Vertical Exaggeration and Perceptual Models

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ABSTRACT: *By computing the vertical exaggeration in the geometric stereoscopic model, it is shown that the geometric model and the perceptual model experienced by the observer are radically different. It is concluded that an experimental study to determine the metric of the perceptual model is required before accurate knowledge can be had as to subjective estimates of the model's characteristics.*

THE properties of the stereoscopic model have almost always been examined in photogrammetry by a geometric treatment. This has been satisfactory so long as we dealt with properties subject to physical measurement in the model, as with floating mark and scales. When subjective properties are to be treated, requiring measurement or characterization of the model by estimation, such as occur in interpretive applications, the geometric model is quite the wrong thing to use. Instead, a model termed here the *perceptual* model must be used. This is the model as perceived by the operator, and is quite different from the geometric model.

This has been illustrated by the volume of papers in the last few years dealing with vertical exaggeration. It so happens that the concept of vertical exaggeration is an ideal tool for demonstrating that there is a difference. Hence the balance of this paper is concerned with vertical exaggeration. The point, that there is a difference between the geometric and perceptual models, having been made, however, it is apparent that the difference will extend to many properties other than vertical exaggeration. The perceptual model is not characterized in this paper; unfortunately, not enough is known about it yet. An experimental investigation of the perceptual model is called for, and this should prove to be an interesting and fruitful field.

In photogrammetric practice there are several useful concepts of vertical exaggeration. Three which come to mind immediately are:

1. Gross vertical exaggeration. Ratio of vertical to horizontal scale for large objects (e.g., mountains) or for large distances in the space as a whole.
2. Local vertical exaggeration. Ratio of vertical to horizontal scale for small objects (e.g., buildings), or differential scales.
3. Slope exaggeration. Ratio of vertical angles, or of their tangents, or other functions.

Undoubtedly there are more concepts. Each of these concepts is useful and practical in some phase of photogrammetric practice. Under some conditions two or more concepts may be the same. For example, if the Jacobean (or the indicatrix) of the transformation from space to model is a constant, then gross and local vertical exaggeration are the same by any reasonable measure. But slope exaggeration may be different, and it may not be possible to find any suitable measure which would make it the same as the other two.

Because photogrammetry includes a large number of different applications and a variety of taking and viewing conditions which seldom result in a simple space-to-model transformation, we must expect no single formula for the general concept of vertical exaggeration, but instead a different formula for each specific concept. This, of course, should cause no difficulty, and is just what one might expect.

The trouble comes when one starts to be specific and to define measures for the distances, angles, etc., that enter into the general concept. There is just one case that presents no difficulty. This occurs when physical measurements in both the space (survey methods) and the model (floating mark with scales) are valid for the application. Most ordinary mapping applications, such as flight planning to get the desired scale and contouring interval, lie in this category. Here a straight-forward geometric computation may be made to compare the actual motion of the floating mark in the model with its corresponding fictional motion in real space, and the result will be valid for the purpose intended regardless of the appearance of the model to the operator.

Trouble arises when one or both of the spaces is not subject to physical measurement—that is, when one of the entities being considered is neither real space nor a geometric model of real space, but instead is a *subjective* space formed by the *perception* of an operator. Two examples of such applications are:

a) vertical exaggeration of culture or terrain in a stereoscopic model as compared with the actual view of an unaided observer at the same altitude as the pictures, such as may occur in certain navigation or intelligence problems,

b) estimation of slopes in a stereoscopic model, to compare with true ground slopes, such as may occur in geological studies.

In each case at least one of the spaces is perceptual, and the measurement made is subjective.

As of now the metric for perceptual spaces is unknown. We do know the following things about them:

1. The distance at which an object seems to lie in stereoscopic viewing is *not* given by the intersection of the principal rays from the eyes. This is simply common experience—witness the ability to fuse a model even under divergence.

More elegantly, it is known that convergence is only a rough clue to distance, is easily over-ridden by other clues, and is accepted to the exclusion of stronger forces only if extreme enough to be difficult. It does contribute to and sharpen other non-conflicting clues.

2. When a strange scene is presented visually, other sensory clues being excluded and closer examination (as by motion of the observer through the object space) prevented—all of which apply to photogrammetric viewing—the operator's perception is determined by his experience, and he tends to give the scene the most familiar interpretation, even though this may sometimes result in some intellectual (but not visual) absurdities.

