The Stereomodel, How It Is Formed and Deformed

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I. Introduction

Several articles have appeared lately concerning the estimation of the vertical exaggeration[2] of stereomodels. The published quantitative approximations are either based on geometric-mathematical assumptions, which are only of formulistic value, because they do not correspond whatsoever to physical reality;[3] or they are in complete disagreement with well known facts established by experience.[4] It is the author's opinion that approximation formulas which are not founded on a basic understanding of stereoscopic vision, will not bring us much closer to the goal. It is essential to know how stereoscopic models are formed, especially those observed with parallel or divergent visual axes.

Any approximation of the vertical exaggeration factor, based upon a thesis that does not explain at least one of the most common ways of achieving stereoscopic vision and that is not in agreement with elementary optical laws, has to be considered with great distrust. In the following pages the author presents a thesis that can explain all possible cases of stereoscopic vision and that adheres closely to the results of modern research in optical physiology. This thesis will give a solid basis for approximating the vertical exaggeration of any stereomodel formed under any conditions.

[1] Permission for publishing this material was obtained from the Honorable Pedro Nel Rueda Uribe, former Minister of Mines and Petroleum, and Dr. Enrique Hubach, Director of the National Geologic Institute.

II. Natural Binocular Vision in Everyday Life

The author's thesis of stereoscopic vision of stereograms, to which vertical aerial photographs also belong, adheres closely to natural stereoscopy in everyday life. This field has been investigated quite thoroughly by optical physiology. The main results will be briefly summarized in this section, so that they may serve as a starting point for developing the author's thesis.

1. Fixation, Accommodation, and Their Link

When we look intently at an object (Figure 1), the visual axes will be directed towards this object, i.e. they meet at some point P of its surface, thus forming an angle γ. The author will call P the fixation point, γ the angle of convergence (of the visual axes) and E_m-P the distance of convergence. At the same instant the eyes will accommodate for the distance $el-P$, but for easy reference E_m-P will be called the accommodation distance.

Convergence and accommodation are coupled: as soon as we have fixed our eyes on a certain object, they accommodate correspondingly. Their association is ruled by the equation

$$\tfrac{1}{2}\tan\gamma = \frac{E}{2D} \tag{1}$$

when E is the eye base and D the distance of the object (E_m-P of Figure 1). The connection between the two functions of the eyes is not very rigid. According to von Tschermak-Seysenegg it may be summarized in the following way: "To every state of convergency belongs a certain

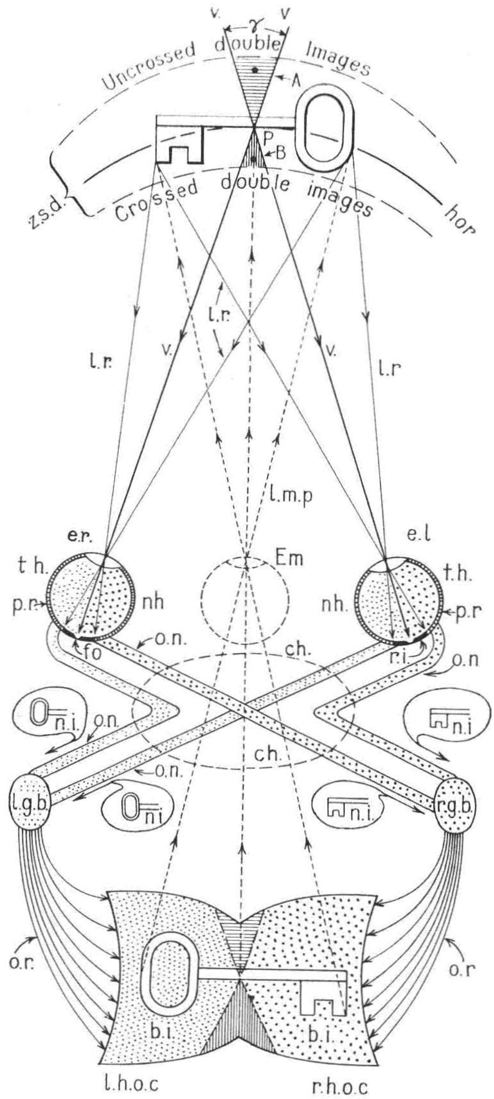


FIG. 1. Abbreviations: *b.i.*, brain image; *ch.*, chiasma; *e.l.* & *e.r.*, eye lenses of left and right eye; E_m, mental or cyclopean eye; *e.r.*, see *e.l.*, *fo.*, fovea; *hor.*, horopter; *l.g.b.* & *r.g.b.*, left and right geniculate bodies; *l.h.o.c.* & *r.h.o.c.*, left and right halves of the occipital cortex (brain center for vision); *l.m.p.*, imaginary lines (dashed) of mentally projected image; *l.r.*, light rays; *n.h.*, nasal hemispheres of retinas; *n.i.*, nerve impressions, travelling from retinas trough primary centers of vision (geniculate bodies) to brain center for vision; *o.n.*, optical nerves; *o.r.*, optic radiation; *P*, fixation point; *p.r.*, photosensitive retinas; *r.g.b.*, see *l.g.b.*; *r.i.*, retinal images of the observed object (in black); *r.h.o.c.*, see *l.h.o.c.*; *t.h.*, temporal halves (hemispheres) of retinas; *v.*, visual axes, forming an angle γ; *z.s.d.*, zone of stereoscopic depth.

range of accommodation possibilities, and for every state of accommodation we can vary the convergency angle only within certain limits."

The author, who carried out experiments concerning this connection, came to the same conclusions.[5]

2. THE BRAIN IMAGE

By accommodating, the eye lenses obtain the proper defraction power for converging the pencils of divergent rays, emitted by each material point of the object (for this detail see Figure 7*c* and *d*), exactly on the photosensitive cones of the retinas (*pr* of Figure 1) where a point for point (optical) projection of the object, or retinal image (*ri*) is formed. Every cone, stimulated by the light of a fraction of the retinal image, will send a corresponding "nerve impression" (*ni*) through its optic nerve fibre to a certain cell (or group of cells forming a unit) in the brain center for vision (striate area of the occipital cortex).

By the combined activity of all cones, a point for point physiologic projection of the retinal images is formed on the brain center for vision; the projection will be referred to as "brain image" (*bi*).[6]

3. PSYCHICAL EXTERIORIZATION OF THE BRAIN IMAGE

It is a fact, which can be deduced from many observations, that the brain image is projected outside as soon as it is formed. This projection is purely mental or psychical, as nothing material or physical is exteriorated out of the brain. The mentally projected image is automatically provoked by the light rays that reach our retinas and induce the brain image.

If previous fusion of the brain images has occurred, the mentally projected image has three-dimensional qualities, and we may speak of it as a "model." Without previous fusion two flat, eventually superimposed, images are projected outside by the mind.

4. THE PROJECTION THEORY

The question now arises: What is the geometric relation between reality and the mentally projected plastic model?

Hatched areas of occipital cortex and field of vision (with points *A* and *B*): fusion is obtained by two brain-halves.

The projection theory[7] claims to establish this relation. According to this theory we have to reproject the retinal images outside along straight lines through the nodal points of the eyes; that is, along the same lines but in opposite directions from those traveled by the principle light rays when they cast the images on the retinas. At the intersection of a pair of corresponding straight lines, each one coming from a different eye, the mental projection of the observed detail should lie. As we have to reproject the complete retinal images, the two pencils of lines from both eyes describe the complete mentally projected model.

The projection theory actually assumes that this model is geometrically identical to the real, observed object. This is in agreement with what most people believe, i.e. that we receive a "true" picture of reality.

5. FLAWS IN THE PROJECTION THEORY

It should be emphasized that, in a certain sense, we do not "see" the outside physical world at all, but only our mentally projected space impressions. That these two generally coincide makes the foregoing statement none the less true. However, this coincidence is not always perfect and considerable differences may occur between the physical world and our mental projection, or the one which is obtained with the geometric constructions of the projection theory.

The mental image is projected to a wrong spot if we look into mirrors, through prisms or other defracting media, as happens when a stick is partly submerged in water. The projection theory and mental projection in such cases give us a wrong impression, which can only be eliminated by experience and reasoning.

It can also happen that the mental projection differs both from the physical world and the result of the projection theory. This can be determined by the following experiments: Fix the eyes on a distant object and hold a finger some 10 inches in front of the nose; two images of the finger will then be observed. Now fix them on the finger, held at the same place; distant objects (a window pane, etc.) will be seen double.

In these two experiments we obviously project our images outside in a way which does not correspond to reality.

6. DOUBLE VISION—CORRESPONDING AND DISPARATE RETINAL POINTS

The phenomenon of double images is adequately explained by the thesis of corresponding and non-corresponding (or disparate) retinal points, which is explained in several textbooks on optical physiology. A few indications as to how the thesis works out are therefore sufficient, but at the same time necessary, because of its importance in understanding stereoscopic vision of stereograms.

A certain foveal cone of the nasal half of the left eye and a certain foveal cone of the temporal half of the right eye are connected, by their individual nerve fibres, with the same individual brain cell (or group of brain cells forming a unit) in the right half of the occipital cortex. They are corresponding retinal cones or points. Similarly there are many pairs of corresponding retinal points in the left and right eye.[8]

Experiments have demonstrated that a given small object, projecting images on corresponding retinal points, is observed as such (single), but a double image is experienced if light rays cast it on disparate points. Our eyes appear to be built in such a way that corresponding retinal points are at (approximately) the same angular distance from the visual lines. This conclusion can be reached because every point lying (approximately) on a circle (the horopter) through the nodal points of the eyes and the fixation point P, is visually experienced as a single point, whereas points outside or inside the horopter are observed as double images (Figure 2).

