Elements of Long-Focus Photographic Astrometry[*]

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1. INTRODUCTION

ACCURATE positional measurements, obtained from photographic plates taken with astronomical telescopes, furnish astronomers with essential astrometric data, which form the basis of studies in the field of celestial mechanics, the physical properties of stars, and the structure of our Milky Way system. No other technique exists that permits the observer to obtain as highly accurate a mapping of the sky. Less than a century old, *photographic astrometry* continues to be of the utmost importance in current research and in the planning for future research, and shows no sign of impending obsolescence. However, one must reckon with the possibility of appreciable reductions in exposure times through the development of image tube techniques.

Experience has shown that in general, for positional work, refracting telescopes are superior to reflectors. This paper shall be limited to a description of the astrometric technique as applied to the jewels of optical astronomy,—the long-focus achromatic refractors. These are refracting telescopes with focal lengths of about 8 meters or more, and commonly of large aperture—about 40 cm. or more—resulting in focal ratios up to $f/20$. The power of these instruments lies in the large-scale portrayal in the focal plane, of an observed star field of limited angular extent, usually less than a degree in diameter.

The long-focus refractor is of particular importance in the study of small angular shifts of stars, such as the star's parallactic displacement due to the earth's annual motion around the sun, as distinguishable from the star's own angular motion across the sky (" proper motion"). Another fruitful application lies in the study of the orbital motion of double star components and of the orbital motion or "perturbation" of a "single" star, due to the presence of an invisible companion star.

2. THE LONG-FOCUS REFRACTOR

The long-focus refractor normally has a focal-ratio of between $f/15$ and $f/20$. The achromatic objective usually consists of a convex crown-glass lens and a concave flint-glass lens, which may be very close together, or separated by several centimeters. For example, here are the specifications for the objective of the (visual) Sproul refractor, given in a letter of October 15, 1924 from Charles S. Hastings to my predecessor, Dr. John A. Miller. See tabulation.

The aperture of the Sproul refractor is 24 inches (61 cm.), the focal length 36 feet (1,093 cm.), the focal ratio is $f/18$ (Figure 1).

The scale value in the focal plane is defined as the number of seconds of arc per millimeter; for a focal length of F mm., the scale value is $206{,}265/F$. For

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SPECIFICATIONS OF 24-INCH OBJECTIVE OF SPROUL VISUAL REFRACTOR

	Radii of Curvature		
front lens (flint)	+284.20 inches +101.82 inches	central thickness	1.33 inches
		spacing between lenses*	.005 inch
rear lens (crown)	+101.82 inches −457.15 inches	central thickness	1.85 inches
	+ . . . convex toward source − . . . concave toward source		

Wavelength	Refractive Indices	
	Crown	Flint
C	1.510484	1.605533
λ5614	1.514290	1.612663
F	1.518980	1.621876
G	1.524140	[G′ 1.631756]
h	1.526685	1.637993

* The original aluminum cell was replaced on March 17, 1949 by a cast iron cell, in which the two lens components are separated by three strips of lead foil .006 inches thick.

the Sproul refractor the scale value is 1 mm. = 18.87″, or .053 mm. for one second of arc. To illustrate this, the focal image of the moon is nearly four inches across and fills the greater part of the useful field, portrayed on the photographic plates of 5×7 inches (13×18 cm.).

The telescope has an equatorial mounting, the telescope following the earth's diurnal rotation through a gravity-operated clockwork. However, during exposures of more than several seconds, manual "guiding" is necessary and is

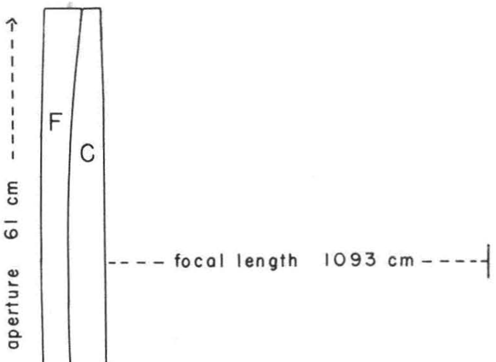

FIG. 1. Diagram of Sproul 24-inch objective.

accomplished by using a double-slide plate carrier, permitting an accurate slow motion of the plate holder. A guide star is kept centered on the cross-wires of a small eyepiece attached to the plate carrier; the observer bears the greater part of the responsibility for the accurate guiding of the telescope that is necessary to obtain good photographic records.

