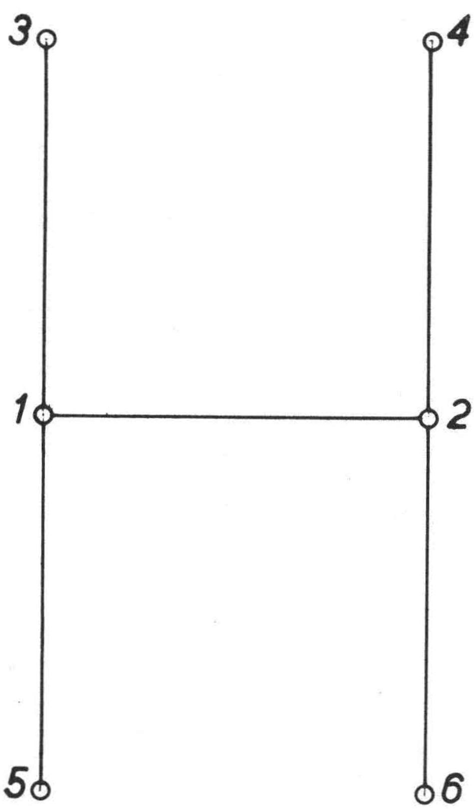

FIG. 1

where n is the over-correction factor previously computed by equation (1), p^* the y-parallax measured for the first time at point 5, and Δp^* the y-parallax measured at point 5 for the second time, when the relative orientation procedure is repeated. If the photographs have large tilts, if the stereo model contains considerable topographic relief; or if the y-parallax cannot be observed at symmetrically located points, then the initial value for the ω-motion $n \cdot p^*$, may be extremely erroneous when computed according to equation (1). In such cases, equation (2) will not be a good result, and y-parallaxes at the corner points may even be increased with repeated orientation procedures or their signs and magnitudes changed in an unpredictable manner. Even trained operators will then need much time to find the correct value of the over-correction factor. These difficulties are increased if one or more of the six standard points are situated in a water surface or the like, where no y-parallax can be observed; thus necessitating the choice of other points for the elimination of the y-parallaxes.

Accordingly, it is the purpose of this paper to describe a simple graphical method of determining the over-correction factor required for vertical photographs to remove lateral tilt under general conditions of relief and location of orientation points. This method is based upon an equation derived by H. Gänger, and reproduced by R. Finsterwalder in his textbook "Photogrammetrie" published in Berlin in 1939. The form of Gänger's formula was rather complicated, but it can be simplified and interpreted geometrically in a very fine form that lends itself to a simple graphical determination of the over-correction factor.

This simplified formula can be derived from the y-parallax equations for relative orientation in the following manner—the left projector being considered as fixed: for instance if that cross-section through points 1, 3, and 5 on Figure 1 is chosen (see Figure 2) the specific y-parallax equations will then be

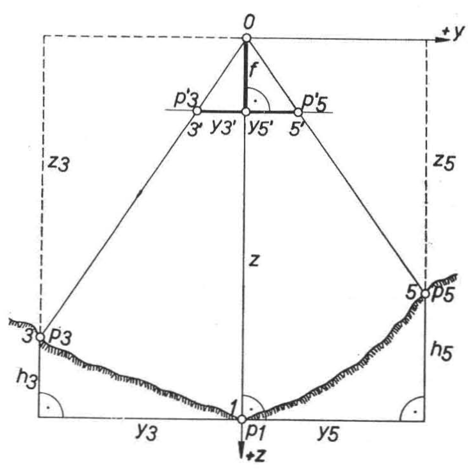

FIG. 2

$$\left.\begin{aligned}
&1)\ (b\cdot d\kappa + dby) + z_1\cdot d\omega = p_1\\
&2)\ (b\cdot d\kappa - dby) + z_3\left(1 + \frac{y_3^2}{z_3^2}\right)d\omega + \frac{y_3}{z_3}(b\cdot d\phi - dbz) = p_3\\
&3)\ (b\cdot d\kappa - dby) + z_5\left(1 + \frac{y_5^2}{z_5^2}\right)d\omega + \frac{y_5}{z_5}(b\cdot d\phi - dbz) = p_5
\end{aligned}\right\} \tag{3}$$

