*The Graphic Construction of Controlled Stereoscopic Models**

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ABSTRACT: *A line drawing may be constructed so as to appear as a three-dimensional image when viewed stereoscopically. This stereoscopic image will have the same radial and parallactic displacement, as well as vertical exaggeration, that a stereoscopic view of a similar subject would have if represented on two vertical aerial photographs of specific scale, focal length, and photo base. By appropriate construction the three-dimensional geometry of the image rays of a stereoscopic model is reduced to a two-dimensional drawing from which the right and left members of a stereoscopic pair can be obtained.*

INTRODUCTION

IN STEREOSCOPIC MODELS from vertical aerial photographs the vertical scale generally is exaggerated. The relationship of this exaggerated vertical scale to the true vertical scale is especially important in photogeologic studies where the true slope (dip) of outcropping rock strata is desired. In recent experiments measuring the exaggerated slopes in stereoscopic models of areas of known true slopes, it becomes apparent that the study could be greatly facilitated if a method of constructing controlled stereoscopic model slopes to predetermined conditions of simulated flying height, focal length, and photo base could be devised. A method of constructing line drawings that will appear as three-dimensional images when viewed stereoscopically is presented herein. The model, at any scale chosen, will have inherent in it all the parallactic and radial displacement that a stereoscopic pair of vertical aerial photographs would have.

GENERAL METHOD OF CONSTRUCTING A CONTROLLED STEREOSCOPIC MODEL

With the following known or selected information—photo base, scale, flying height, focal length, and dimensions of the subject to be represented in the stereoscopic model—a two dimensional diagram can be made from which the right and left members of a stereoscopic model can be obtained. The general method of construction is as follows:

1. Lay out a hypothetical photo-base distance (in inches or millimeters); this line will also represent the flight line. The end points of this line establish the hypothetical photograph centers.
2. Select location of subject to be represented in the stereoscopic model. At this location an orthographic projection of the subject is drawn at the desired scale.
3. Assume elevations for selected points above the hypothetical ground plane.
4. Determine the parallactic displacement of all points that will be above the hypothetical ground plane. With a selected or desired scale, flying height, focal length, and photo base, this can be done by the simplified parallax formula:

$$\Delta p \equiv \frac{bh}{H - h}$$

where

Δp = Change in parallax measured in units of 0.01 millimeter.

H = Height of camera expressed in feet above the ground.

b = Photo base measured in units of 0.1

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millimeter (the average distance between the center and the conjugate center of each member of the stereoscopic pair).

h = Difference in elevation, expressed in feet, between the hypothetical ground plane and the elevation point.

The above formula can be used instead of the more complicated

$$\Delta p = \frac{hp_1}{(H - h_1) - h}$$

because the lower points of all elevation differences are assumed to lie on the hypothetical ground plane, which also contains both photograph centers and conjugate centers.

5. Construct radial lines from the hypothetical photograph center points through all points of the orthographic drawing that are at elevations above the ground plane.

6. Using the proper parallax change for an elevation above the hypothetical ground plane as determined in (4), plot a line representing this distance in such a manner that it will be parallel to the flight line, and its end points will intersect the respective diverging radial lines. (The line representing parallax change will always be at a distance farther away from the flight line than the intersection point of the radial lines.) Such lines should be plotted for all elevations above the hypothetical ground plane.

7. These points establish the radial displacement of elevation points from their respective hypothetical photograph centers. With these points established, the drawing of the subject can be constructed:

(a) as it would appear in the right member of the stereoscopic pair.

(b) as it would appear in the left member of the stereoscopic pair.

As the base of the subject is on the hypothetical ground plane the base points will not be displaced, and they will have the same relative location on both members of the stereoscopic pair.

8. Transfer the right and left members to separate sheets, either by tracing on an overlay sheet or by pin pricking the points through the diagram to a medium under the sheet. In transferring the right member:

(a) Transfer the two photograph centers.

(b) Transfer all points of zero elevation in reference to the hypothetical ground plane.

(c) Transfer all points of elevation above the hypothetical ground plane that have been displaced along radial lines drawn from the right photograph center.