Since no one has ever seen the earth from 20,000 feet with eyes 12,000 feet apart, it is questionable just what the perception may be. Probably it is different for different observers, but there may be enough common characteristics to derive, eventually, a "standard observer's" perceptual space.

3. Whatever the metric of a perceptual space may be—and there have been attempts to metrize such spaces—it is almost surely different from the Euclidean metric of the geometric model, especially if head motion is prevented, as in photogrammetry.

Witness the frontal plane horopters and similar phenomena. Items 2 and 3 above make it extremely unlikely that any theoretical studies will be able to characterize the subjective appearance of the model to the observer at present, for vertical exaggeration or for many other useful properties of models. What we need right now is carefully collected, controlled experimental data. We can then begin to learn the metric of perceptual space. This seems to be a proper and necessary activity for a field so dependent on the properties of vision.

4. Although we do not know the actual subjective metric, it appears from experiment that the perception is approximately invariant under changes of scene which cause all parallax angles to change by an additive constant, and leave polar angles unchanged.

The normal variables in photogrammetric viewing practice are viewing distance, photo separation and lateral position. Changes in the last two change parallax angles by an additive constant, approximately. Viewing distance does not. Changes in lateral position of photos which do not change the centering of the pair leave polar angles unchanged. Hence we should expect the perceptual model to be sensitive only to viewing distance and centering—but not, perhaps, completely indifferent to large changes of the other variables.

In order to show the marked difference between geometric and perceptual models the transformation between real space and the geometric model is needed. Obviously, according to what has been said, this model will *not* provide any information as to the subjective model exaggeration. The transformation is given for vertical photos only. It may be made general by inserting the tilt transformation but this is not necessary now. Details of derivation are omitted since this is not new, and the work can easily be checked by the reader.

Let two vertical photos with principal distances f be taken from stations

$$\left(\frac{b}{2}, 0, h\right) \quad \text{and} \quad \left(-\frac{b}{2}, 0, h\right)$$

in a space coordinate system. The image points, in the usual coordinate systems of photographs, of a space point (x, y, z) are:

$$\text{Left: } \frac{f}{h-z}\left(x+\frac{b}{2}, y\right)$$

$$\text{Right: } \frac{f}{h-z}\left(x-\frac{b}{2}, y\right)$$

Let these photographs be arranged in the $u-v$ plane of a coordinate system for the viewing space, with their centers at

Left: $(d, 0, 0)$ Right: $(c, 0, 0)$

and viewed with the eyes at

$$\left(\frac{e}{2}, 0, k\right) \quad \text{and} \quad \left(-\frac{e}{2}, 0, k\right).$$

The point of intersection of rays from the eyes through the photo images is

$$u = e\frac{fx + \left(\frac{d+c}{2}\right)(h-z)}{fb-(c-d-e)(h-z)}$$

$$v = e\frac{fy}{fb-(c-d-e)(h-z)} \qquad (1)$$

$$w = k\frac{fb-(c-d)(h-z)}{fb-(c-d-e)(h-z)}.$$

Of course, if the photos are presented to the eyes with a change of scale, the quantity f used in (1) must be the principal distance for the projection.

Equations (1) represent the geometric model. If a physical model were to be constructed using the transformation (1) and viewed from the specified eye-points, the visual coordinates of all points would be the same as in the binocularly viewed photographic images. This does *not* mean that (1) is a valid representation of the perceptual model. But we proceed to derive the scale properties of this model as if it were.

The horizontal scale for two points at the same elevation is

$$H = \frac{u_2-u_1}{x_2-x_1} = \frac{v_2-v_1}{y_2-y_1} = \frac{e}{bD}$$

where

$$D = 1 - \frac{(c-d-e)(h-z)}{bf}.$$

The vertical scale for two points at elevations z_1, z_2 is

$$V = \frac{w_2-w_1}{z_2-z_1} = \frac{ek}{bfD_1D_2},$$

D_1 and D_2 being the expression D at z_1 and z_2 respectively.

The gross vertical exaggeration is an expression of the general form V/H. Since H is a function of z unless $c-d-e=0$ (which means the principal points of the photos are separated by the *i. d.* of the operator—a condition seldom used) the question arises as to what H to use. Suppose we use some mean scale

$$H^* = \lambda H_1 + (1-\lambda)H_2 = \frac{e}{b}\,\frac{D_1+\lambda(D_2-D_1)}{D_1D_2}$$

Then the gross vertical exaggeration is

$$E = \frac{V}{H^*} = \frac{k}{f}\,\frac{1}{D_1+\lambda(D_2-D_1)}.$$

Evaluating the denominator we find that if

$$h = (\lambda z_2 + (1-\lambda)z_1) = \frac{bf}{c-d-e} \qquad (2)$$

then the denominator is zero and E is infinite.

