*The Stereo-Slope Comparator—An Instrument for Measuring Angles of Slope in Stereoscopic Models**

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ABSTRACT: *This paper describes a new instrument, the Stereo-slope Comparator (also known familiarly as the Super Duper Dipper), and a method for rapidly determining angles of slope in the stereoscopic model from two vertical aerial photographs. Two additional devices used with the instrument are: (a) The Supplementary Slope Model and (b) The Slope Conversion Chart.*

In measuring angles of true slope with the Stereo-slope Comparator, the exaggerated angle of the Supplementary Slope Model is first determined. According to the focal length and photo base of the photography, the true angle this slope represents is read from the table that accompanies the Supplementary Slope Model. With these two angles known, an exaggeration factor is determined from the Slope Conversion Chart. Angles of exaggerated slope are then determined in the stereoscopic model of the photographs. These angles of slope are then referred to the Slope Conversion Chart, where, by using the exaggeration factor established above, they are converted to angles of true slope.

This method also provides a means of demonstrating the existence of vertical exaggeration and the factors that affect it.

INTRODUCTION

IN MANY fields of photo interpretation, such as geologic and engineering studies, an accurate determination of angles of slope is essential. These angles are generally exaggerated in the stereoscopic model of two vertical aerial photographs. The true angle can be determined by photogrammetric measurements, or an interpreter can estimate the exaggerated angle of slope and by various methods convert it to a true angle. The first method is accurate but time consuming, whereas the second is fast but not always reliable. The method described in this paper takes a middle course between these two. It approaches the accuracy of photogrammetric measurements but at the same time considerably reduces the time involved. By use of the Stereo-slope Comparator ("the Super Duper Dipper"), the exaggerated angle of slope is measured in the stereoscopic model. This angle is then converted to a true angle by the use of two other devices, the Supplementary Slope Model and the Slope Conversion Chart.

The Stereo-slope Comparator was designed for use under a stereoscope that accommodates an image separation of 6.5 to 10.5 inches. The stereoscope should be of the straddle type, or one mounted on a projecting arm which is bolted to the desk or work board. The field beneath the stereoscope should be unobstructed by the supporting stand.

In using this method, the only required information is the focal length of the camera used in taking the pictures, and the average photo base of the two pictures in the stereoscopic pair.

DESCRIPTION OF INSTRUMENT

The Stereo-slope Comparator (Figure 1) consists of two gear housings joined to a central block by sliding tubes. A shaft on which a target is mounted projects from

* Publication authorized by the Director, U. S. Geological Survey.

each housing. Movement of the targets is synchronized by a gear arrangement. Control knobs on either housing and on the central block control the three motions of the synchronized targets. These are: (a) Separation, (b) Horizontal swing, and (c) Slope angle.

The separation motion is controlled by two knobs, *A* and *B*. (Figure 1). Knob *A*, when loosened, permits rapid separation of the two gear housings. The amount of the separation can be increased or decreased. Knob *A* must be tightened after the approximate separation is obtained. Knob *B* controls a screw for use in securing a finer adjustment.

The horizontal swing motion of the shafts, parallel to the base of the instrument, is controlled from either housing by knob *C* or *C′*. By adjusting either, the two shafts can be simultaneously swung to any desired position within an arc of 180 degrees. By rotating the instrument 180 degrees in reference to the viewing board or desk and by a similar adjustment of knob *C* or *C′*, the shafts can be swung another 180 degress, completing a 360 degrees arc of movement.

The slope-angle motion is controlled by knob *D* or *D′*. By adjusting either the targets are simultaneously set at any desired angle of slope. The angle is read on a protractor-like dial mounted on each shaft. The dials are calibrated in units of 1 degree and range from 0 degrees, which is parallel to the base of the instrument, to 90 degrees, which is perpendicular to the base.

The instrument is equipped with six pairs of targets, of which only one pair is used at a time. The targets are of different sizes and should be selected in accordance with the surface expression of the slope to be measured as it appears in the stereoscopic model. The target fits into a slot cut in the end of the shaft so that the target pairs are easily interchangeable. The targets shown in Figure 1 were designed for photogeologic work.

Supplementary Slope Model

The Supplementary Slope Model (Figure 2) is a line drawing in two parts. When the two are viewed steresocopically, two sloping rectangular planes are visualized in space.