The apparent dimensions, even for the nearest and for the largest stars, expressed in their angular diameters, are so small (less than 0″.1) that no photographed star image bears any relation to the star's angular diameter. Any apparent size, as seen with the telescope, is caused partly by the star's diffraction image, partly by turbulence due to the atmosphere. The photographic image is appreciably enlarged by photographic action in the emulsion, and increases with the brightness of the star.

3. AT THE TELESCOPE

In one group of long-focus problems the material consists of photographic plates on which the position of the star in which we are interested,—the "central" star, —is referred to a "background" of three or more reference stars. The classic example is the parallax work done with the largest existing visual refractor (focal-length 19.37 meters, aperture 102 cm.)—at the Yerkes Observatory—at the beginning of the century. The necessary techniques of observing, measuring, and calculating were developed by Frank Schlesinger (1871–1943), who succeeded in obtaining parallaxes with an accuracy not earlier achieved. His methods of long-focus photographic astrometry are basic and complete; only minor improvements remained possible.

The annual or heliocentric parallax is defined as the angle under which an observer, located at the star, sees the unforeshortened radius of the Earth's orbit. In other words, the annual parallax is the semi-axis major of the small ellipse which the star seems to describe on the sky in the course of a year. Stellar parallaxes are now measured with an accuracy of ″.01 or better, resulting in percentage errors of less than 5 per cent,—even as low as 1 per cent —for the parallaxes, and hence for the distances of the nearest stars.

Then there is the study of relative positions of the components of double stars, first developed by Hertzsprung. Both Schlesinger and Hertzsprung had visual refractors; they used panchromatic emulsions and a yellow filter, thus eliminating the blue light, and obtaining sharp images in the color for which the objective was corrected.

The present paper shall be limited to star positions measured on a background of several reference stars.

Magnitude (brightness) and color differ from star to star; it is of particular importance, therefore, to be aware of the effects on the photographed positions due to the magnitudes and colors of the stars. Residual guiding error is minimized by aiming at magnitude compensation between central and reference stars. This may be accomplished by reducing the brightness of the central star by means of a small rotating sector in front of the plate. Sector openings of less than about 2.5% (extinction 4 stellar magnitudes) lead to a slight increase in positional error, and generally should be avoided. "Coarse" diffraction gratings, in front of the objective, may be used to provide fainter companion images, symmetrically placed on each side of the star images. Both sectors and gratings play an important role in reducing differences in the apparent sizes of star images, with a resulting increase in the ultimate accuracy of measurement.

Color effects are primarily due to dispersion in our atmosphere, although imperfect collimation of the objective may contribute its share also. Atmospheric refraction depends on the wave-length and on the zenith distance. The refraction at zenith distance ζ may be represented to a high degree of approximation by the formula $(\mu-1)\tan\zeta$, where μ is the index of atmospheric refraction at the observer's location. We write the above relation as $R\tan\zeta$, where R, the atmospheric refraction at $\zeta=45°$ is the so-called *refraction constant;* this is tabulated below, together with the dispersion per 100 Å. At moderate zenith distances the refraction, and hence the dispersion, varies proportionally to the tangent of the zenith distance.

λ	*R*	*Dispersion per 100* Å
4,000 Å	61″.34	−.108″
4,500	60.89	−.072
5,000	60.58	−.050
5,500	60.33	−.037
6,000	60.19	−.028
6,500	60.06	−.021
7,000	59.96	−.017
7,500	59.89	−.014
8,000	59.83	−.011

Except at the zenith, each star appears as a spectrum, whose blue end is closer to the zenith than the red end. For stars of different spectral types, the energy distribution is different; moreover, the spectra differ in brightness. For positional work, the spectral range should be reduced in order to have as nearly "monochromatic" images as possible. The approach to monochromatism is obtained by the triple combination of objective, filter, and emulsion. Sharp, round star images are obtained as long as the effective radiation is within Rayleigh's (visual) criterion for focal accuracy. According to the latter, all images obtained within $4f^2\lambda$ of the focus of a theoretically perfect objective are equally good. In this expression λ is the wave-length that corresponds to minimum focal-length and f is the ratio of focal-length to aperture; the same criterion holds closely for photographs as well. Generally, for long-focus refractors, Rayleigh's limit is less than one millimeter.