The tilt error $d\omega$ can easily be determined from these equations, if the first equation is subtracted from the second and third, $d\kappa$ and dby are eliminated, and two new equations are then obtained. If they are each divided by their respective coefficients y/z of the terms $(bd\phi - dbz)$, and if resulting equations are substracted from each other, a single equation is then obtained. This equation may be transformed to:

$$d\omega = \frac{(p_5 - p_1)\dfrac{z_5}{y_5} - (p_3 - p_1)\dfrac{z_3}{y_3}}{y_5 - y_3 - \left(\dfrac{z_5}{y_5}h_5 - \dfrac{z_3}{y_3}h_3\right)} \tag{4}*$$

If it is now assumed that the y-parallaxes at 1, 2, 3 and 4 are eliminated by the above described orientation method, the principal points as well as one edge of the model are free of parallax and the y-parallaxes p^* appear at the opposite edge.

The values $p_1 = p_3 = 0$ and $p_5{}^*$ are now introduced into equation (4) and there is obtained:

$$d\omega = \frac{p_5{}^*\dfrac{z_5}{y_5}}{y_5 - y_3 - \left(\dfrac{z_5}{y_5}h_5 - \dfrac{z_3}{y_3}h_3\right)} \tag{5}$$

From the third equation in (3) one obtains

$$p_5{}^* = z_5\left(1 + \frac{y_5^2}{z_5^2}\right)\cdot d\omega^*, \tag{6}$$

if the y-parallax $p_5{}^*$ is measured only with the tilt motion in an amount equal to $d\omega^*$. In order to obtain the correct tilt $d\omega$ from $p_5{}^*$, the latter has not only to be eliminated with the tilt motion, but it has to be corrected n times. For point 5, one finds therefore that:

$$n_5 = d\omega/d\omega^* \tag{7}$$

If (5) and (6) are introduced in the equation (7), one obtains the following value for the over-correction factor:

* This equation can also be found in a publication by E. Gotthardt entitled "Rechnerische und Zeichnerische Hilfsmittel der Gegenseitigen Orientierung von Senkrechtaufnahmen Gebirgigen Geländes." Diss. Berlin 1938 (a.e. Analytical and Graphical Means for the Relative Orientation of Vertical Photographs in Mountainous Terrain, Thesis, Berlin. 1938).

$$n_5 = \frac{y_5\left(1 + \frac{z_5^2}{v_5^2}\right)}{y_5 - y_3 - \left(\frac{z_5}{y_5} h_5 - \frac{z_3}{y_3} h_3\right)} \tag{8}$$

The above formula can also be obtained by a minor transformation of the formula originally determined by Gänger.