To reconstruct our visual impressions geometrically, the retinas are thought to be shifted to a place halfway between the eyes, after rotating them half the convergency angle ($\frac{1}{2}\gamma$), Figure 2 *b*, *c*, *d* and *e*. Corresponding points of the retinas will then cover each other (Figure 2*c*), but not so the disparate points (Figure 2*e*). The two retinal images of the transferred eyes should now be projected outside along straight lines through the common nodal point. This geometric operation corresponds to the mental projection, and the combined eye in the middle of the forehead may be referred to as the *cyclopean* or *mental eye* (E_m). When the construction is carried out, it will be found that all points

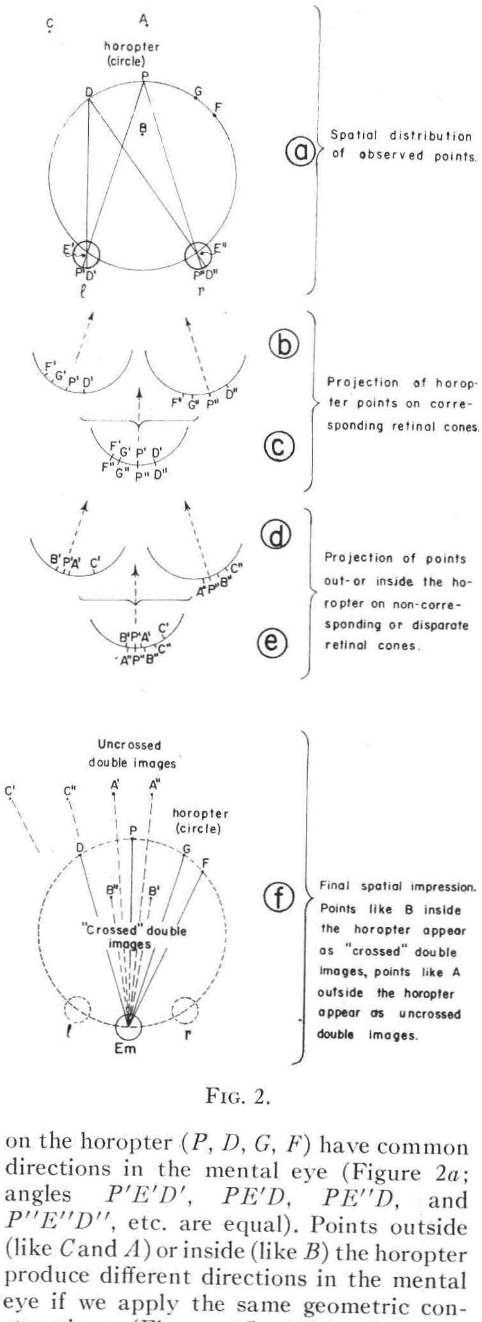


FIG. 2.

on the horopter (P, D, G, F) have common directions in the mental eye (Figure 2*a*; angles $P'E'D'$, $PE'D$, $PE''D$, and $P''E''D''$, etc. are equal). Points outside (like C and A) or inside (like B) the horopter produce different directions in the mental eye if we apply the same geometric constructions (Figure 2*f*). With this construction we can explain that objects inside the horopter are observed as "crossed" double images, i.e. the image observed with the right eye is mentally projected opposite the left one and vice versa. B will be seen as a single image when the fixation point P is moved nearer.[9]

7. AREA OF PANUM, PANUM ACTIVITY, FUSIONS AND PLASTIC VISION

The thesis of corresponding retinal points needs some corrections. It is not completely correct that any amount of retinal disparateness is experienced as a double image. Although only a certain foveal cone K' of the left eye is anatomically related to the corresponding cone K'' of the right eye, the brain is also capable of establishing relations between K' and other retinal cones lying in a small area around K''. This area, called *area of Panum* by von Tschermak-Seysenegg, has a diameter of 7 minutes of arc approximately (measured from the nodal points of the eyes), corresponding more or less to 7 cones in the foveal region.

The author suggests the term *Panum's activity* to refer to the mind's *active fusion* of disparate fractions of the brain images, to distinguish it unambiguously from the *passive fusion* of corresponding visual impressions or from simple *superposition* when two essentially different images enter the eyes.[10]

The activity of Panum or active fusion is the basis for stereoscopic or "plastic" vision; it is the mind's three-dimensional interpretation of a limited amount of disparateness in the retinal and brain images.

8. CORRECTNESS OF THE PROJECTION THEORY FOR A LIMITED AREA; WITH SOME RESTRICTIONS FOR THE WHOLE VISUAL FIELD: IMMEDIATE AND INTELLECTUAL SPACE IMPRESSIONS.

The areas of Panum define the depth of the zone of stereoscopic vision for a certain fixation point. With elementary geometry it can be determined that the zone of stereoscopic vision extends from 2.78 *m* to 3.30 *m* when the fixation point occurs to be at 3 meters.

Within this zone we may apply the projection theory, but outside of the stereoscopic area we must complete the picture with constructions based on the thesis of corresponding retinal points (Figure 2*f*). With both we can approximate geometrically what we actually see.

These constructions, combining the two theses, correspond to the *momentaneous* or *immediate* visual space impression. In a practical case we will move our visual axes several times, i.e. move the fixation point P back and forth over the landscape or observed object. By doing so we join into one spatial continuum various consecutive zones of stereoscopic depth, as the fixation point travels through space in a series of jumps. This final space impression which we so obtain, is in reality a *construction* of the intellect and therefore we may call it *intellectual* space impression. This intellectual space impression is identical with the one we obtain from the projection theory.

9. WHY DO WE PROJECT THE MENTAL IMAGE AT THE CORRECT DISTANCE?

It is of itself not obvious that we project our mental image to the right place, that is, where the object is generally situated in reality. After some thought it becomes clear that this phenomenon of the localization of the mentally projected image, is a different one from that of plastic vision which is based upon a limited disparateness of the retinal images.

As we always direct our visual axes to the object of interest, the latter will invariably be projected on the same areas of the retinas and consequently on the same brain cells. Therefore we cannot attribute special telemetric qualities to the brain cells, and it is apparent that they cannot account for the proper localization of the mental image.

Optical physiology generally connects mental projection with the degree of tension in the muscles that converge the eyes (internal recti). As the internal recti have to overcome the pull of the external recti, which tend to diverge the visual axes, a greater muscular tension is present when the eyes are converged for a near object, than is the case for a distant object. This tension would work as a telemetric regulator, defining the correct distance for the mental projection of the brain images.[11]

10. CONCLUSIONS

To explain natural binocular vision, optical physiology has built up a rather flawless system, which is very suitable for geometric treatment. What should now be investigated is how this system works for unnatural binocular vision of, for instance, stereograms, and how it has to be complemented.

III. Unnatural Binocular Vision of Stereograms

11. THE THESIS OF WHEATSTONE. UNNATURAL STEREOSCOPIC VISION CANNOT BE EXPLAINED WITH PRESENT KNOWLEDGE OF OPTICAL PHYSIOLOGY

It is obvious to assume, first of all, that the localization of the mentally projected plastic image, obtained with stereograms, occurs according to the telemetric principle of the internal recti, which shall be referred to as *convergency principle*, or briefly as *CP*. If dominated by this principle, the image is expected to lie at the intersection of the visual axes. Such an opinion seems to be rather old, because von Frijtag Drabbe (op. cit.) attributes it to Ch. Wheatstone (1833) and speaks of the *thesis of Wheatstone.*

Wheatstone's thesis, however, cannot be correct, as it cannot explain stereoscopic vision with parallel and divergent visual axes. This was already observed by von Frijtag Drabbe, Goodale, as published in their papers, quoted before, and further by Beltman (see note[12]). It led the first named author to present a complicated thesis, which cannot withstand the test of critical analysis and the second to a purely formulistic-mathematical approach (Treece: "pseudo-geometric development") of the vertical exaggeration ratio.

Treece, apparently not satisfied with such an approach, which does not reflect any physical reality, fell back again on the old thesis of Wheatstone. It seems that Walter Treece was more intent on finding a logical, natural base for developing his formulas, than to observed facts. He consequently ignores stereoscopic vision with parallel and divergent visual axes, and even goes so far as to say that it is impossible to see the photographs clearly when they are held in the focal plane of the stereoscope lenses because no virtual image would be formed (by these lenses)![12]

However, observed facts cannot be denied! Here we have reached the heart of the problem, the stumblestone in explaining un-natural stereoscopic vision.

12. THE WAY OUT: ADDITIONAL TELEMETRIC PRINCIPLES.

The author believes that the results of optical physiology, as presented in the previous section, are, at least basically, correct. However, little effort has been given, until now, to explain un-natural vision of stereograms and this may be the reason that a few image-localizing principles have been overlooked. The author wants to present additionally the following two:

The *accomodation principle* (briefly *AP*:) when accommodating for a near object, the ciliary muscle has to contract. Thus, similarly to a muscular feeling for convergence, we have another for accommodation.