4. Photovisual Technique

Proper choice of filter and emulsion keeps the range of light close to the wave-length corresponding to the minimum focal-length of the color curve of the objective. The photographic position still depends on the residual energy distribution of the star's spectrum, as "filtered" by the objective (transparency and color curve), filter and emulsion. The effective wave-lengths of these star images depend on the spectrum, and to some extent the magnitude; however, with proper choice of filter and emulsion this dependence may be reduced to a minimum. Even with the best possible spectral compensation, small differences in effective wave-length are likely to remain.

We have noted the rapid decrease of atmospheric dispersion toward longer wave-lengths, which gives an advantage to the photographic technique as applied with visual refractors,—referred to as photovisual technique. Take, for example, the photovisual astrometric technique as employed with the Sproul visual refractor (Figure 3). The aperture of the achromatic objective is 61 cm., the minimum focal-length is 1,093 cm. for λ5,607. A minus-blue (No. 12) Wratten filter is used in contact with the 5 × 7-inch plate, eliminating practically all radiation on the blue side of approximately λ5,100. A suitable range of radiation is admitted to the photographic plate by using the Eastman G-type emulsion, for which the sensitivity is greatest at λ5,650, but extends hardly beyond λ6,000.

Even with the best possible spectral compensation, small differences in effective wave-length are likely to remain. However, sharp images are obtained with effective wave-lengths, ranging only from about λ5,480 for a blue star of spectral type A to about λ5,525 for a red star of spectral type M. This corresponds to a small difference in refraction constant of $\Delta R = ''.017$ at an altitude of 45 degrees, the A star being comparatively that much closer to the zenith than the M star. Variations in the flexure of the objective and possible relative motion of the two lens components may further cause prismatic effects which have the same effect as refraction. It is desirable to limit the photographic observations, as much as possible, to the same position of the telescope, preferably close to the meridian.