The parallactic displacement, and thus the steepness of slope, is fixed for a selected focal length and photo base. However, for other focal lengths and photo bases this parallactic displacement represents different known angles of slope. These different angles of slope are shown in the table that is a part of the Supplementary Slope Model. It covers only 6- and $8\frac{1}{4}$-inch focal length photography with photo bases from 2.5 to 4.5 inches.

The two line drawings are mounted on a rigid backing and joined together by a sliding arm in such a way that they can be moved in the *X*-direction. The image separation distance may be read to the nearest 0.1 inch from the scale mounted on the sliding arm.

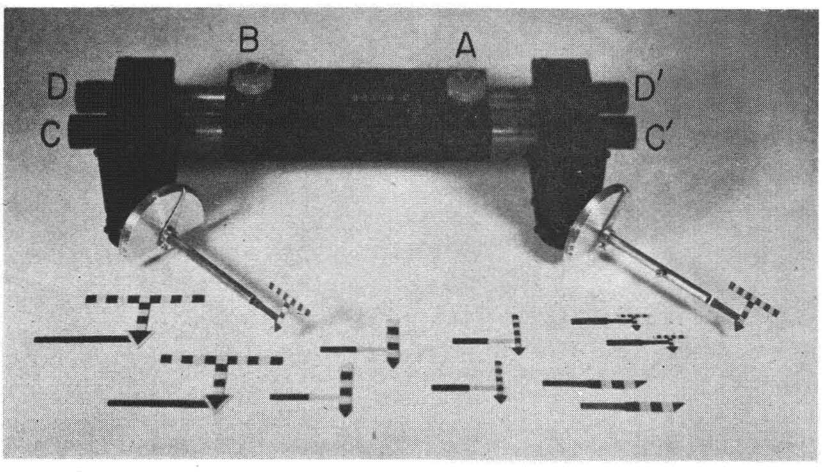

Fig. 1. The stereo-slope comparator.