The principle of the *magnifying glass experience or MGP*. When we observe an object, for instance a finger, under a magnifying glass, it will of course appear enlarged, but apart from this, we will have a definite impression of its location in space. The remarkable fact is that we see the finger more or less at the real distance from the eye, that is, slightly farther than f, the focal length of the lens, supposing that we hold the latter close to the eye and the finger in its focal plane. In this case the eye lenses will accommodate for infinity, but instead of seeing the finger infinitely far away, it appears to be very near. The principle of the magnifying glass experience is therefore independent of the accommodation principle. It is also independent of lenses and still present when we see with naked eyes. That the phenomenon exists is obvious, but the author did not find any reference to it in literature, nor can he give further indications to explain it.

Summarizing, he thinks that we have three telemetric principles, one binocular, *CP*, and two monocular (Figure 3, left half), *AP* and *MGP*.[13]

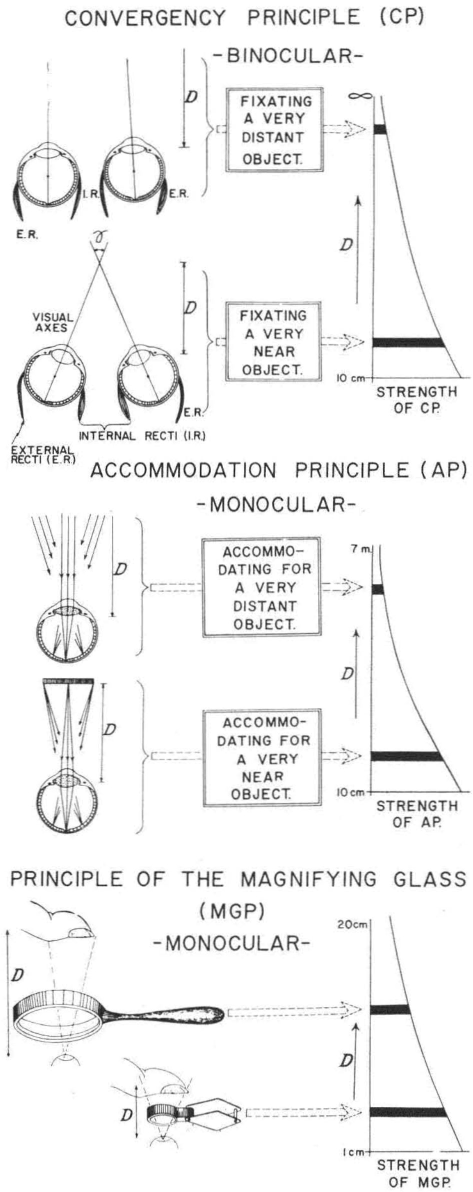


FIG. 3.

13. PHYSIOLOGIC STRENGTH OF THE PRINCIPLES

According to the author, the place of the mentally projected image is determined *through competition of the three principles.* It is therefore important to know their relative (physiologic) strengths, which, in the first two cases, are thought to be proportional to the muscular tensions.

It has been mentioned before that the muscular tension is greater when converging for near objects. The same holds for accommodation because of the following reasoning: When accommodating for a near object the ring-shaped ciliary muscle contracts; the diameter of the muscular ring is reduced; the zonular fibres, which attach the lens to the ciliary muscle, slacken and the eye lens takes a more spherical form because of the elasticity of the lens capsule. The contrary happens when we accommodate for a distant object: the muscular tension will be less.

The strength of the third principle

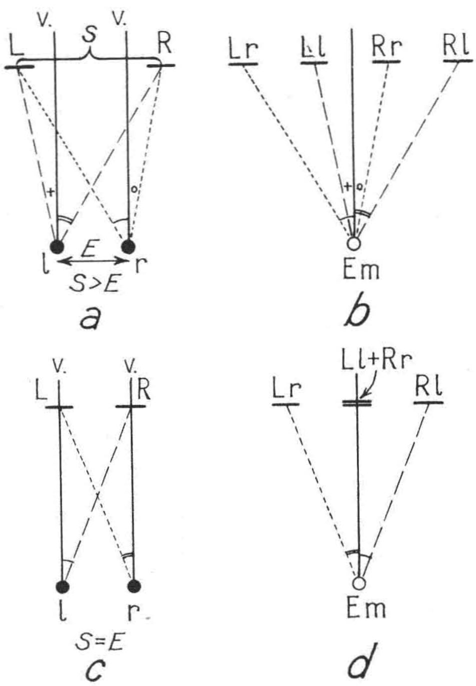


FIG. 4.

(*MGP*) is similarly supposed to be greater for "proximal" than for "distal" vision. There is no convincing reason for this assumption, but it seems to be confirmed in practice.

See Figure 3, right half.

14. VIEWING STEREOGRAMS WITH THE NAKED EYE.

A few examples will be presented to show how stereoscopic vision of stereograms is performed in specific cases.

Suppose that the visual axes are parallel and that the photographs have, at the beginning, a separation (S) which is larger than the eye base E (Figure 4*a*). Four images of the photos will then be observed (Figure 4 *b*) which can be explained with the construction of the thesis of corresponding retinal points (paragraph 6).[14]

The visual axes do not have to be revolved over half the convergency angle. They need only to be shifted to the center of the forehead (E_m, mental eye), but the angles between visual axes and principle light rays should not be altered by this operation. The four images *Lr*, *Ll*, *Rr*, and *Rl* have the following meaning: *Lr*, image of the left photograph (L) observed with the right eye (r); *Rl*, image of the right photo (R) observed with the left eye (l), etc.

When we shift the photographs together, the images *Ll* and *Rr* will approach each other and, finally, one will be seen superimposed on the other. The separation (S) between the photos is then equal to the eye base (E): corresponding details of each photograph will then be projected on corresponding retinal points of the eyes (Figure 4*c*, *d*). Accommodation will be adapted for the distance between photographs and eyes (let us say 50 cm.), passive, and eventually active fusion (Panum activity) will take place and then a plastic model will be observed.[15]

To find out where the model is lying, the following reasoning may be applied: *CP* suggests that the stereoscopic image is lying infinitely far away. The other two principles, *AP* and *MGP* however, will try to convince us that the plastic model is rather near (50 cm.). The latter support each other, but they are in conflict with the tendency of convergence. For this position of the visual axes, the muscular tension in the internal recti is rather weak and consequently also the strength of *CP*. The two other principles are, however, rather strong on account of the nearness of the photos. As a result of the competition between the three physiologic principles, the stereomodel will be projected between 50 cm. and infinity, but rather close to 50 cm., let us say at 2 meters.

When we increase the separation (S) of the photographs, the visual axes will become divergent, because corresponding points of the photos must continue to fall on corresponding retinal points. *CP* will then suggest that the stereo-model is, so to speak, even farther away than infinity.[16] But convergence and accommodation are coupled (paragraph 1), and under these conditions of extreme conflict between the two we are not able to maintain the accommodation for 50 cm. The eyes will then accommodate for let us say 100 cm. and as a result of all this the model will move farther away.

With increasing separation the conflict between convergence and accommodation can grow to such an extent that the latter is pulled so far away from its original value that unsharp images of the photographs are formed in the retinas. That is, the model becomes blurred. Finally, when we still increase the separation, our firm intention to see the model as sharp as before, will induce the ciliary muscles to accommodate for a shorter distance. Because of their link with the ciliary muscles, the internal recti will consequently be pulled together and diminish divergence. The result is that details of the stereograms will no longer fall on corresponding retinal points and hence stereoscopy is lost.

From these sketchy indications it will be obvious that stereoscopic vision of stereograms without the use of lenses, is based on a loosening of ties between convergency and accommodation.[17] Therefore the author called it "un-natural" binocular vision, because under natural conditions all three telemetric principles are in complete harmony: for instance, when we observe a real object at 50 cm. distance, all three principles will suggest that it lies at 50 cm. and as a consequence the mentally projected image will appear to be at 50 cm. This harmony has been disturbed when we look at stereograms and the "place" of the mental projection will be determined by the resultant of the three physiologic components.

When accommodation and convergence deviate appreciably from their natural association, defined by the equation 1) of paragraph 1, a certain "strain" is felt and we promptly become tired. This strain can be eliminated when we use lenses, or in other words: stereoscopes.

15. VIEWING STEREOGRAMS WITH THE AID OF STEREOSCOPES

Let us suppose that we observe the stereograms with a separation equal to the eye base and that they are moreover in the focal plane of the stereoscope lenses. Pencils of divergent light rays coming from points of the stereograms are then transferred by the stereoscope lenses into pencils of parallel rays which enter the eyes from various directions. The latter accommodate now for infinity. Both *AP* and *CP* suggest that the model is infinitely far away. The *MGP*, however, maintains that the image is in the focal plane of the lenses. This last named principle is very strong (because the photographs are near), much stronger than the combined physiologic forces of the other two. Thus, the stereomodel will be mentally projected between infinity and the focal plane of the lenses, but much closer to the latter.

When the separation of the photographs is increased, *CP* will pull the model farther away, as the tendency of the other two principles remain the same. Similarly the model moves closer to our eyes when the photo-separation is diminished.

Looking through a stereoscope with the visual axes parallel and accommodating for infinity, then *AP* and *CP* are in harmony. As far as it concerns these principles, it is as if we were scanning a distant landscape, and for this reason it is considered to be the most restful way of observation.