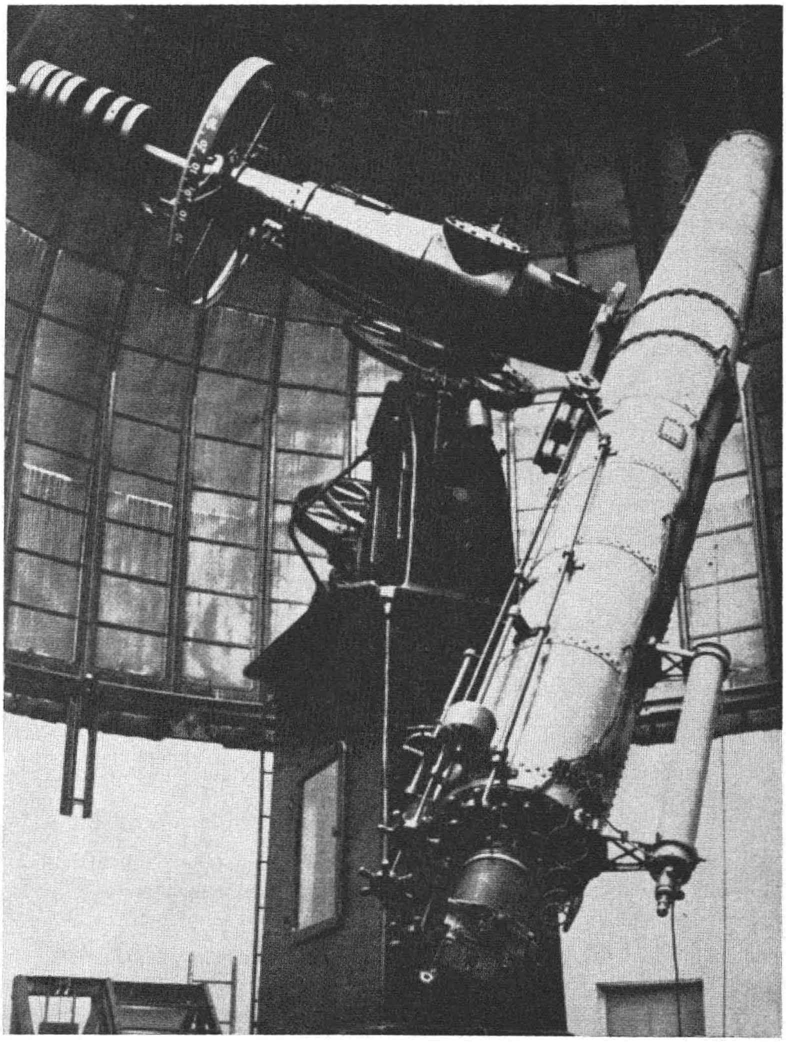

FIG. 2. The long-focus visual refractor of the Sproul Observatory; the aperture of the objective is 24 inches and the focal length is 36 feet.

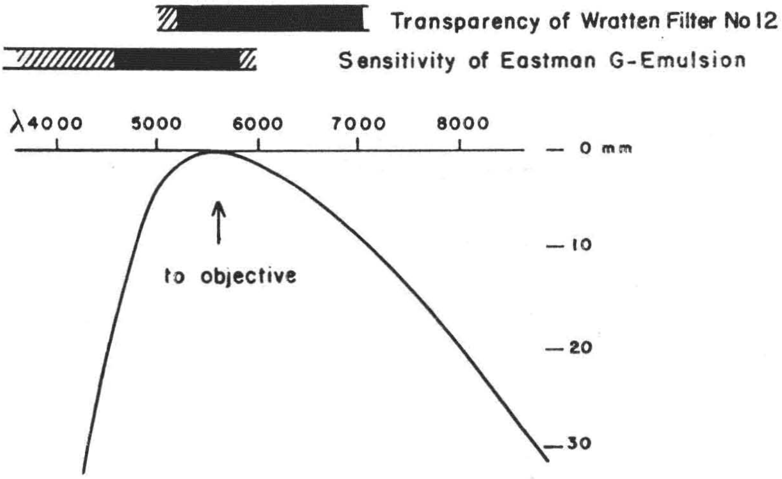

FIG. 3. Focal curve of the Sproul 24 inch objective.

5. MEASUREMENT

Long-focus astrometric problems are customarily studied in rectangular coordinates x and y, which coincide closely with the directions of right ascension and declination of the equatorial coordinate system. For an angular extent α on the celestial sphere, a third order distortion $\alpha^3/3$ between plane and angular portrayal exists, which is ordinarily negligible because of the limited areal extent of the field used. For an extended stellar path, an increasing disparity between rectangular and spherical coordinates will develop which may be represented to a high approximation as a slowly changing orientation of the equatorial with respect to the rectangular coordinate system, amounting to

$$\Delta\theta = + \Delta x \sin \delta$$

where Δx is the projection of the path on the x-coordinate and δ the declination.

Because of atmospheric refraction, any vertical angular distance suffers a minute contraction, for which the first order term is represented by the factor $R(1+\tan^2\zeta)$; any horizontal distance, measured along the great circle suffers a contraction R. For a wide range in wave-lengths in the visual spectrum, R is close to $60''$, hence the scale reduction in the zenith amounts to .00029.

The positions of star images in any one coordinate may be conveniently obtained with the aid of a long-screw measuring machine. The plate is mounted on a carriage which can be moved up and down in a plane, at an angle with the horizontal, and is oriented by two or more star images. The plate is viewed with a low-power (say 20×) microscope mounted on a carriage by means of a nut which fits a horizontal long-precision screw having a pitch of, say, one millimeter; by turning the screw and moving the carriage up and down, all portions of the plate may be viewed. The microscope contains a vertical wire (and a horizontal one to indicate the center of the field), for bisecting the star images. At one end the screw is provided with a graduated dial, so that the amount of turning, and hence the distance moved, can be read and recorded to .001 millimeter (one micron). It is customary to carry out averages and further calculations to .0001 mm.

Two successive settings are made on each image; the central star is usually measured twice. The plates are measured in 4 positions, differing by 90 degrees each, representing the "direct" and "reversed" directions of the rectangular celestial coordinates; the averages of the direct and reversed values in each coordinate are thus reasonably free from systematic errors of bisection, varying with the measurer and the intensity of the photographic image.

