Some Preliminary Results of the Determination of Radial Distortion in Aerial Pictures

B. HALLERT, *Stockholm 70, Sweden*

IN ACCORDANCE with a method, published in [1] and [2], some preliminary investigations of aerial pictures from test areas have been performed.

Some pictures have been taken by the U. S. Coast and Geodetic Survey, Washington D. C., USA, over a test area in Ohio, USA. Two different aerial cameras were used for aerial photography from two different altitudes. Another picture, taken with a third camera, was obtained from the Ohio Highway Department, Columbus, Ohio. The measurements and computations have been performed at the Division of Photogrammetry of the Royal Institute of Technology, Stockholm, Sweden.*

The test area is rather flat and contains a great number of signalled control points, the coordinates and elevations of which have been determined by the U. S. Coast and Geodetic Survey. The pictures were taken nearly vertically and approximately over a central point, around which the control points could be combined into circles with varying radii. The displacements due to the elevation differences on the ground were computed and corrected.

The image coordinates of all points were first measured in an ordinary coordinatograph, the accuracy of which was checked with the aid of measurements of high precision glass grids. The standard error of the measurements according to the adjustment was found to be around 0.01 mm. The accuracy of this measuring device is consequently not very good.

In order to obtain two different sets of coordinate measurements the U. S. C. & G. S. pictures were also measured in the Autograph A7. The preliminary values of the tilts were set into the projector and the coordinates were measured in a fixed projection plane. The radial standard error of single image coordinate measurements in the A7 has been determined to be around 0.005 mm in the image, according to test measurements with high precision glass grids.

The flying altitudes above the ground for the U. S. C. & G. S. pictures were in the vicinity of 10,400 and 14,000 feet. Some identification difficulties appeared, particularly in the pictures from the higher altitude but also in the lower altitude pictures. Therefore some points had to be omitted, and the distortion curves could not be determined as accurately and completely as wanted.

In Diagrams 1–3 the distortion amounts for the different radii are plotted. By estimation the distortion curves are then drawn as the fully drawn curves. Unreliable parts of the curves are dotted. The discrepancies between the individual distortion determinations and the chosen curves are obviously not too large, and the accuracy of the determination of the distortion seems in general to be rather good. If the standard errors of the measurements are computed, according to the condition of the central projection as indicated in [2], rather large amounts are found, however. As an average of several determinations the value of around 0.04 mm was found. This obviously means that there are present other systematic errors that cannot be compensated by the ordinary six elements of orientation or degrees of freedom in the adjustment set up. But, according to the satisfactory shape of the obtained distortion curves, the determined distortion amounts are probably not afflicted by these up to now undetermined systematic

* The measurements and computations were primarily performed by L. B. Ottosson, E. Rehnlund, L. E. Lycken and G. Palmgren, all assistants at the Division of Photogrammetry.

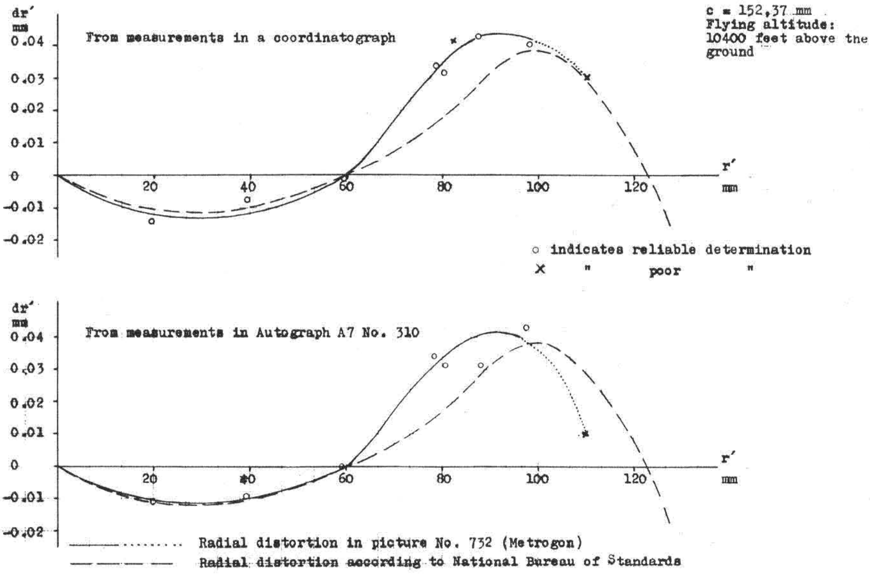

DIAGRAM 1.—Distortion curve of a metrogon lens in the air.

errors. Investigations are being carried on in order to find out the reasons for the unexpected high values of the standard error of the measurements, as computed from the existing conditions of the central projection.