We can also examine the stereograms when they are closer than the focal distance of the lenses, as W. A. Treece considers the only and proper way of stereoscopic vision (K. Schwidefsky expresses the same opinion, *op. cit.*). Divergent rays of light then enter the eyes, which accommodate for the distance of the virtual images, distance (d_i) that can be determined with the lens formula mentioned in Treece's paper (p. 523). When the photographs are kept at such a separation that the visual axes intersect at the same distance d_i, then we imitate the natural vision of an object at a distance d_i ("proximal" vision). This too is an easy way of viewing, but not as comfortable as "distal" vision. But also when we accommodate for the distance d_i we may independently vary the convergency by enlarging or reducing the separation of the photos. The model will then move farther away or come nearer. However a certain discomfort for the eyes will appear.

16. THE PROJECTION THEORY FOR UN-NATURAL VISION. THE VIRTUAL FIXATION POINT

The projection theory is useless when the visual axes are parallel or divergent because no proper fixation point is present and therefore principal rays do not yield intersections. Nevertheless we do observe a distinct model at a certain distance. As a first step the author introduces here the use of a *virtual fixation point* and it is obvious to choose at this point some detail of the stereomodel upon which we have concentrated our attention. The introduction of the virtual fixation point has the following advantages:

1. *Difficult determinations of unknown*

physiologic factors and their mutual influences are avoided.

2. *By estimating the total effect of the three principles AP, CP and MGP, the relation of stereomodel and photographed original can be based on experimental data.*

By way of illustration the author presents schematically in Figure 5 the estimated distances of the virtual fixation points for stereoscopic vision of two stereograms observed with the naked eyes at a distance of D cm. Stereoscopic vision without lenses is so interesting, because we can pass through all principal cases of stereoscopic vision by gradually changing the separation S of the photographs from strongly negative to positive and greater than the eye base (E). Along the vertical direction of Figure 5 are plotted the distances of the virtual fixation points, F_v, i.e. the estimated distances between the eyes and a chosen point or reference plane of the stereomodel. The separation S is plotted horizontally. The 5 principal cases are:

1. *Stereoscopic vision with "crossed visual axes,"* "squinting vision," or briefly "X-vision." In this case the left photograph should be placed opposite the right eye and vice versa, to prevent seeing inverted relief. Therefore the separation is called negative. The visual axes intersect above the surface of the photographs, forming the letter "X" between eyes and photos.
2. *Anaglyphic vision.* This case cannot be realized with photographic copies, because they would cover each other completely, but, for instance, with printed anaglyphs observed through goggles. The separation here is zero, except for parallactic displacements. It will be referred to as Δ (delta)-vision.
3. *Stereoscopic vision with the visual axes intersecting behind the photoprints,* briefly "A-vision." The separation here is positive, but smaller than the eye base.
4. *Stereoscopic vision with parallel visual axes,* briefly "H-vision." The separation is equal to the eye base.
5. *Stereoscopic vision with divergent visual axes:* "V-vision." The separation is greater than the eye base.

In all cases, except the second, CP is in conflict with the remaining two principles which support each other mutually. This causes the stereomodel to be pulled away

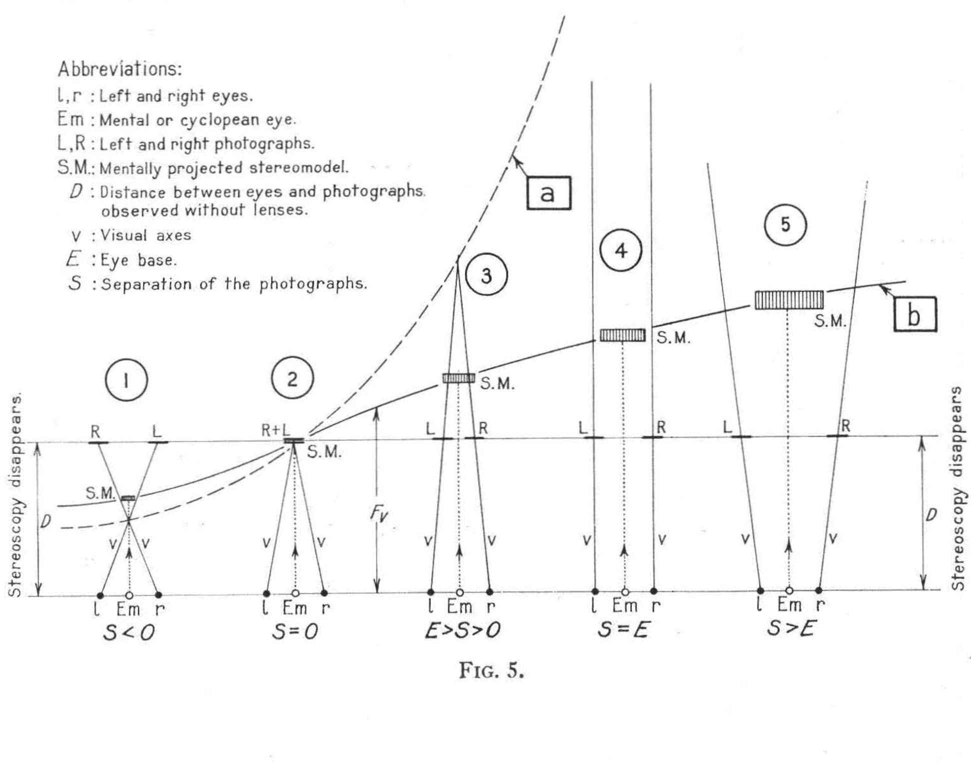


FIG. 5.

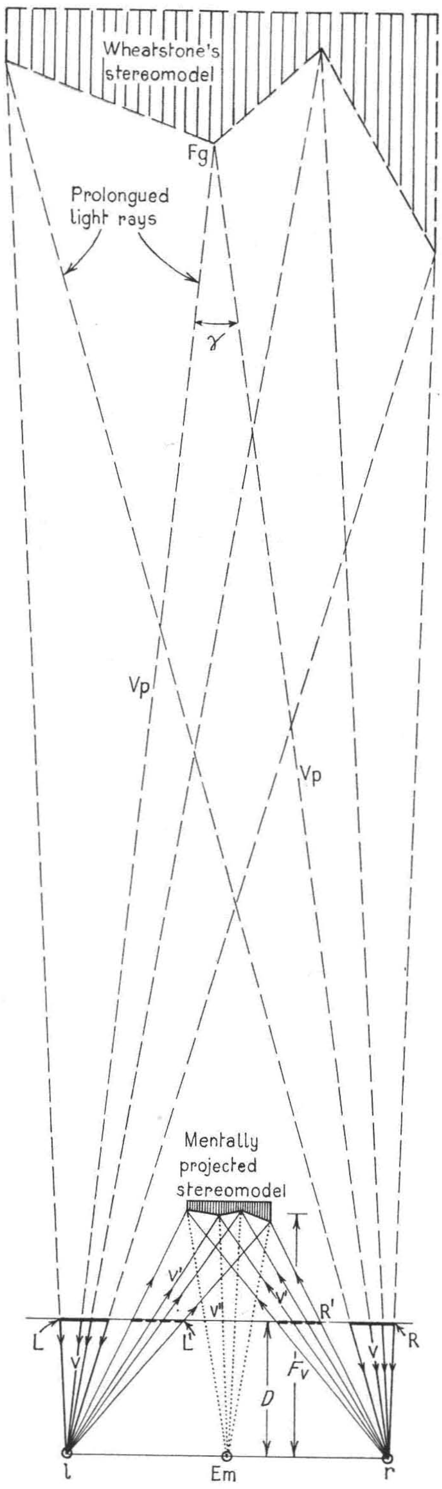


FIG. 6.

from the site of the photographs or stereograms and, by increasing separation, successively to occupy the positions indicated by line "*b*."[18] Only in the case of anaglyphic vision (2 above) does the model appear to be located in the plane of the anaglyphs (that is: some datum plane; high points are above and low points beneath it, depending on the way that the anaglyphs are printed). In this case all three principles are in harmony, and it could be qualified as natural vision by artificial means or pseudo-natural vision.

Curve "*a*" indicates the places where the visual axes intersect, or where the model should lie according to Wheatstone's thesis. This line approaches asymptotically a vertical line at $S=E$.

Now that, at least in principle, the distances of the virtual fixation points are defined, it remains to be indicated how the author thinks that the constructions of the projection theory should be carried out for these cases. See Figure 6. For this reconstruction the photos should be shifted from their actual viewing positions L and R, into the positions L' and R', in such a way that the lines uniting the observed detail with the nodal points of the eyes, intersect at the estimated virtual fixation point F_v. The visual axes V, the prolongations (V_p) of which meet in the fixation F_g (the example applies to A-vision; the construction for X, H and V vision is similar), are thought to be rotated to the positions V'. The visual line V'' represents the combined visual axes in the mental eye E_m. Rays of light travelling from the photographs into the eyes are drawn as heavy lines, their prolongations are interrupted. Thinly drawn are those lines used in the construction of the projection theory; dotted lines represent the imaginary routes of the mental projection.[19]

17. THE OPERATION OF SIMPLE LENSES

Considerable confusion exists concerning the effect of simple lenses upon the stereomodel. Formulas for the vertical exaggeration often contain a factor (m) for the magnifying power of the stereoscope lenses. This factor is generally defined by the following equations:

$$m = \frac{L_d}{4} = \frac{25}{f}$$

L_d being the refractive power of the lenses in diopters and $f(=100/L_d)$ the focal length of the lens in cm.[20] The use of these equations presupposes that the object is held in the focal plane of the lens, which should be thin and near to the eye. This is the *conventional magnification*. The size of the object, viewed enlarged with the lens, is compared with the size of the object seen without a lens at a distance of 25 cm.