These points, excluding the center and conjugate center points, are then joined together in appropriate manner to construct the displaced subject as it would appear in the right member of the stereoscopic model. In a similar manner this same procedure is followed in transferring and constructing the left member of the stereoscopic model.

9. With completion of the right and left members the subject can now be viewed stereoscopically. Some shading for shadow effect will add a natural touch to the three-dimensional image.

Principle Used in Constructing a Stereoscopic Model

The following is the principle used in constructing a stereoscopic model: In the perspective view (Figure 1) $IJKL$ is the hypothetical ground plane. Points A and B are camera stations; G and H are points on the ground vertically below these respective camera stations. CD is a pole at random location on the ground plane. CE is the projection of the pole on the ground plane as seen from B. This projection falls on a radial line drawn from H through C. In a similar manner CF is the projection of the pole on the ground plane as seen from A, and falls on a radial line drawn from G through C. The line EF is the

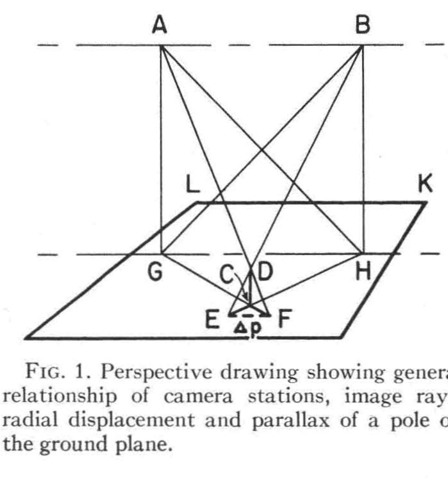

FIG. 1. Perspective drawing showing general relationship of camera stations, image rays, radial displacement and parallax of a pole on the ground plane.

parallax change and represents the displacement of the top of the pole on the hypothetical ground plane in moving from A to B. Triangles GCH and FCE are similar, and EF is parallel to GH.

Figure 1 shows how the pole is displaced radially from the two points G and H and that EF, the parallax distance, is parallel

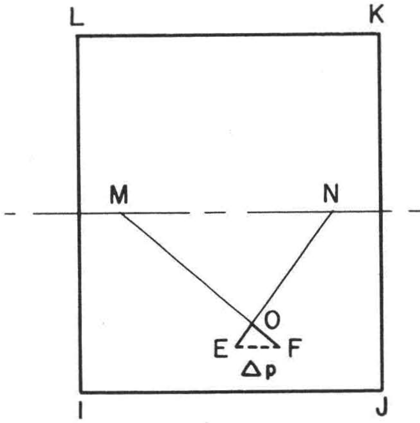

FIG. 2. Plan view of Figure 1. showing relationship of photo centers, radial displacement, and parallax distance of a pole on the ground plane.

to the flight line GH. This perspective view can be represented in an orthographic projection (Figure 2). In this diagram all lines as shown in Figure 1 have been projected to the plane $IJKL$.

The camera station A and the point G coincide to become point M. The camera station B and the point H coincide to become point N. The pole CD becomes point O. The change in parallax, EF, remains the same, and all the rays from points A and B coincide with the respective radial line from the points M and N.

In viewing this diagram it can be seen that if the distance MN, the location of the pole, and the parallax change are known, by the use of radial lines from the points M and N through the point O, and by plotting the distance EF parallel to MN and intersecting the diverging radial lines, the radial displacement and projection of the pole can be plotted in a two-dimensional drawing.

RELATION TO VERTICAL AERIAL PHOTOGRAPH

Now all points on the hypothetical ground plane and all projected points to this plane, as seen from the camera station A, would, at a reduced scale, have the same geometric pattern that a vertical aerial photograph of the area from the point A would have. In a similar manner, all points on the hypothetical ground plane and all projected points to this plane as seen from the camera station B would, at a reduced scale, have the same geometric pattern that a vertical aerial photograph of the area from point B would have. All angular relations in the horizontal plane would remain the same. Figure 2 would then also represent at a reduced scale the geometry of a stereoscopic pair of vertical photographs.