Astrometric series of plates often extend over several decades; it is not always feasible nor advisable to measure all the plates of a series during a limited span of time. Where measurements are made at different times, the stability of the measuring technique is of prime importance. Experience with the long-screw measuring machines of the Sproul Observatory has shown that the measuring technique is generally quite stable. Repeated measurements seldom have revealed systematic differences over one micron in the measured positions of central star on reference background; most of these systematic differences are below half a micron. The average of two measurements of a plate increases the effective plate weight by a factor 1.15, three measurements by 1.25. The resulting gain is limited, and, generally, the additional effort is not warranted, apart from its value as a test of the stability of the measuring machine.

The accuracy of the positions is much higher than the photographic resolving power; the smallest star images are rarely below 1″ in diameter. Although the diameters of star images ordinarily range from .04 mm. to .20 mm., or even more the relative location of two stars may be deduced with an uncertainty which is only a small fraction of the size of the image. With the Sproul refractor, the relative location of two star images is obtained with a probable error of about .002 mm., or less than ″.04. Experience has shown that the positional accuracy is hardly dependent on atmospheric turbulence or "seeing," provided inferior definition is avoided. Apart from such obvious image qualities as symmetry, the most important requirement for high positional accuracy is that the exposures shall be well blackened, i.e., give a good representation of the position of the star; the sharpness of the image is of secondary importance.

Increase in positional accuracy on any one night may be obtained by increasing the number of exposures per plate, and the number of plates. *Plate errors*, common to all exposures on one and the same plate, are to a great extent due to emulsion shifts, which rarely amount to as much as 10 microns or more. Generally, therefore, no more than three or four exposures per plate are taken. Appreciable reduction of film errors is obtained by the use of "double plates," obtained by turning the same photographic plate 180 degrees in its own plane between the two successive sets of exposures, representing the two "single" plates. This procedure is an application of the general principle of eliminating systematic errors by reversal. The two sets of exposures for the central star are close together on the emulsion; here the effects of a general film shift are virtually equal and are opposite for the two successive single plates. The double-plate procedure is especially desirable for large configurations, where this source of error is to a great extent responsible for the decreased positional accuracy of single sets of exposures.—The number of plates on any one night is limited by the *night errors*, which are probably due to refraction anomalies. These positional night errors are assumed constant during all the exposures of one and the same field, within the same night.

The law of diminishing returns operates here through the plate and night errors. Generally it is not warranted to take more than four exposures on any one plate, and more than four plates on any one night. The resulting positional accuracy (probable error) is about .001 mm. (one micron), or about ″.02 for plates taken with the Sproul refractor. This accuracy for any one night may be regarded as the limit, beyond which it is not easy to go. Measurements of relative positions of stars close together, such as visual double stars, are less affected by plate and night errors, and an appreciably higher accuracy may be obtained.

6. REDUCTION OF PHOTOGRAPHIC PLATES

The long-focus photographic method is now applied to the study of the path of a star. The star may be under observation for parallax, in which case useful information may be obtained in a few years. Or the star may be under observation for orbital motion, and hence, a prolonged interval of time,—several decades, or even centuries,—may be necessary to obtain the required observational material. In all cases the path of the star is referred to and measured on a background of virtually "fixed" reference stars. While the largest known parallactic displacement for a star is less than 1″, there are numerous stars whose annual proper motion exceeds 1″, the largest known annual motion being 10″.3 for Barnard's large-proper-motion star. Appreciable displacements are the rule, at least for the more interesting nearby stars. Hence the measured positions must be properly adjusted or "reduced" so as to permit a precise analysis of the star's path. There is need only to consider differences in origin, scale, and orientation, when comparing the different plates of a series of observations of one and the same field. All that is necessary is to reduce the measurements by a linear transformation to a common origin, scale, and orientation. The effect of plate tilt and other higher order terms is generally negligible for long-focus instruments.