Well then, for the case where we do not use lenses, simple geometric proportionality teaches us that the retinal image of an object observed at 100 cm., or at 5 cm., is one fourth the size of, respectively five times as big, as when seen at 25 cm, or in general $m=25/d$ for an object at d cm distance from the eye. Or, viewing through a lens, the object may be held at a distance d_0, which is smaller than the focal length f of the lens and we may accommodate for a virtual image (d_i) of 25 cm. The object distance d_0 may then be determined with the lens equation quoted by Treece

$$\left(\frac{1}{d_0}+\frac{1}{d_i}=\frac{1}{f}\right),$$

and the enlargement which is now bigger than the conventional, is accordingly $25/d_0$. Thus, the geometric proportionalities

$$\frac{25}{f},\frac{25}{d} \quad \text{or} \quad \frac{25}{d_0}$$

depend only upon the distance of the object from the eyes, whether we use lenses or not.

The fact is, however, that our eyes are not able to accommodate properly for distances shorter than 25 cm, and certainly not for so short a distance as 5 cm. Without the lens we would not see the object sharply. The function of the lens is apparently to ease accommodation or to increase our accommodation possibilities by artificially adding a few diopters to the refractive power of the eye lens.

We may wonder if this is the only function of a lens. A sketchmaster can give an answer to this question. This instrument is in essence a half transparent mirror (M, Figure 7, a), which permits one to see two scales S_1 and S_2 superimposed. A lens (L_e) introduced between the eye (O) and the second scale, enables one to observe the effect of the lens by comparison of the two scales, if the distances $O-S_1$ and $O-S_2$ are kept equal. The author used a German sketchmaster ("Luftbildumzeichner" of Zeiss Aerotopograph, of a construction very similar to Fairchild's "Rectoplanograph") that permitted him to vary the distances $O-S_1$ and $O-S_2$ independently. Moreover it has slits to hold the lenses. For different lenses that go with the instrument the author found the magnifications listed in Table I.

TABLE 1

Power of Lens in Diopters	Focal Length in Lens in Cm.	Magnification Factor	Distance O–Le in Percentages of f
+3	33.3	1.13×	16.5%
+4	25.0	1.18×	22.0%
+5	20.0	1.25×	27.5%
+6	16.7	1.30×	35.0%
+7	14.3	1.33×	38.0%
+8	12.5	1.37×	44.0%
+9	11.1	1.38×	50.0%
+10	10.0	1.39×	55.0%

It seems that the magnification factors are the same for different observers, but slight variations may be found for other kinds of lenses. Thus, apart from the conventional magnification, there is an additional one, which the author suggests be called *optical magnification* or *OM* for short. Stronger lenses have a greater *OM*.

The author also observed that the *OM* increases when the lens is removed from its slit and held farther away from the eye. This is proved by the observations given in Table II. The distance $(O-L)$ between eye and lens is expressed in percentages of the focal length of the lens.

TABLE II

LENS+3	$O-L$ *in % of* f	16.5%	27.0%	39.6%	46.2%
	magnification	1.12×	1.23×	1.30×	1.33×
	$O-L$ *in % of* f	22.0%		39.7%	
	magnification	1.18×		1.32×	

It is of more practical interest to know the OM values when the lens is held somewhat closer to the eye than the construction of the instrument allows. These values can only be approximated by extrapolation of the observed data. For this reason the author plotted the observations of Tables I and II in a graph (Figure 7b), which shows remarkably little spreading between the curves for different lenses (only three curves, for lenses of +3, +4 and +10 are drawn). The tendencies of the curves for lenses of +3 and +4 diopters suggest that these lines will pass through the origin of the graph.

This is a very logical result because it means that the OM would be 1 (that is no magnification at all) if the lens could be moved so close to the eye that there would be no separation between the two.

It may now be understood that the OM is introduced by the marginal parts of a lens, working as a wedge (Figure 7 c). Therefore the OM increases when the lens is held farther from the eyes, because then the outer parts are used. OM reduces when the lens is near to the eye, because light rays pass only through the central part.

In optics it is generally supposed that *thin* lenses are used *very near* to the eyes. Theoretically, the center of the lens should coincide with the nodal point of the eye as indicated by Figure 7 d and it should be infinitely thin. Only one ray, the principle ray (*pr. r*), of a pencil of diverging light rays (of which the conical angle θ depends upon the momentary aperture of the pupil) passes undeflected through the lens.

Thus, the OM exists only because we cannot comply with the theoretical suppositions: there is always some separation between eyes and lenses and the latter are not thin enough.

From the graph of Figure 7 b we may read that a rather strong lens of 10 diopters has an OM of only 1.1 × approx., if used at a distance of 1 cm., or at 10 per cent of its focal length from the eye. For most

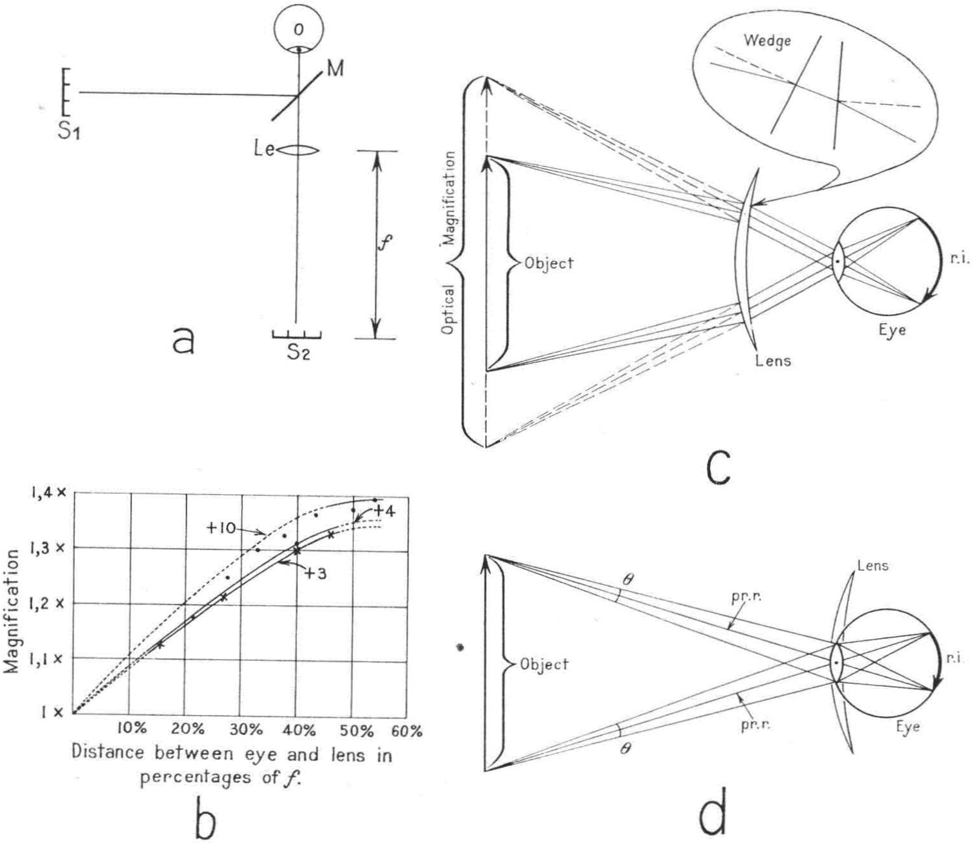


FIG. 7.

cases the OM can be disregarded completely.

The author's conclusion is therefore: In approximations of the vertical exaggeration we may introduce a small correction for the optical magnification, but the conventional magnification has no influence whatsoever upon the geometric pattern of principal rays of light. The power of the lens affects only the accommodation principle, and only in this way does it influence the distance of the virtual fixation point, and consequently the mentally projected stereomodel.

18. FORMULAS FOR THE HORIZONTAL AND VERTICAL SCALE AND THE VERTICAL EXAGGERATION OF THE STEREOMODEL

The author's formulas are a direct consequence of the projection theory adapted for un-natural stereoscopic vision and his concept about the working of lenses (paragraphs 16 and 17). They are deduced from Figure 8, which represents the right half of a drawing that is symmetrical along the line M-M.

Abbreviations used in Figure 8:

- SP: surface of the photographs.
- h: height of point P above datum plane in the hypothetical, reduced model.
- p: parallax of point P measured on the photographs.
- f: focal length of photography, nf should be introduced if the photographs are n-times enlarged.
- D: distance between eyes and photographs.
- E: eye base.
- m_0: optical magnification of the stereoscope lenses.
- F_v: estimated distance of virtual fixation point.
- h': height of point P above datum plane in the observed stereomodel.
- h_0, w, x: line symbols in intermediate phases of the equations.

Abbreviations used in the equations:

- S_v: vertical scale of the stereomodel
- S_h: horizontal scale.
- E_v: vertical exaggeration.