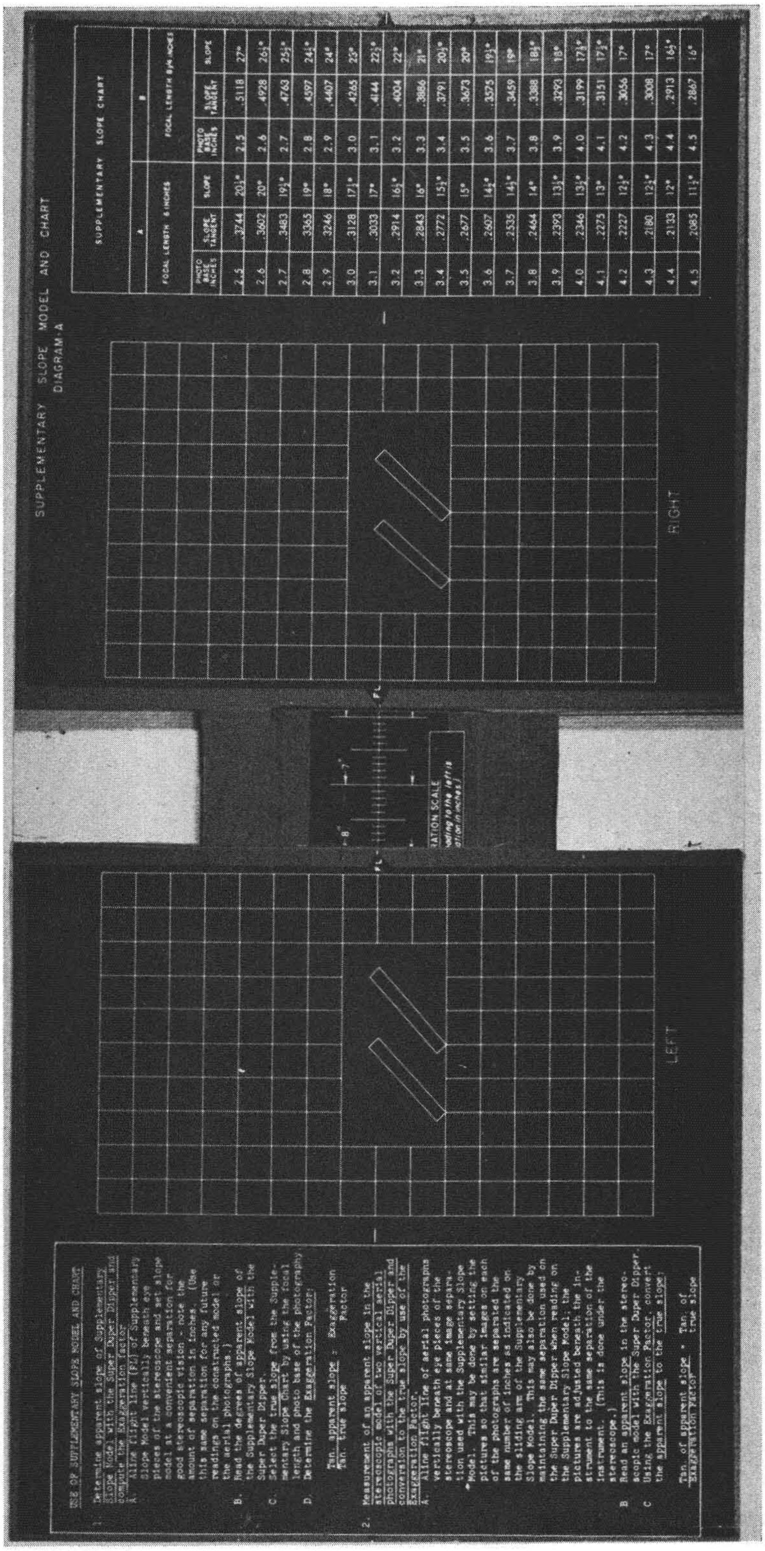

FIG. 2. The supplementary slope model.

The Slope Conversion Chart

The Slope Conversion Chart (Figure 3) incorporates the relationship of exaggerated angle of slope, true angle of slope, and exaggeration factor into a graph. By use of this graph, with any two of the three quantities given, the third can be rapidly determined.

Operational Procedure

By using the Stereo-slope Comparator exaggerated angles of slope can be determined. However, to convert these angles to angles of true slope, it is necessary to compute an exaggeration factor from a constructed stereoscopic model such as the Supplementary Slope Model. In the following paragraphs the step-by-step procedure of computing true angles of slope, using the Stereo-slope Comparator, the Supplementary Slope Model, and the Slope Conversion Chart, is explained.

1. Determine the true slope represented by the Supplementary Slope Model for the focal length and average photo base of the stereoscopic pair that you are using. The true angle of slope may be determined from the table on the right hand side of the model.

2. Measure the exaggerated angle of slope of the Supplementary Slope Model by means of the Stereo-slope Comparator. The Model should be aligned under the stereoscope so that a line connecting the centers of the Model is vertically below and in alignment with the centers of the eye pieces of the steresoscope. Shift the right and left members of the slope model in the X-direction until good steresoscopic vision is obtained. Read the amount of this separation from the sliding arm of the slope model. This is the image-separation distance and this same separation must be maintained when viewing the vertical aerial photographs under the stereoscope.

Place the Stereo-slope Comparator over the Supplementary Slope Model, one target over the right image and one over the left. Adjust the separation, the horizontal swing, and the slope angle motions until the inclination of the targets (seen stereoscopically as one target) is in visual agreement with the exaggerated angle of slope in the constructed stereoscopic model. This visual agreement can be made more accurate by adjusting the fine-thread separation knob B, which changes the separation of the targets in reference to the model and causes the stereoscopic target to appear raised or lowered in reference to the plane of the model. By this adjustment the steresoscopic sloping target can be placed in, below, or above the plane of the selected slope in the model. The angle of slope is read in degrees on the instrument dial. Several readings should be taken to provide a good average, as this is a critical measurement.

Step 2 may be omitted in all further operations as long as the same image separation is maintained and the same stereoscope used. As the stereoscopic slope of the Supplementary Slope Model has a fixed parallax, any instrument measurements of this exaggerated angle of slope will be