In order to permit a comparison of measured positions on different plates, a reduction is made to a *standard frame* as defined by the reference stars. Use is made of rectilinear coordinates closely oriented to the celestial directions of right ascension and declination. Let X', Y' and x', y' be the measured positions of central and reference stars as recorded at the measuring machine. The zero point is arbitrary, the scale and orientation are close to that of the adopted standard frame given by the configuration of n reference stars, at least three, and seldom more than four in number. The coordinates of the standard frame shall be denoted by the subscript s; the positions x_s, y_s, defining the standard frame of reference, are relative to their mean position, i.e.,

$$[x_s] = [y_s] = 0.$$

All measured positions can now be reduced to the scale, orientation and origin of the reference frame (x_s, y_s) through *plate constants* a, b, and c, which are given by the linear equations of condition,

$$a_x x_s + b_x y_s + c_x = x_s - x'$$
$$a_y x_s + b_y y_s + c_y = y_s - y'$$

For the central star the position, X, Y reduced to the standard frame is given by

$$X = X' + a_x X_0 + b_x Y_0 + c_x$$
$$Y = Y' + a_y X_0 + b_y Y_0 + c_y$$

where X_0, Y_0 are values of X, Y rounded off to a sufficient number of significant figures.

7. DEPENDENCES

For the case of linear plate constants, considerable time may be saved and insight gained by expressing the reduced position as an explicit linear function of the measured coordinates. The resulting reduction statement, regardless of the zero-point of the measured coordinates, is

$$X = X' + [D_i(x_s - x')_i]$$

$$Y = Y' + [D_i(y_s - y')_i]$$

where the numbers D_i, $i=1,2, \cdots n$, are Schlesinger's so-called *dependences*. It is obvious that $[D]=1$, and in keeping with least-squares procedure, that $[D^2]$ is a minimum. In the plate-constant method X and Y are implicit functions of (x') and (y'); the dependence method provides a direct expression that greatly simplifies the reduction calculations.

The dependence method leaves only a small segment uncorrected, the so-called *plate solution*, also called *offset*:

$$\xi = X' - [Dx']$$

$$\eta = Y' - [Dy']$$

Because of their explicit use in the dependence reduction method, it is convenient to substitute the plate solution ξ, η in the reduced position so that

$$X = [Dx_s] + \xi$$

$$Y = [Dy_s] + \eta$$

The position $[Dx_s]$, $[Dy_s]$ defines a point close to the central star; it is called the *dependence center*; $[Dx']$, $[Dy']$ is the, measured, *dependence background*. The primed symbols refer to the measured coordinates, their zero-point is eliminated; the coordinates, x_s, y_s used in the computation of the dependence center, however, refer to their mean. The calculations are carried to .0001 mm. In general, if the plates are carefully oriented, the plate constants are factors of less than .0002 and their effect on the plate solutions is well below the observational errors, if the plate solutions are kept sufficiently small, say about .2 to .3 mm. Linear plate constant reduction is obtained within the errors of observation for the duration of the dependence set, and a number of plates can be reduced with one set of dependences calculated for one position X_0, Y_0 of the central star. For example, in the case of parallax determinations extending over a few years only, all plates, as a rule, are reduced by one dependence set. The dependence method has another advantage; it reveals the significance, or weight of the different reference stars. The dependence method gives no information about the plate constants, which are eliminated; in general, however, the plate constants are of no particular interest.

For the case of appreciable displacement of the central star, successive dependence sets are so chosen that the dependence center is kept close to the central star. Since many measured positions are reduced by the same set of dependences, the economy of the dependence method is maintained. The dependences are computed to four decimals, but rounded off to three always taking care that their sum shall rigorously equal unity.

The rounding off to three places in the dependences leads to values of the plate solutions which are generally well under .1 mm near the dependence epoch. By spacing the dependence centers not more than about .50 mm. (nearly 10″) apart in each coordinate, the solutions seldom exceed .25 mm. and errors due to extreme orientation (and scale) constants are rarely above .00005 mm.

8. CHOICE OF REFERENCE STARS

The choice of reference stars is guided by various considerations. Generally one need have little concern about the proper motions and parallaxes of the reference stars; almost any set of faint stars represents an acceptable close approximation to a fixed background. The choice depends of course on the exposure time and limitations due to required magnitude compensation. In any long-term astrometric problem it is important to study carefully any possible choice of reference stars, so that one will not be faced with early obsolescence, but instead will have a well-planned foundation for the present, and possibly future, configurations of reference stars. As to the number of reference stars, even for a central star at the origin, the accuracy does not increase much with the number of reference stars. Considering the extra work involved, generally not much accuracy is gained by using more than four reference stars. Graphical methods are very useful for an initial exploration and evaluation of the dependences for different configurations of reference stars; they are particularly effective for three-star combinations.