With similar triangles the following may be obtained:

$$h = \frac{pf}{b+p}, \qquad x = \frac{\frac{1}{2}m_0pF_v}{D},$$

$$w = \frac{\frac{1}{2}ED}{F_v}, \qquad h_0 = \frac{\frac{1}{2}m_0pD}{\frac{1}{2}m_0p + w}$$

introducing the value for w:

$$h_0 = \frac{m_0pDF_v}{m_0pF_v + ED}$$

$$h' = \frac{xh_0}{\frac{1}{2}m_0p},$$

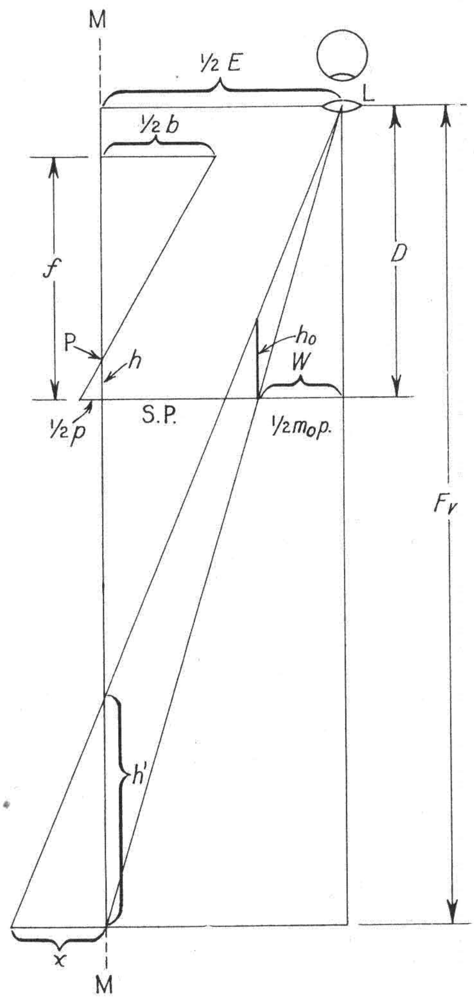


FIG. 8.

introducing the values for h_0 and x:

$$h' = \frac{m_0 p F_v^2}{m_0 p F_v + ED}$$

$$S_v = \frac{h'}{h} = \frac{m_0 F_v^2 (b+p)}{f(m_0 p F_v + ED)} \quad (2)$$

$$S_h = \frac{x}{p} = \frac{m_0 F_v}{D} \quad (3)$$

$$E_v = \frac{S_v}{S_h} = \frac{D F_v (b+p)}{f(m_0 p F_v + ED)}. \quad (4)$$

For small parallaxes, that is, near to the reference plane, we may write:

$$S_v = \frac{m_0 F_v^2 b}{EDf} \quad (5)$$

$$E_v = \frac{F_v b}{Ef}. \quad (6)$$

On a quick inspection it may appear strange that equation 6 does not contain the factor D and the separation between the photographs, which is also missing in the other equations. These factors are implicitly included in the estimated distance of the virtual fixation point (F_v).

Near the reference plane, to which F_v refers, the *OM* has no influence at all upon the vertical exaggeration, because it affects equally the vertical and the horizontal scale.

The values for S_v, S_h and E_v are with reference to the Hypothetical Reduced Model (*HRM*). This model is obtained by uniting corresponding points of each of the superimposed photographs with their focal points. The intersections of corresponding (projective) lines describe the *HRM* point for point. If the photographs are superimposed in the proper way, the scale of the *HRM* is $f:A$ (A = altitude of camera above reference plane) or equal to the scale of the photographs with respect to this reference plane. When the scale is for instance 1:20,000, S_v and S_h should be multiplied by 1:20,000 to obtain numerical scale values.

If anaglyphs are printed with the above mentioned "proper separation" ("standard anaglyphs") and if then the photobase happens to be equal to the eye base, and if moreover the eyes can be kept exactly above the center points of the photos at the focal distance f, then the observed anaglyphic model would be identical to the *HRM*. For this case of pseudo-natural vision without lenses $F_v = f = D$ and as $b = E$, $m_0 = 1$, all values for S_v, S_h and E_v would be 1.[21] Equation 4 suggests that the vertical exaggeration is not constant throughout the model, and more in particular, that it varies with increasing distance from the reference plane. This result is not so surprising because photographs of mountainous terrain are not of a uniform scale either. A geometric analysis of equations 2, 3 and 4, however, would not pay by reason of the following considerations.

The diameter of the areas of Panum indicates that we can appreciate stereoscopically only a retinal disparateness that corresponds on the photos to a parallax difference of approximately 0.2 mm for a pocket stereoscope and 0.55 for a mirror stereoscope like Fairchild's.

This means, that with these stereoscopes, we see in one single moment a zone of stereoscopic depth corresponding only to 12 m, respectively 33 m, if we use photographs of 9″ taken with a camera of 6″ focal length, at a scale of 1:40,000 and approx. 60 per cent forward overlap.[22] Scanning the whole model we constantly move our visual axes, at the same time changing the angle of convergence. In reality we make a three dimensional construction of the stereomodel adding new zones of stereoscopic depth to the previous one, very similar to the intellectual space impression described in paragraph 8. Every one of these zones has a different virtual fixation point and hence a different E_v.

Of greater importance is that we furthermore move the stereoscope over the photographs. This causes additional distortions.

19. DEFORMATION OF THE STEREOMODEL CAUSED BY THE CENTRAL PERSPECTIVE OF VERTICAL AERIAL PHOTOGRAPHS AND DISPLACEMENTS OF THE STEREOSCOPE

In the following deductions it is assumed that the bases of eyes, stereoscope and photographs always remain parallel when the stereoscope is displaced. This is common practice in the examination of vertical aerial photographs. In general, a vertical line ($V'V$, Figure 9a) is projected radially displaced on the photographs ($V'V_1$ & $V'V_2$). This is the reason that the vertical line ($V'V$) is observed inclined ($V'V_0$) in the stereomodel.[23]

When we look straight down into the stereoscope, that is when V_1 and V_2 are in the central part of the field of vision, then the inclined line $V'V_0$ will lie in a vertical plane which goes through the middle points of eye base and photobase. If each principal point (P_1' and P_2') of the photographs is a *perspective center for vertical lines*, then M' halfway on the photobase could be called the *perspective center* (of vertical lines) *for stereoscopic vision.*

In Figure 9a the following may be obtained with similar triangles:

$$y = \frac{hr}{f}$$

if $V'V_0' = y$; $VV' = h$; $V_0'M' = r'$ and $MM' = P_1P_1' = f$, the focal length of the camera.

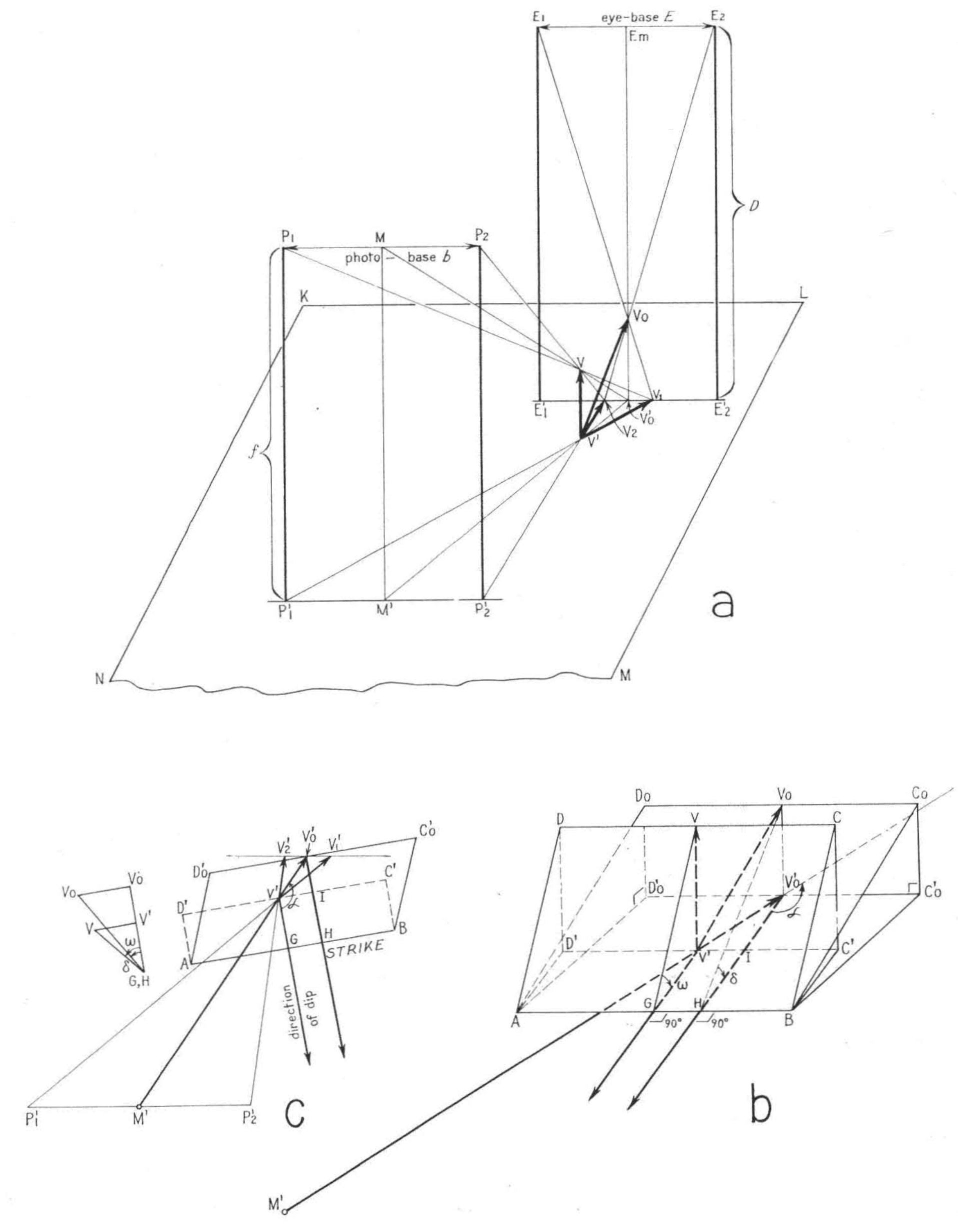


FIG. 9.

