*Vertical Aerotriangulation Adjustment Utilizing Electronic Computers**

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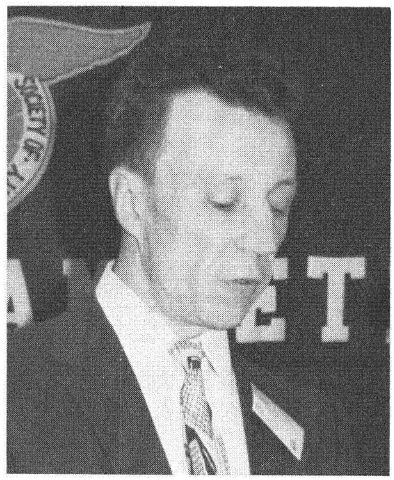
ABSTRACT: *For more than two years at the AMS, the UNIVAC has been used for the horizontal adjustment of aerotriangulation. It was decided that the use of a similar approach would be advantageous for the vertical adjustment. In this analytical method the coefficients of the terms in the general equation are automatically determined by the method of least squares. The corrections are then computed for the elevations on the photogrammetric passpoints and the final results are printed in tabular form. For comparison, this method has been tried on a project previously adjusted by graphical methods. The accuracy of the results obtained and the efficiency of the method justify its continued application.*

THE value of aerotriangulation is well-recognized as an integral part of photogrammetric mapping. This is particularly true in areas where it is uneconomical or virtually impossible to establish adequate geodetic control for the absolute orientation of individual stereoscopic models. As a result, search is constantly being made for methods to improve the accuracy and efficiency of spatial aerotriangulation. One attempt toward this objective at the Army Map Service is the automatic adjustment of photogrammetric extensions with an electronic computer.



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It has been acknowledged by the photogrammetrists and instrument manufacturers that maximum precision from first-order stereoplotting instruments can be obtained when the analog measurements from the instrument are recorded in digital form. Although it is highly desirable to utilize this precision in aerotriangulation, the task of adjusting these data from a photogrammetric extension is time-consuming and laborious due to the computations involved. A possible solution to this problem was realized when a Univac was purchased by the Army Map Service. After considerable experimenting, suitable equations were derived for the horizontal transformation and quadratic adjustment of a photogrammetric extension. A program was written and coded for the adaptation of the problem to automatic solution by means of the UNIVAC. Consequently, the horizontal adjustment for photogrammetric extensions has been performed in this manner for the past two years. The success of this procedure suggested the use of a similar approach to the vertical adjustment of the photogrammetric extension.

When stereoscopic models are successively oriented in a photogrammetric extension, systematic errors in elevation are accumulated due to earth curvature and the optical-mechanical system of the stereoplotting instrument. A plot of these

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vertical errors along any longitudinal section of the extension can be closely approximated by a parabolic curve. The difference in elevation error between two such parabolae drawn for two longitudinal sections of a given extension will vary linearly along the strip. In other words, this pattern of vertical error can be described by the general equation of a ruled surface, if earth curvature is ignored in the transverse direction. The surface is expressed with the difference in elevation between geodetic elevation converted to the scale of the photogrammetric extension and the instrument z reading, as a function of the instrument x and y coordinates. The general equation of the surface is of the form $\Delta h = Ax^2 + Bxy + Cx + Dy + E$. This analytical approach to the vertical adjustment to a stereophotogrammetric extension has been previously discussed in various papers on the subject of spatial aerotriangulation; however, the method is usually regarded as too time-consuming to justify its use, when compared with graphical methods.

While planning to program the vertical adjustment, it was decided to introduce an x^2y term into the general equation. It was assumed justifiable, since considerable experience with the graphical adjustment has indicated the presence of non-linear accumulation of tilt about the x axis in the successive transverse sections of a photogrammetric extension. Thus, the x^2y term would permit more flexibility in fitting the equation of the surface to the actual pattern of vertical error; however, the coefficient of this term in a specific solution must be noted with caution. If it were excessively large, it would indicate an error on one of the vertical control points used in the solution or an error in the instrumentation.