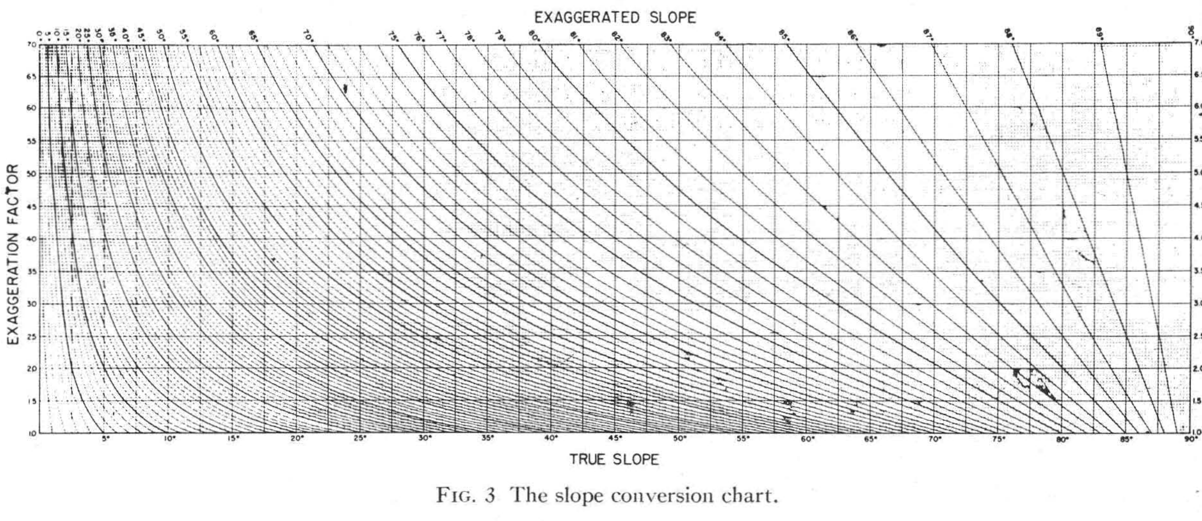

FIG. 3 The slope conversion chart.

similar. It is only the angle of true slope that this model represents in relation to different focal lengths and/or photo bases that will vary. This change of angle of representative slope can be obtained from the table on the right member of the Supplementary Slope Model.

3. Determine the exaggeration factor by referring the selected true angle of slope of the Supplementary Slope Model and the exaggerated measured angle of slope to the Slope Conversion Chart. On the chart all vertical lines represent degrees of true slope, all curved lines exaggerated slope, and all horizontal lines various exaggeration factors. To determine an exaggeration factor, locate the intersection of a line representing a true angle of slope with one representing an exaggerated angle of slope; and read the horizontal line representing an exaggeration factor.

4. Measure an exaggerated slope in the stereoscopic model from the aerial photographs, with the Stereo-slope Comparator. The photographs must be aligned under the stereoscope in a manner similar to that used for the Supplementary Slope Model. It is essential that a line connecting the centers of the stereoscopic pair be vertically below and in alignment with the centers of the eye pieces of the stereoscope. Care must be taken to maintain the image separation that was used when measuring on the Supplementary Slope Model. The Stereo-slope Comparator is then placed on the photographs with one target over each image. The separation, the horizontal swing, and the slope angle motions are adjusted until the inclination of the targets, seen stereoscopically as one target, is in visual agreement with a selected angle of exaggerated slope in the stereoscopic model from the photographs. The number of degrees of the exaggerated angle of slope is then read on the instrument dial. Several readings should be made to assure consistency in setting the targets at the proper inclination.

5. Determine the true angle of slope from the Slope Conversion Chart, using the exaggeration factor determined in step 3 and the exaggerated angle of slope obtained in step 4.

Compensation for Vertical Exaggeration

When two vertical aerial photographs are viewed stereoscopically, vertical exaggeration is generally present. This causes altitudes to appear higher than they are, in reference to the horizontal scale of the model. Therefore, all angles of slope appear steeper than they actually are. This vertical exaggeration can be expressed as the ratio of the vertical scale to the horizontal scale and is called the Exaggeration Factor.

In considering only angles of slope, the Exaggeration Factor may be expressed by the following formula:

$$\text{Exaggeration Factor} = \frac{\text{Tangent of exaggerated angle of slope}}{\text{Tangent of true angle of slope}}.$$

Five factors that affect vertical exaggeration are: 1. The focal length of the camera; 2. The photo base of the stereoscopic pair; 3. The image separation of the photographs; 4. The eye base of the person viewing the pictures; 5. The viewing distance. (The distance between the photographs and the viewer's eyes.)

In determining angles of slope by means of the Stereo-slope Comparator and the Supplementary Slope Model, these five factors of vertical exaggeration are compensated in the following manner.