The deflection of the vertical causes a slope (Figure 9b) $ABCD$, including an angle ω (VGV', *o*mega = *o*riginal angle in terraine) with the horizontal datum plane, to appear inclined with another angle δ (slope ABC_0D_0; angle $V_0HV_0' =$ *d*elta = *d*eformed angle) in the stereomodel.

Apparently: $\tan\omega = (h/x)$, when $x = GV'$ and $h = VV'$.

From the perspective center of stereoscopic vision (M') a vector will be traced towards the observed slope, the *radial vector*. The *direction of dip* will be indicated by another vector, the *dip-vector*, which is perpendicular to the strike of sloping surface. The dip vector is not an ambiguous characteristic of the slope as, for instance, the strike. Let the angle between the two vectors be α(*a*lpha, a relative *az*imuth).

From Figure 9b it is deduced that:

$$\tan\delta = \frac{V_0V_0'}{HV_0'}$$

and from Figure 9b, and c:

$$HV_0' = HI + IV_0' = GV' + V'V_0' \cdot |\cos\alpha|.$$

Considering that α in Figure 9b and c is greater than 90 degrees and $\cos\alpha$ is negative:

$$HV_0' = GV' - V'V_0' \cdot \cos\alpha.$$

As V_0V_0', GV' and $V'V_0'$ in the stereomodel correspond to h, x and y in the HRM, considering the horizontal and vertical scale of the stereomodel with reference to the HRM and the values encountered for y and $\tan\omega$:

$$E_v \cdot \cot\delta = \cot\omega - \frac{r}{f} \cdot \cos\alpha. \qquad (7)$$

For point M' midway between the photo-centers, where $r = 0$, the equation is reduced to:

$$\tan\delta = E_h \cdot \tan\omega \qquad (8)$$

This is the conventional application of the vertical exaggeration ratio. The second term on the right side of equation 7, $-(r/f)\cos\alpha$, is apparently a correction which has to be applied when using the vertical exaggeration factor outside of the perspective center for stereoscopic vision. This correction is zero if $\alpha = 90$ degrees or $= 270$ degrees, that is when the strike of the slope passes through M'.

The correction factor has two extremes when $\alpha = 0°$ or $180°$. The estimated slope will be less steep than it is found to be with the vertical exaggeration formula 8 if the dipvector and the radial vector point in the same direction ($\alpha = 0°$); it will, however, be steeper when these vectors have exactly opposed directions ($\alpha = 180°$).

Similar corrections were described by R. F. Thurrell Jr, although it seems that Thurrell had only an experimental basis for his conclusions. He does not give a formula, nor does he indicate how to apply the corrections for any arbitrary angle α between dip-vector and radial vector. The quantitative values in his Table 2 are unfortunately erroneous, as can be proved without doubt by the following analysis.

It is assumed that a natural slope parallel to the photo-base has such an inclination (ω_0) that the prolonged slope will pass through the nodal points of the camera lenses in the positions in which the photographs were taken. All points on the surface of this slope will project into a single line, the strike line. Such a slope gives the impression of being vertical, whatever stereoscope is used, if employed according to the assumptions set forth in the beginning of this paragraph.

For dip-angles larger than ω_0, similar slopes parallel to the photobase, will appear overturned. Now, for the 8″ focal length photography, upon which Thurrell's table is based, and slopes at distances of 2″, 3″ and 4″ from the midpoint, the values of $\tan\omega_0$, in the same order, are

$$\frac{8.25}{2}, \frac{8.25}{3} \text{ and } \frac{8.25}{4}.$$

The corresponding angles ω_0 are 64°, 70°, and 76°. Hence the corrections for these angles are 26°, 20° and 14° and not 14.5°: 13° and 8.5° as can be read from Thurrell's Table 2. Consequently the accuracy of nearly all other values of this table is doubtful.

For an apparent angle (δ) of 90° in the stereomodel the author's formula 7° yields that $\tan\omega = f/r$, which is the correct value in this case in which the strike is parallel to the principal points ($\alpha = 0°$).

IV. Conclusions

E. R. Goodale, commenting on Treece's paper[24] makes the following statement: "In my opinion we have had enough theorizing. What is needed are good, sound experimentation and *proofs*."

Serious and rather extensive experiments were however made by Thurrell, but it was demonstrated in paragraph 19, that the results of his experiments, presented in his Table 2, are obviously incorrect.

Apparently it is not enough just to experiment, but a leading idea is necessary. As long as stereoscopic vision is not basically understood, experiments will not bring the desired results.

In this paper the author has interconnected all possibilities of stereoscopic vision. In fact the equations of paragraphs 19 and 20 apply equally well to X-vision[25], Δ-vision, A-vision, H-vision and V-vision, whether lenses are used or not. With slight adaptations they can also be used for 3D projections, stills or movies.

The strengths of the three physiologic principles can be determined only in a well equipped laboratory. However, for practical purposes it is only necessary to know their resultant, the virtual fixation point; this value has to be obtained by direct estimation, especially for H-vision and slightly convergent A-vision with a lens stereoscope. From figure 5 it is clear that the virtual fixation point moves away when the separation of the photos is increased. This is also the case when the prints are held at a greater distance from the eyes. The vertical exaggeration will then increase correspondingly.[26] As a consequence, vertical exaggeration is greater for long focal length stereoscopes than for those of short focal length and it may happen that E_v is smaller than 1 for certain pocket stereoscopes. By means of the latter small differences in altitude are appreciated more easily, because of the conventional magnification, but natural slopes seem less inclined than, for instance, when using the bigger stereoscopes. The influence of other factors, such as eye base, photo base and focal length of the aerial camera are defined by equation 6.

The reader may notice that the author's results are in agreement with those of Victor C. Miller's qualitative study, summarized on pp. 603 and 604, *op. cit.*

The author has refrained from giving any values for the distances of virtual fixation points, but in a future paper he intends to come back to this particular detail. He hopes that other photo-interpreters will contribute with their observations and that some discussion or exchange of results will be possible through PHOTOGRAMMETRIC ENGINEERING.

AUTHOR'S NOTES

[2] C. M. Aschenbrenner's *relief stretching* or *relief flattening*; cf. "A review of facts and terms concerning the stereoscopic effect," PHOTOGRAMMETRIC ENGINEERING, Vol. XVIII, No. 5, Dec. 1952, pp. 818–823. It seems to the author that "vertical exaggeration," an already popular term in photo-interpretation, will not cause confusion.

[3] Goodale, E. R., "An equation for approximating the vertical exaggeration in a stereoscopic view," PHOTOGRAMMETRIC ENGINEERING, Vol. XIX, No. 4, pp. 607–616, Sept. 1953.

[4] Treece, Walter A., "Estimation of vertical exaggeration in stereoscopic viewing of aerial photographs," PHOTOGRAMMETRIC ENGINEERING., Vol. XXI, No. 4, pp. 518–527, Sept. 1955.

[5] Page 174 of von Tschermak-Seysenegg's "Einführung in die physiologische Optik," Vienna, Springer, 2nd ed., 1947.

The author's Spanish paper, which deals more extensively with the problems related to stereoscopic vision, "Investigación de la vision estereoscópica," is published in "*Revista de la Academia Colombiana de Ciencias exactas, físicas y naturales*," number 36 & 37, a publication of "Ministerio de Educación Nacional" of Colombia.

See also the interesting paper of E. L. Rabben, "The Eyes have it," PHOTOGRAMMETRIC ENGINEERING, Vol. XXI, No. 4, Sept. 1955, p. 574.

[6] In Figure 1 the well known anatomic peculiarities of our optical nerves are presented. As each eye is divided by a vertical plane through the foveas (*fo.*) into two hemispheres, and as the optic nerves coming from cones of the nasal hemispheres (*n.h.*) cross in the chiasma (*ch.*), the right halves of the retinas (corresponding to the left half of the field of vision) are connected with the right half of the brain center for vision (*r.h.o.c.*). Left retinal halves corresponding to the right half of the field of vision, are similarly connected with the left half of the occipital cortex (*'l.h.o.c.*).

Fusion of those parts of the brain images coming from the right half of the object, takes place in the left half of the brain center and vice versa, but points like B, immediately before, or like A immediately behind, the fixation point P, will project on temporal and nasal retinal hemispheres respectively, and their fusion will take place instead by an activity between two brainhalves. This, as a matter of interest, may explain the special acuteness of stereoscopic vision in the direction of our attention.