The geometrical accuracy of the reduced position of the central star depends on the distribution of the dependences for the reference stars. In case of a central star of appreciable proper motion the dependences change and result in a corresponding change in accuracy for the changing dependence background. In that case, the error squared, or inverse weight of the position measured on the dependence background is proportional to $1+[D^2]$.

For any investigation spread over a limited time interval, greatest accuracy is maintained if the position of greatest dependence accuracy is reached about the middle of that interval. The absolute minimum value of $[D^2]$ in the configuration (x_s, y_s) exists for the origin defined by $[x_s]=[y_s]=0$ where each of the dependences equals $1/n$. For any central star, therefore, to insure a satisfactorily small $[D^2]$ it is important to choose a configuration whose origin will not lie too far off the path of the star.

9. STELLAR PARALLAXES

Probably the most important single application of long-focus photographic astrometry is the determination of stellar distances, since these are basic in comprehending the structure of the stellar universe. The history of photographic determinations of parallaxes, as developed by Schlesinger and others, is well known. Long-focus refractors at several observatories are engaged in the determination of parallaxes of various types of stars. As a rule, plates are taken near extreme parallactic displacement (shortly after dusk and before dawn) of the parallactic ellipse. Some twenty or thirty plates, containing two to four exposures each, spread over several years, yield a parallax determination with a probable error of about $\pm''.010$. Gradually a tendency has developed to increase the accuracy of individual parallax determinations by increasing the number of plates.

As an example of a parallax determination of very high accuracy mention is made of the faint, twelfth magnitude star Ross 248 (Figure 4). A total of 350 plates, on 164 observing nights were taken with the Sproul refractor over the decade 1937–1946. An analysis of the measured positions yields a parallax of $+''.320\pm''.003$ (probable error), relative to the background of four faint, distant reference stars.

The photographic method has yielded measurable parallaxes for thousands of stars. The 1952 edition of the Yale Catalogue of stellar parallaxes lists determinations for some six thousand stars. The nearest star, Alpha Centauri, has a parallax of $0''.76$. By a simple computation the distance to Alpha Centauri is found to be 270,000 times that of the sun, or 270,000 *astronomical units*. The corresponding light time is 4.3 years and the distance is thus conveniently expressed as 4.3 light years (1 light year = 63,300 astronomical units $= 9.46\times 10^{12}$ kilometers, or 5.88×10^{12} miles).

The annual parallax is slightly over ″.03 at a distance of 100 light years, the approximate limit of distance penetration by the photographic method.

10. NEARBY STARS

The parallaxes of the nearest stars have yielded both individual and statistical results of interest. Consider, for example, the known 56 stars within the arbitrary distance limit of 16 light years. Of this cosmic sample fewer than a dozen stars are visible to the unaided eye. The majority are faint telescopic objects which were first recognized as being close to us because of their large proper motion. Not all 56 stars are isolated like the sun. Of these 31 appear single, 20 are grouped,—two by two—in 10 binary systems, while 6 are grouped —three by three—in two triple systems.