But $c-d-e$ is the photo-base corresponding to stereoscopic viewing with zero convergence. In much usual photogrammetric practice (except multiplex and Kelch, for example) equation (2) is nearly satisfied, and it requires only a very slight adjustment of the print separation $c-d$ to cause it to be satisfied. Thus, (2) says

> *whatever mean horizontal scale is used to to define vertical exaggeration, adjustment of the print separation for zero convergence, at a corresponding mean altitude, causes vertical exaggeration, as defined in the geometric model, to become infinite.*

Since it is a matter of experience that

small adjustments of the photo separation do *not* cause large changes in apparent vertical exaggeration, and infinite vertical exaggeration is never observed, this is a proof that

the geometric model does not represent the perceptual model.

If for local vertical exaggeration E is defined as

$$\frac{\frac{\partial w}{\partial z}}{\frac{\partial u}{\partial x}} = \frac{\frac{\partial w}{\partial z}}{\frac{\partial v}{\partial y}}$$

the proof goes through exactly the same. A similar proof for slope exaggeration requires more manipulation but comes to the same result.

To show the point made in item 4 above, for models which are viewed with near zero convergence, a very good approximation to the parallax angle is

$$\frac{e}{k-w} = \frac{1}{k}\left(\frac{bf}{h-z} - c + d + e\right).$$

Thus changes in c, d and e change all parallax angles by the same amount. Changes in k cause proportional changes in parallax angles, under which the perceived model is not invariant.

The tangents of the polar angles are

$$\frac{u}{k-w} = \frac{1}{k}\left(\frac{fx}{h-z} + \frac{d+c}{2}\right)$$

$$\frac{y}{k-w} = \frac{1}{k}\,\frac{fy}{h-z}.$$

Hence the perceived model should be relatively insensitive to changes in e, and to changes in c and d which keep $d+c$ fixed.

NEWS NOTES

Relief Map of Seattle Area

A big, $6\times7\frac{1}{2}$ ft. relief map of the Seattle area will help officials of that city with their planning problems. The new map is a lightweight plastic model produced recently by Aero Service Corporation. Map scale is 1 inch $=\frac{1}{2}$mile. The map covers 1,656 square miles. Below 1,000 feet, elevations have a vertical exaggeration of 3:1, and above that height a vertical exaggeration of 2:1. Highest point is five inches above the lowest point on the relief map

The corporate limits of Seattle include some 88 square miles of land and 3 square miles of water surface. Its water supply and waste channels reach into the mountains and tides beyond the city limits. Therefore, when a new water supply line or power line is brought into the city, the problem is complex and difficult. Successful planning calls for a good visual presentation of the geographic areas and physical problems involved. The new relief map will help greatly to solve such problems. With it an engineer can demonstrate water flow problems by pouring water on a mountain divide and watching it flow down creek and river into Puget Sound.

The map was produced in 75 working days and is made of a durable non-inflammable plastic. Map information is shown in 4 colors. The surface of the map has been plastic coated to protect these inks. Weight of the map is only 65 pounds, including a rigid plywood mounting.

For more information write to Robert Sohngen, Aero Service Corp., Philadelphia 20.

Geophoto Announces Soils and Engineering Geology Section

Geophoto Services, Denver, Colorado, has recently expanded its services to include a soils and engineering section.

Geophoto was organized in 1946, and during the past decade has specialized in photogeologic evaluation and detailed surface mapping in the field of petroleum exploration. The Geophoto Group includes Geophoto Services for projects in the United States; Geophoto Services, Ltd., Calgary, Canada, for Canadian work; and Geophoto Explorations, Ltd. for foreign work. Combined staff is over 100 and includes geologists, professional engineers, draftsmen, and other technical personnel. Geophoto has operated in 15 countries besides the United States.

The new soils and engineering geology section is headed by Mr. James G. Johnstone, formerly Assistant Professor of Engineering Geology and Highway Engineering, Purdue University.

The new section will be concerned with numerous applications of air photo interpretation in soils and engineering studies such as highway, pipe-line and transmission line route locations, dam sites, industrial site selection, water supply problems, and general geological engineering.