[7] See von Tschermak-Seysenegg, *op. cit.*

[8] In the region surrounding the fovea (angular diameter 2° 25′), the cones are less densely distributed, alternating with rods (function: vision in darkness). Several cones are united

there by a common nerve fibre. All this causes the zone of acute stereoscopic vision to be limited, and to be dissolved laterally.

[9] C. A. J. von Frijtag Drabbe certainly would not have introduced his thesis of the "broadcasted image" and the "active or preferential eye" to explain this phenomenon, had he consulted the findings of optical physiology! cf. "Some new aspects in stereoscopic vision," *Photogrammetria*, VIII, No. 4 Special Congress Number, 1951–1952.

[10] This is for instance the case of von Frijtag Drabbe's "Experiment No. 4," his figure III, *op. cit.*, where the left eye receives an image of a coin and the right one the image of a drawing pencil. Both images, superimposed, are mentally projected outside. The pencil (p) seems to touch the coin in a certain part (x), determined by the thesis of corresponding retinal points (p and x).

[11] Some comparison could be made with stereotactile observations; that is, the recognition of form and three-dimensionality (sphere or ellipsoid, etc.) of objects by the touch, as with the fingers and the whole hand when the eyes are closed. The recognition and interpretation of form cannot depend simply upon the stimulation of cutaneous receptors, but also on the position, relative distance, etc. of the fingers, i.e. muscular tensional feelings are decisive complementary factors; cf. Sahli, H., "Lehrbuch der klinischen Untersuchungsmethoden für studierende und praktische Aerzte," Vol. III, Leipzig-Vienna, 7th ed., 1932, p. 189.

[12] Treece had to maintain this and also that the photographs should be viewed with a separation which is slightly less than the eye base, because otherwise his formulas would be reduced to absurdity. An opinion similar to Treece's can be found in K. Schwidefsky's "Grundriss der Photogrammetrie," 4th ed., V. F. W. Vielefeld, 1950, p. 41. Goodale, von Frijtag Drabbe and E. L. Rabben (*op. cit.*, p. 573) already found that photos under a stereoscope are generally observed with parallel visual axes. Most optical instruments are intentionally constructed in such a way that the object (or a real image) is in the focal plane of the (last) lens. Parallel *light rays* thus enter the eyes, which accommodate for infinity, the most restful manner of observation. This should not be confused with convergency, parallelism or divergency of the *visual eye axes!*, cf. Treece, *op. cit.*, p. 522, under 4: ?? "to see stereoscopically by divergent *rays*." ? and p. 523, middle of left column: "Goodale has the separation equal to the eye base, and hence he has all *rays of light* to the datum plane as non-converging." Goodale apparently referred to the visual eye-axes, but Treece, confusing one phenomenon with the other, speaks about rays of light, to which the eye-lenses react by accommodation and to which his lens formula on p. 523 applies.

[13] Other factors that play a role in the appreciation of distance, such as perspective, relative sizes, background blocking, increase of haze with distance, relation between objects and their shadows, subconscious pattern of double images, etc. are not considered by the author to be physiologic factors. They are supposed to be quick, subconscious deductions of the mind.

[14] To simplify the figure, non-stereoscopic parts of the photos are thought to be cut off.

[15] Active fusion, of course, only sets in if a certain disparateness is present, i.e. if the stereograms show parallactic displacements between several details.

[16] The author definitely assumes that the muscular tension of the internal recti is *not* zero when the visual axes are parallel; in this case the pull of internal and external recti is only equal. This means that the telemetric property of the internal recti still extends towards the region of divergence and, therefore, also here he continues to speak of the "convergency principle," considering divergence of the visual axes as a negative convergence.

[17] Cf. E. L. Rabben, *op. cit.* p. 575.

[18] R. A. Goodale speaks of a "plane of fusion." Fusion of course takes place in the brain. But apart from the wording, the author has the impression that Goodale also meant the place in space where the stereomodel is experienced, or more precisely the distance of the virtual fixation point. Thus, curve "b," using Goodale's phrasing, represents the distances of the "planes of fusion" for different cases of stereoscopic vision. C. A. J. von Frijtag Drabbe noticed for the case of A-vision without lenses (match experiment), that the stereomodel is lying between the fixation point and the photographs, but much closer to the latter.

The author is of the same opinion.

[19] The author based the projection theory for stereograms upon a *translation* of the photographs, because with this assumption a straight line perpendicular to the viewing direction remains straight. This line becomes curved and the stereomodel receives deformations which do not correspond to reality, if we base the projection theory on a pure rotation as R. A. Goodale has done. In the process of fusion of two stationary brain images, governed by physiologic tensions, there is nothing that suggests a rotation. The translation thesis is after all an approximation; more exact results may perhaps be obtained by a geometric-mathematical analysis of the *empirical horopter*; for the latter see v.T. Seysenegg, *op. cit.*

[20] Similar equations, for inches instead of cm, are used by Goodale, *op. cit.*, p. 612, and accepted by Treece.

[21] The same conclusion was reached by Victor C. Miller, see p. 593–594, "Some factors causing vertical exaggeration and slope distortion on aerial photographs." PHOTOGRAMMETRIC EN-

GINEERING, Vol. XIX, No. 4, Sept., 1953, pp. 592–607. The vertical exaggeration is only 1, if the mentally projected stereomodel is identical to the hypothetical, reduced model. It is *not* sufficient that principal rays of light enter into the eyes with the same angles as they did into the camera. In the case of the stereoscope with "natural depth impression" proposed by B. J. Beltman (Comments on "The interpretation of Tri-dimensional form from stereo pictures," PHOTOGRAMMETRIC ENGINEERING, Vol. XVIII, No. 5, Dec. 1952, pp. 823–825) the eyes accommodate for infinity. The visual axes will be convergent, parallel, or divergent, according to the value for the photobase. Only exceptionally will F_v be equal to f and E_v equal to 1.

From the working of the 3 telemetric principles one arrives at the conclusion that Beltman's stereoscope in most cases would not give the desired result.

Except for the scale, the *HRM* is equal to C. M. Aschenbrenner's *instrument model* if the photographs are perfectly vertical and the lens imperfections are negligible (*op. cit.*, 1952).

[22] Outside of this zone double images appear.

[23] This item is also discussed and illustrated by C. M. Aschenbrenner, "The interpretation of tridimensional form from stereo pictures," PHOTOGRAMMETRIC ENGINEERING, Vol. XVIII, No. 3, June 1952, pp. 469–472.

[24] Discussion of Paper by Walter A. Treece, PHOTOGRAMMETRIC ENGINEERING, Vol. XXI, No. 4, Sept. 1955, pp. 527–528.

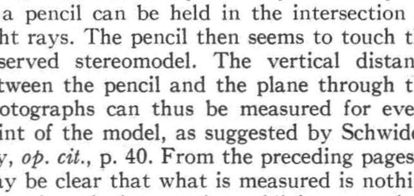
[25] *X*-vision has the peculiarity that the point of a pencil can be held in the intersection of light rays. The pencil then seems to touch the observed stereomodel. The vertical distance between the pencil and the plane through the photographs can thus be measured for every point of the model, as suggested by Schwidefsky, *op. cit.*, p. 40. From the preceding pages it may be clear that what is measured is nothing more than the intersections of light rays, where Wheatstone thought that the stereoscopic model would lie. This model lies, however, nearer to the photographs and is larger (in this particular case). The pencil point substitutes two floating dots, which always seem to touch the model if they are placed in corresponding light rays. With floating dots Aschenbrenner's instrument model (or the *HRM*) can be investigated, as well as the intersection of light rays (Wheatstone's phantom model), but never the really observed, mentally projected stereoscopic model. The latter can be studied by direct and subjective estimation.

[26] This is not in agreement with the paper of M. H. Salzman, "Note on stereoscopy," PHOTOGRAMMETRIC ENGINEERING, Vol. XVI, No. 3, June 1950, pp. 475–477. The author does not think, as Salzman seems to, that the eyes measure stereoscopic depth by the exact amount of retinal disparity, similarly as a thermometer indicates quantitatively the temperature by the displacements of the meniscus of the mercury column. Salzman's statement is in general true within one viewing position, when the distance D between eyes and photographs is kept constant; but it does not apply in comparing different viewing position with varying distances D.

The fact is that the mind interprets the rays of light as a *meaningful picture*, i.e. *as if* the light rays were coming from a real object.

Therefore the author thinks that the concept, represented by Salzman's Figure 1, is a more correct interpretation of parallactic displacements (and corresponding retinal disparity) than Salzman's too formalistic-mathematical conception, illustrated graphically by his other figures.

Parallactic displacements and retinal disparities of course increase when the photos are enlarged, but so does, to an equal degree, the stereoscopic model. Whether enlargement of the photos represents an increase or a decrease of E_v, can only be determined with the proposed constructions of the projection theory applied to the distance F_v, defined by the way the photographs are observed.



A NEW CAMERA MOUNT

A new heavy-duty, lightweight camera mount designed for precision adjustment of elevation and azimuth has been developed by Gordon Enterprises, camera manufacturing firm of North Hollywood, California.

An ingenious locking method allows the camera to be removed for service and later replaced in its exact former position without disturbing the boresighting. Designated the 80GE, the mount will take such heavy cameras as the Mitchell, Fastex, Maurer, Bell and Howell and heavy photoinstrumentation and data-recording cameras. First units of the new camera mount are being supplied to the National Advisory Committee for Aeronautics for use in mounting cameras in aircraft and in wind tunnels.