ACTUAL CONSTRUCTION OF A SPECIFIC STEREOSCOPIC MODEL

A stereoscopic model of a simple pyramid-shaped structure (Figure 3A) is desired. The pyramid has a square base with sides 2,000 feet long and a height of 500 feet, and the following specifications of vertical photography are assumed:

Flying height = 10,000 feet
Focal length = 6 inches
Scale = 1:20,000
Photo base = 92 millimeters

These are the necessary data for constructing the stereoscopic model.

On a sheet of chart paper a line representing the flight line is drawn, and on this line the two photograph centers, p^1 and p^2, are plotted 92 millimeters (photobase) apart. A desired or random location for a plan view of the pyramid is then assumed. The pyramid is constructed at a scale of 1:20,000, the base being represented by the square $ABCD$ and the vertex by point E of Figure 3B.

Next radial lines are drawn from the respective photograph centers, p^1 and p^2, to any key point (here point E) that is to be represented above the hypothetical ground plane.

The parallax change (Δp) for the point E, 500 feet above the hypothetical ground is now computed using the following formula:

$$\Delta p = \frac{bh}{H - h}$$

where

$h = 500$ feet, the change in elevation from the hypothetical ground plane to point E,

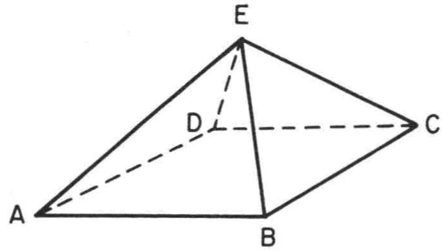

Figure 3A.

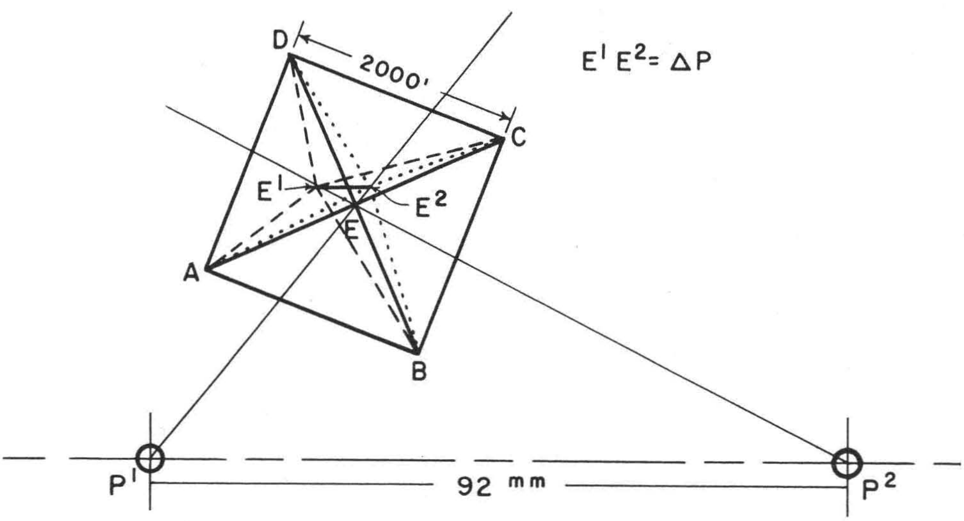

Figure 3B.

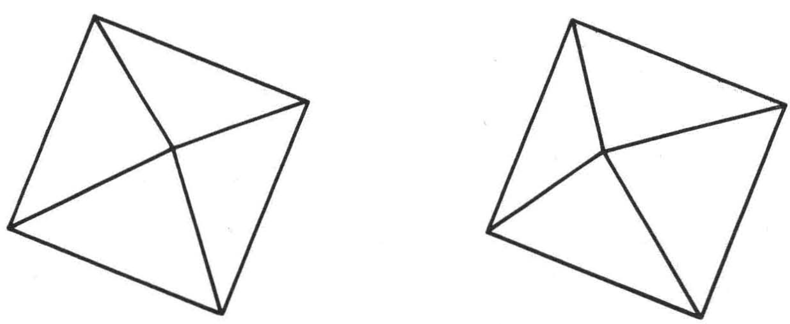

Figure 3C.