The distortion curves of the lenses from laboratory determination are also demonstrated in the Diagrams 1–3 as dashed lines. The determination was performed by the National Bureau of Standards in Washington D. C., USA. Obviously there are rather large differences.

For the Aviogon lens in question, another laboratory distortion curve was received from the manufacturer, the Wild Company in Switzerland. This is shown in Diagram 2 as the dashed-dotted curve.

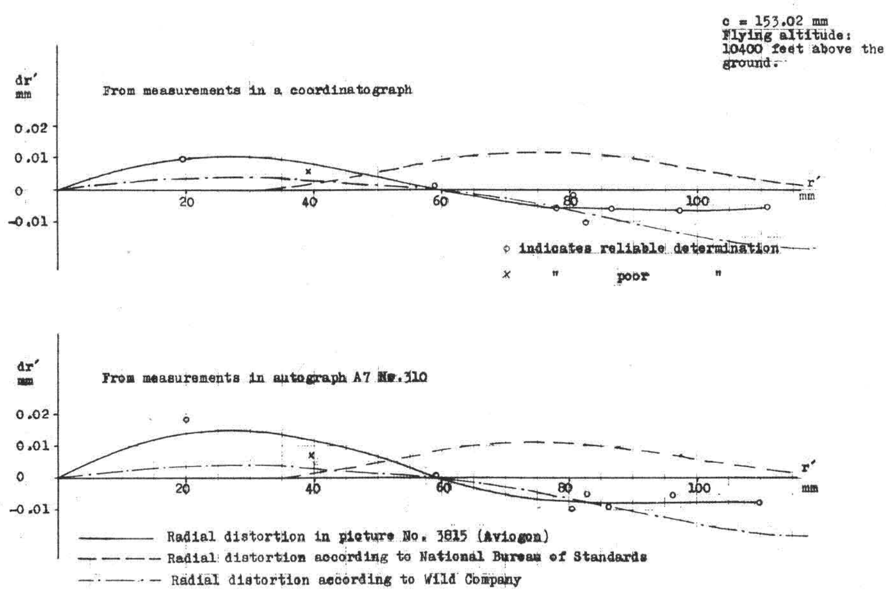

DIAGRAM 2.—Distortion of an aviogon lens in the air.

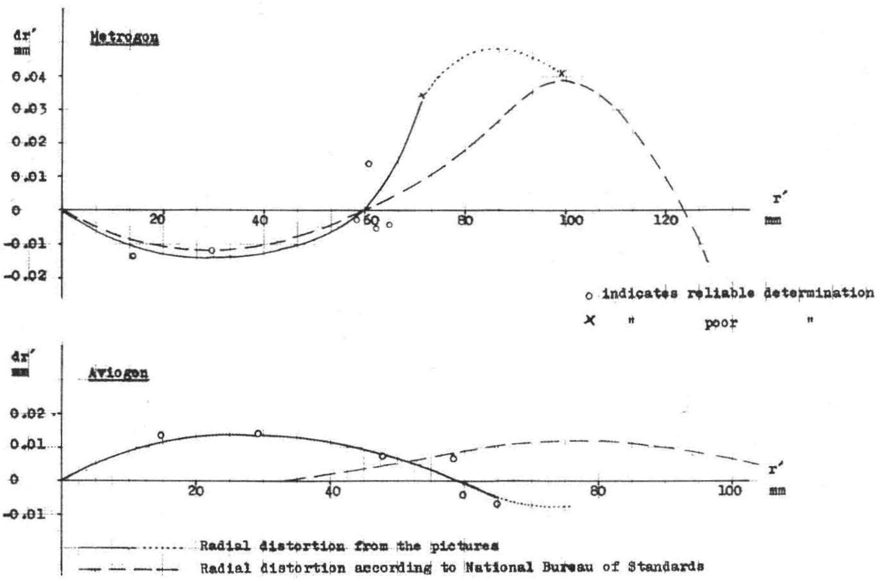

DIAGRAM 3.—Distortion curves from 14,000 feet.