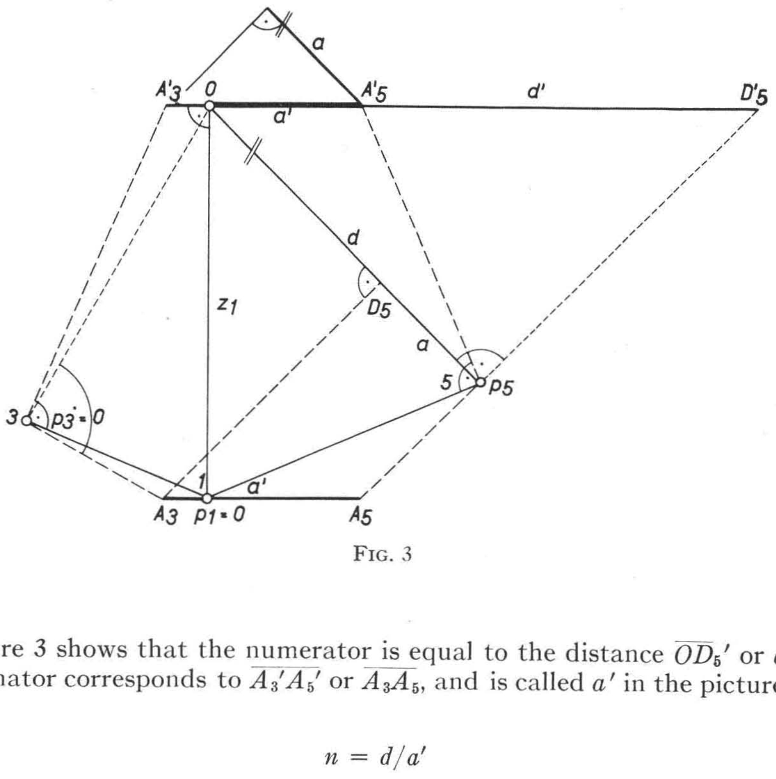

FIG. 3

Figure 3 shows that the numerator is equal to the distance $\overline{OD_5}'$ or d'. The denominator corresponds to $\overline{A_3'A_5'}$ or $\overline{A_3A_5}$, and is called a' in the picture. Thus we find

$$n = d/a' \tag{9}$$

If d' is projected onto $\overline{O_5}$, the projection is called d. If a' is projected onto $\overline{O5}$ or onto a parallel line to $\overline{O5}$, and if the length of this projection is called a, the ratio d/a will equal be to d'/a'. Thus

$$n = d/a \tag{10}$$

If all the auxiliary construction lines are omitted in Figure 3, one obtains the simplified Figure 4. This figure shows how the over-correction factor can be interpreted geometrically and how the problem can be generalized. Point 1 need not necessarily be a nadir point, but like the points 3, and 5, may be any point in the cross section. The over-correction, therefore can always be easily determined with three perpendiculars to the projection rays.

If point 5 is located on the critical circle of $a=0$, (see Figure 4) the relative orientation cannot be accomplished with the three chosen points. In practice, this is also true when a is very small because the factor of over-correction will

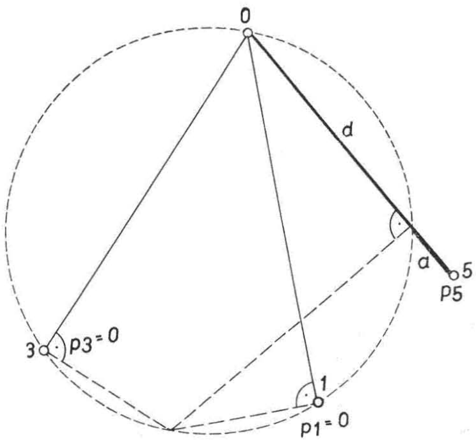

FIG. 4

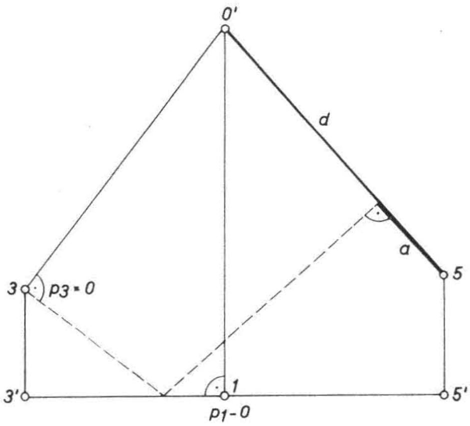

FIG. 5

then be very large. If point 5 is situated inside of the circle determined by 1, 3, and 0, the over-correction factor will be negative.

The factor of over-correction can be determined by drawing just one perpendicular if the terrain is flat, as shown in Figure 6.

In spatial aerial triangulation, carried out according to the method of consecutive pairs, the shape of the cross section normal to the base is always known in the common overlap region with the previous stereoscopic model. The coordinates and heights of at least three points with nearly equal x-values are always determined in these regions of common overlap in order to make the scale setting and height comparison in the next model possible. The position of the center of projection being also known, it is easy to draw a cross section. The over-correction factor $n = d/a$ then is determined with sufficient accuracy by only two perpendiculars, as shown in Figure 5. This figure is valid if the y-parallaxes have been eliminated by any method in the points 1 and 3.

If it is desired to increase the accuracy of the separation of ω (tilt) and by, the procedure described above may be repeated in different profiles (for different x-values) and the mean taken from the ω-values thus obtained.* Even if this method is applied only for a quick, rough estimation of the over-correction factor, it will seldom be necessary to repeat the relative orientation more than once. A very good approximate value of the over-correction will also be obtained in an unfavorable model.