Focal length and photo base.—The effect on the amount of vertical exaggeration is compensated in the computed true angles of slope shown in the table on the right-hand side of the Supplementary Slope Model.

Image separation.—By maintaining the same separation when measuring the angle of slope of the Supplementary Slope Model and angles of slope on the stereoscopic model from the aerial photographs, the effect of image separation on vertical exaggeration remains constant and can be disregarded.

Eye base and viewing distance.—The eye base of an individual is a constant distance. The viewing distance of any one stereoscope at a fixed magnification is also a constant factor. Any vertical exaggeration caused by these factors is believed to equally affect the stereoscopic target of the Stereo-slope Comparator, the Supplementary Slope Model, and the stereoscopic photo model. In connection with the method of slope measurement described in this paper these factors can thus be disregarded.

Compensation for Radial Displacement

All points above a plane passing through the lowest elevation represented on a vertical aerial photograph, are displaced radially from the center of the photograph. The amount of this displacement varies with the altitude of the points above the lowest point of elevation, their distance from the center of the photograph, and the focal length of the taking camera. This displacement causes a change in the angle of slope seen stereoscopically on a pair of vertical aerial photographs. The amount of this apparent change will depend on the location and direction of the slope in the model. The apparent inclination of slopes known to be similar will appear different at different locations and orientations within the stereoscopic model.

The targets of the Stereo-slope Comparator are displaced in a similar fashion, but from a perspective point at the eye pieces of the stereoscope. If the photographs are properly aligned beneath the stereoscope, displacement of the targets in a large part compensates for the displacement of the photographic images.

Application and Accuracy

Measuring angles of slope in the stereoscopic model using the Stereo-slope Comparator has two excellent applications in photo interpretation work: a means of rapidly measuring angles of slope in the stereoscopic model and the training of new personnel in recognizing the existence of vertical exaggeration and understanding the factors that cause it.

A complete evaluation of the Stereo-slope Comparator method of measuring angles of slope is in progress. To date the results are favorable. Several hundred determinations of true angles of slope were made on a constructed stereoscopic model. The model contained 108 angles of slope ranging from 2 degrees to 90 degrees. The slopes were at random locations and directions. Measurements of the exaggerated angles and their conversion to true angles gave the following results: 2 to 4 degree angles of true slope were read with a maximum error of plus or minus 1 degree; 20 to 25 degree angles of true slope, with a maximum error of plus or minus 3 degrees; 40 to 50 degree angles of true slope, plus or minus 7 degrees; and 80 to 90 degree angles of true slope, plus or minus 10 degrees.

The figures above indicate an increase in error as the angle of slope steepens. This increase is due primarily to the relatively large increase of the tangent value of steep slopes. The result of this increase is shown in the following: With an exaggeration factor of 3.5, a 17 degree exaggerated angle of slope would represent a 5 degree angle of true slope. If the exaggerated angle of slope were erroneously determined as 16 or 18 degrees, the true slope would be read as $4\frac{3}{4}$ or $5\frac{1}{4}$ degrees. The plus or minus 1 degree error in reading this low exaggerated angle of slope would still give approximately the correct answer for the true angle of slope. On the other hand, an 85 degree exaggerated angle of slope would represent a true angle of slope of 73 degrees. If this angle of exaggerated slope were erroneously determined as 84 or 86 degrees, the true angle of slope would be read as $76\frac{1}{4}$ or $69\frac{3}{4}$ degrees. Thus a plus or minus 1 degree error in this steeper exaggerated slope would cause an error of plus or minus three degrees in the true angle of slope.

The increase in error as the angle of slope steepens is also due to the greater difficulty involved in orienting the stereoscopic target. As the angle of slope increases the plan view of the slope in the stereoscopic model commonly decreases, making proper alignment of the target less accurate.

Another Expansion for Gordon Enterprises

Expansion of facilities currently housing the purchasing department has been announced by Alan Gordon, President of Gordon Enterprises, camera manufacturing firm of North Hollywood. New quarters, located at 5401 Cahuenga Blvd. in North Hollywood, were occupied by the firm's purchasing department October 10.

Gordon Enterprises already occupies three plants with a combined square-footage in excess of 100,000. Established in 1950 in modest quarters, the firm now employs more than 155 men and women, 96% of whom are veterans.