This obviously agrees much better with the distortion curve from the air than that which was determined by the National Bureau of Standards. It is interesting to note that for both lenses the distortion curves from the air indicate an increasing positive distortion with increasing radii in comparison with the laboratory curves. It therefore seems most probable that the atmospheric refraction is the main cause of the discrepancies. On higher flying altitudes the earth's curvature will probably become more dominating. See [5]

The results from the picture taken by the Ohio Highway Department are demonstrated in Diagram 4. The camera is a normal-angle camera $f=208.93$ mm. and the picture was taken from about 12,800 feet above the test area. In this picture the identification of the control points on the ground was very good.

The shape of the obtained distortion curve obviously appears to be rather reliable. No information from laboratory tests is yet available.

In this case the standard error of the measurements, as computed from the conditions of the control projection, is also of the magnitude 0.05 mm. Therefore probably present are systematic errors other than those which can be compensated by the ordinary six degrees of freedom. Compare Diagram 5. Investigations concerning these problems are being performed. It would be very interesting to investigate glass negatives and film negatives from the same camera and taken under similar conditions, in order to find the influence of the film shrinkage upon real photographs from the air.

In this way it seems possible to determine systematic deformations of the pen-

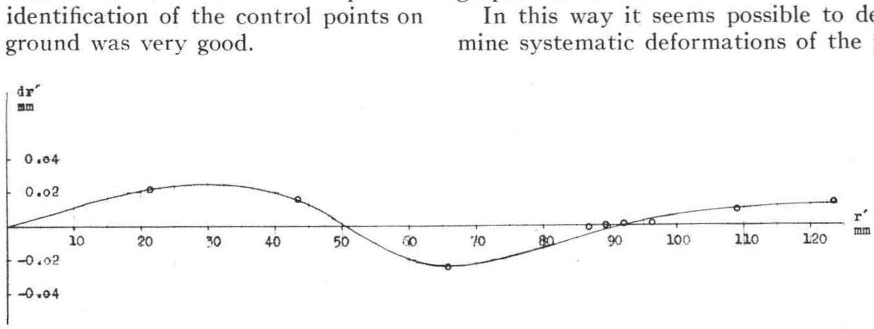

DIAGRAM 4.—Distortion curve of a photogrammetric lens. Manufacturer unknown.
C = 208.93 mm. h = 12,770 feet.