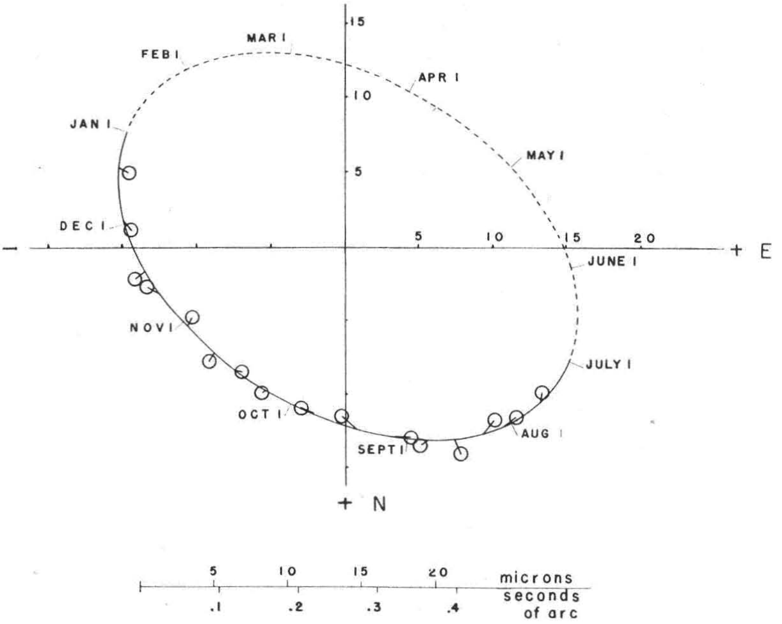

FIG. 4. The annual parallactic motion of the nearby star Ross 248 (distance 10.2 light years), as caused by the earth's orbital motion around the Sun. The photographic observations have been grouped into 16 points, whose accuracy (probable error of ±″.006) is represented by the radii of the small circles. These normal points clearly determine the part of the star's parallactic orbit accessible to observation. Since the observations are always made close to the meridian, the other half of the parallactic orbit falls in daylight, and is therefore not observable.

Knowing the distance to a star permits the calculation of the intrinsic brightness, or *luminosity*, which is conveniently expressed in terms of the sun's luminosity. In the present sample only three stars are brighter than the sun, while the faintest star is only 1/63,000 as luminous as the sun.

Stars differ in size and color, or what amounts to the same thing—in temperature. With few exceptions, there appears to be a remarkable relation between luminosities and colors. The fainter a star, the redder its color, so that an arrangement of the nearer stars according to decreasing luminosity also proves to be a sequence of increasing redness, and of decreasing diameter. The astronomer refers to this relation as the *main sequence*. However, the situation is rather more complex, particularly if there is included more distant stars—and the main sequence must be considered only as a first, basic stepping stone in the advancing knowledge and understanding of the stellar population.

A knowledge of stellar distance is a necessary prerequisite in the determination of *stellar masses*. Binary stars are numerous, and they provide virtually the only means for measuring stellar masses. The required data are the space-time dimensions of the orbital motion of the two stars around each other; to ascertain the linear size of these orbits the parallax of the system must be known. Accurate orbital data and distances are known for several dozen binary stars, and have yielded important information. There appears to be a well-established general *relation between mass and luminosity* for (main-sequence) stars. The higher the mass, the higher the luminosity; however, the range in mass is small compared with the range in luminosity; the luminosities vary approximately as the third power of the mass. Again, there are exceptions for certain types of stars—some striking, some very subtle, but all of them helping us to better understand the structure and evolution of stars.

11. PERTURBATIONS

Another useful application of the methods of long-focus astrometry is the study of minute *perturbations* in "single" stars, caused by the presence of unseen companion stars. A number of stars reveal periodic variations in their paths, indicating the presence of companion objects, too faint to be seen by any telescope, yet massive enough to influence visibly the motion of the visible, primary star. The amplitude and period of these perturbations permit a calculation of the masses of the unseen objects.

Of particular interest is the recent visual discovery of one of these objects. For over two decades a perturbation had been observed in the path of the rather inconspicuous eleventh-magnitude, red star Ross 614. By 1950 it was firmly established that the unseen companion of Ross 614 revolves around the primary star in 16.5 years, and that the extent of the perturbation would be greatest in 1955. The companion was then actually seen and photographed with the 200-inch Hale telescope, and proved to be a very faint star, 1/63,000 as luminous as the sun, and only 8 per cent as massive as the sun. This makes the companion of Ross 614 at present the star of smallest known mass. However it is still 80 times as massive as Jupiter, the heaviest planet.

A star is a self-luminous object; a planet, a non-luminous object shining by reflected light. A conventional distinction, on the basis of mass, is that an object with a mass of 1/20 times that of the sun would be a borderline case. Objects with masses over this limit are stars; those below are planets. A simple calculation shows that planetary companions of stars other than our sun must be very elusive; they could not be *seen* with any observational means now available. The situation would not be quite so hopeless if the planet would reveal itself by a perturbation of the primary star. So far, however, there is no conclusive evidence for companions of planetary mass, though a few cases are suspected and are being investigated.

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* Copies of these articles may be obtained on request by writing to the Sproul Observatory, Swarthmore College, Swarthmore, Pennsylvania.