The programming of the vertical adjustment was the responsibility of personnel of the Mathematical Computation Branch of the Geodetic Division. Through their cooperation with the Photogrammetric Division considerable assistance has also been given toward improving the efficiency and application of the routine. In general, the vertical adjustment routine consists of the following: The true elevations are converted to millimeters at the average scale of the extension. When the amount of relief or the length of the extension requires correcting for the change in scale from one end of the strip to the other, the instrument z coordinates for all geodetic and photogrammetric control points are corrected to their equivalent values at the scale or the starting end of the strip. A least squares solution is automatically computed to the best mean fit of the vertical control, whenever the vertical solution of a strip is based on more than six control points. The black solution is applied to the instrument z coordinates of the geodetic and photogrammetric control points, and the adjusted elevations are converted to feet or meters, as required.

The results, as furnished to the Triangulation Branch of the Photogrammetric Division, consist of a tabulation of the designation and instrument coordinates on all geodetic control and passpoints, the adjusted elevations in feet or meters and the residual errors on the control points. The title block of the tabulation includes the coefficients for the terms of the equation. This information is useful in analyzing the solution of the strip and for computing by hand the adjustment to a passpoint which was excluded or erroneously recorded among the raw data furnished for adjustment.

A tolerance must be set within which the observed data will fit the assumed conditions on which the mathematical solution is based. The tolerance presently used, within which the adjusted elevations on geodetic control points will agree with the true elevations on these points, is one-fifth of the contour interval. Geodetic control points which are not in sympathy with the rest of the control are frequently encountered. This lack of agreement usually results from misidentification of the point. Such points can be isolated by analyzing the residual errors on the control points after the initial solution. These control points must then be eliminated and a second forward solution computed; however, the speed and efficiency of the automatic computation make the reiteration practical.

The primary objective in resorting to automatic adjustment is to improve the efficiency of the operation without sacrificing the accuracy of the results. To draw a comparison between the graphical method and the automatic computation method of vertical adjustment, a project previously adjusted by conventional methods was readjusted vertically employing the use of the UNIVAC. The project consisted of five strips of photography, at a scale of 1:10,000, with each strip twenty-two models in length. There were four bands of control points, spaced approximately seven models apart. The bands are considered normal to the line of flight, with the control points located in the center of the side-lap between flights. Thus there were eight control points for each flight. The basis for comparing the accuracy of results was the degree of fit to the vertical control and the difference between independently adjusted elevations for passpoints common to adjacent strips. In the graphical adjustment of the project the correction curves were so constructed as to reduce the errors on the majority of the control points to zero. A mean of the resulting differences on passpoints common to adjacent strips was 0.82 meter. The computed adjustment resulted in residual errors on the control of less than one meter for four of the five strips. On the fifth strip the maximum residual error on a vertical control point was −1.8 meters; however, no vertical control on this strip could be proven to be in error. A mean of the resulting differences on passpoints common to adjacent strips was 0.97 meter. It was apparent from examining the pattern of residual errors that the greater deviation in fitting the assumed error surface to the actual error surface occurs in the longitudinal direction.

A comparison of the economy and efficiency of the two methods indicates the overwhelming superiority of automatic computation. The time and cost figures are based on a twenty-model extension. If the time rate is .8 hour per model and the hourly cost is $2.62, then the total time for the graphical adjustment of a 20-model extension is 16 hours and the cost is $42.00. On the other hand, the total time for the automatic adjustment including preparation of the data on magnetic tape, the computing time, and the printing of the adjusted data would require approximately thirty minutes. The cost would be $8.75. Thus the automatic adjustment method would cost about one-fifth as much as the graphical method.

The application of automatic computation for vertical adjustment is in its infancy at Army Map Service; however, if the results from future use of this method prove equally encouraging, it will undoubtedly be adopted as part of the standard procedure.

*A New Look at Lens Distortion**

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ABSTRACT: *A new approach to the interpretation of distortion is presented. A curve of distortion based on variations of focal length rather than radial image displacement is proposed. A simple method is described relating height errors in the stereoscopic model to lens distortion.*

WHAT does lens distortion mean to a photogrammetrist? What is his conception of the term "distortion," particularly distortion in the photograph? The dictionary defines distortion as a twisting out of shape. If this definition be applied to the distorted photograph, we would say that the images are twisted out of shape relative to the object. This may seem to be a simple definition; however, there are two basic concepts in the definition, namely: "shape" and "twisted." What is meant by the former? The dictionary says: "Shape is the quality of a thing depending on the relative positions of all the points in its outline or external

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