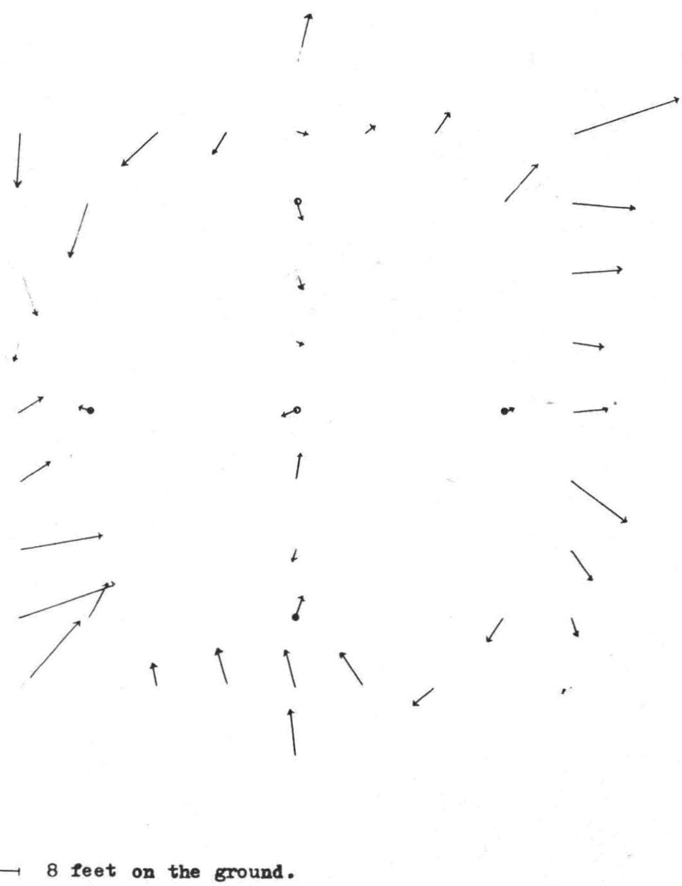

DIAGRAM 5.—Residuals in some points of the image after adjustment of the measurements in 5 points (denoted with the small circles.) The diagram refers to the picture from which the distortion curve, Diagram 4, was determined.

cils of rays at the moment of exposure. The influence of varying temperature and meteorological conditions may be determined from repeated photography, if the existing conditions are registered simultaneously.

Checks of the instrumental devices for the correction of the systematic deformations of the pencils of rays can be performed in the same manner as the camera and photography have been checked. A number of such investigations have given very interesting results. It is quite obvious that the systematic errors of the instru-

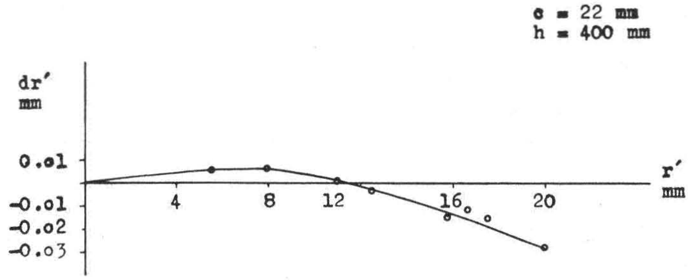

DIAGRAM 6.—Distortion curve of a Multiplex projector (Zeiss No. 201,199.)

ments of today are larger than is normally assumed.

The accuracy and reliability of photogrammetric procedures in general can doubtlessly become considerably increased when the systematic errors of the fundamental pencils of rays are determined. The same principles that have been used above can be applied to all kinds of cameras (microscopes, x-ray instruments, amateur cameras etc.) as well as to all kinds of projectors, [3]. In some cases more degrees of freedom can be used for the adjustment. In projectors with mechanical projection for instance, additional sources of errors as the "Breiten and Schiefen" errors (x-inclination and "latitude distortion") or the lacking intersection of axes can be numerically determined.

The practical treatment of single photogrammetric models and aerial triangulation strips with respect to known systematic errors in the reconstruction of the fundamental pencils of rays has, among other questions, been demonstrated in [4].

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A Counter-Weight for the Multiplex Printer

A. E. BATTISFORE,
U.S.N. Hydrographic Office

THE forty-odd pounds of the multiplex printer dome-assembly can seem like a hundred pounds at the end of a busy day. This is especially true when female photographers are using the printer and when several rolls of film have been used during the course of the day. In addition, valuable space must also be used upon which to place the dome-assembly while changing the rolls of film, and great care must be used when replacing the assembly to bring the precision-ground surfaces of the two parts of the printer together. To alleviate operator fatigue and insure correct alignment, a relatively simple and inexpensive device has been designed and is shown in the figure. To the U. S. Navy Hydrographic Office, the nautical approach, i.e. line, pulleys and counterweight, was a natural one.

The printer shown in the figure is centered $12\frac{1}{2}$ inches from the wall upon a platform 18 inches high. The "pail handle" attached to the head is made of 5/16″ cold rolled steel. The bracket is mounted on 2″×4″ wooden crossmembers which, in turn, are fastened to the wall. The lower 2″×4″ member is about eight feet from the floor and the upper one ten feet. The bracket itself is made of $\frac{1}{4}''\times1\frac{1}{2}''$ strap iron. It extends 12 inches from the wooden crossmembers and has two 10″ crosspieces to support the ball bearing pulleys. The pulleys are hung from universal action swivels.