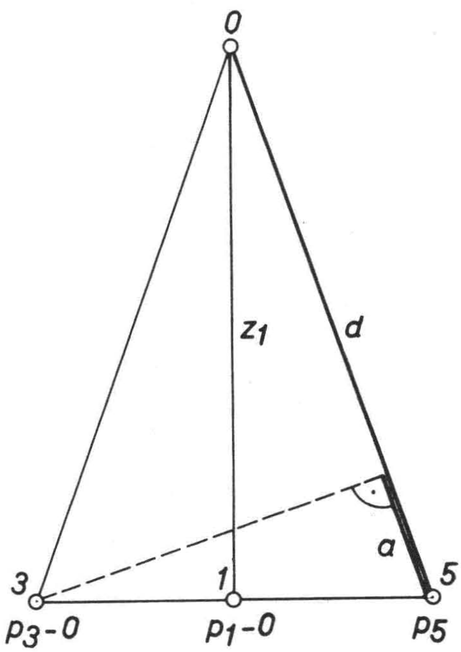

FIG. 6

* Therefore, a correct value for the lateral tilt can easily be determined without tedious trials also when applying the optical-mechanical method of orientation of v. Gruber.

TABLE I. OVERCORRECTION IN NORMAL ANGLE CAMERAS

$$\frac{b}{f} = \frac{y'}{f} = \frac{y}{z} = \frac{1}{3}; \qquad n_5 = \frac{1 - \frac{h_5}{z_1}}{0,2 - \left(\frac{h_3}{z_1} + \frac{h_5}{z_1}\right)}$$

$\frac{h_3}{z_1}$ \ $\frac{h_5}{z_1}$	−0,20	−0,15	−0,10	−0,05	0,00	+0,05	+0,10	+0,15	+0,20
+0,20	6,0	7,7	11,0	21,0	± ∞	−19,0	− 9,0	− 5,6	− 4,0
+0,15	4,8	5,8	7,3	10,5	20,0	± ∞	−18,0	− 8,5	− 5,3
+0,10	4,0	4,6	5,5	7,9	10,0	19,0	± ∞	−17,0	− 8,0
+0,05	3,4	3,8	4,4	5,3	6,7	9,5	18,0	± ∞	−16,0
+0,00	3,0	3,3	3,7	4,2	5,0	6,3	9,0	17,0	± ∞
−0,05	2,7	2,9	3,2	3,5	4,0	4,8	6,0	8,5	16,0
−0,10	2,4	2,6	2,8	3,0	3,3	3,8	4,5	5,6	8,0
−0,15	2,2	2,3	2,5	2,6	2,9	3,2	3,6	4,3	5,3
−0,20	2,0	2,1	2,2	2,4	2,5	2,8	3,0	3,4	4,0

TABLE II. OVERCORRECTION IN WIDE-ANGLE CAMERAS

$$\frac{b}{f} = \frac{5}{8}, \quad \frac{y}{f} = \frac{2}{3}; \qquad n_5 = \frac{13\left(1 - \frac{h_5}{z_1}\right)}{8 - 13\left(\frac{h_3}{z_1} + \frac{h_5}{z_1}\right)}$$

$\frac{h_3}{z_1}$ \ $\frac{h_5}{z_1}$	−0,3	−0,2	−0,1	0,0	0,1	0,2	0,3
0,3	2,1	2,3	2,6	3,0	4,2	6,9	45,5
0,2	1,8	1,9	2,1	2,3	2,8	3,7	6,1
0,1	1,6	1,7	1,8	1,9	2,2	2,5	3,3
0,0	1,4	1,5	1,5	1,6	1,7	1,9	2,2
−0,1	1,3	1,3	1,3	1,4	1,5	1,5	1,7
−0,2	1,2	1,2	1,2	1,2	1,3	1,3	1,4
−0,3	1,1	1,1	1,1	1,1	1,1	1,1	1,1

Table I shows that the differences in elevation in a stereoscopic model affect the value of the factor of over-correction, especially in the case of normal angle photographs. These values have been computed for a base-height ratio of 1:3, assuming the y-coordinates to be the of same length as the base. The variations are smaller for wide-angle photographs, as shown in Table II.