FIG. 3. Construction of a stereoscopic model. A. Subject for construction of right and left members of a stereoscopic pair. B. Plan view showing displacement of vertex and sides of pyramid for the right and left members of a stereoscopic pair. C. Right and left members of stereoscopic pair—ready for stereoscopic viewing.

$H = 10,000$ feet, altitude of photography above the ground plane,

$b = 92$ millimeters, photo base,

then

$$\Delta p = \frac{92 \times 500}{10,000 - 500} \equiv 4.84 \text{ millimeters.}$$

A line 4.84 millimeters is constructed and moved parallel to the flight line toward the point E until its end points E^1 and E^2 intersect the respective diverging radial lines previously drawn from the photograph centers, p^1 and p^2. (The line of parallax change will be at a distance farther away from the flight line than the intersection point of the radial lines.) This operation gives the triangle $E\ E^1\ E^2$, in which E^1 is the displacement of the point E from p^2, E^2 the displacement from p^1, and E^1E^2 the change in parallax. Point E^1 is then joined to the base points, A, B, C, D by short dotted lines. These lines represent the displaced edges of the pyramid surfaces from point p^2. In a similar way point E^2 is joined to the base points, A, B, C, D. The resulting lines represent the displaced edges of the pyramid surfaces from point p^1. Since $ABCD$ constitutes the base of the pyramid and is on the hypothetical ground plane, it appears in both drawings with no displacement.

By means of an overlay or by pin pricking appropriate points of the drawing to an underlying sheet, the information is transferred as the right and left components of a stereoscopic pair. The centers are transferred to both members for relating the model to the hypothetical flight line so that proper orientation may be obtained. (These are not shown in Figure 3C) Points A, B, C, D, and E^1 are transferred to the right-hand drawing and points A, B, C, D, and E^2 are transferred to the left-hand drawing. With the completion of the transfer and joining the proper points together with solid lines as illustrated in Figure 3C, the pyramid can be viewed stereoscopically.

Conclusion

Stereoscopic models may be easily and quickly constructed to represent any angle of slope for specified conditions of flying height, focal length, photo base, and scale of vertical photography. This paper presents a simple method for constructing such models; these models are particularly useful in the study and analysis of vertical exaggeration.

Only two altitudes are represented in the construction described, but there is no theoretical limit to the number of elevation points that can be shown. Although the model constructed involved only straight lines, by selecting enough points of parallactic displacement it is possible to construct models having curved surfaces. Models constructed three or four times larger than the desired scale, and later reduced photographically, give sharper and more accurate stereoscopic pairs.

Constructed stereoscopic models should have significant application in various photo interpretation fields. They are also useful in preparing stereoscopic illustrations.

NEWS NOTE

Expansion of Gordon Enterprises

Plans for construction of 52,000 square feet of additional manufacturing space as soon as a suitable site can be found have been announced by Alan Gordon, president of Gordon Enterprises, North Hollywood. Expansion of the current 41,600 square foot plant is necessitated by increase of orders which will require doubling of personnel and production facilities.

The firm will continue with executive offices, engineering, instrument shops, machine shops and warehousing facilities in the present location at 5362 North Cahuenga Blvd. A planned 400% increase in tooling and production machine shops. Active warehousing and parts department facilities will occupy the new building as soon as available.

Map of Cuba

Photo mapping of the entire Island of Cuba and adjacent Islands of the Republic of Cuba was started recently by Aero Service Corporation. The air photos will be used for geologic studies and general reconnaissance by the oil and mining industries, as well as by engineering companies and others at work on Cuban development. Two specially outfitted P-38's are photographing Cuba's 44,000 square miles from a height of 35,000 feet. Each 9×9 inch photo will cover an area of about 80 square miles, and approximately, 3000 photos will be made at a scale of 1:60,000.