*Sources of Error in Various Methods of Airplane Camera Calibration**†

FRANCIS E. WASHER, *National Bureau of Standards, Washington, D.C.*

ABSTRACT: *Sources of error inherent in several methods used in the calibration of airplane-mapping cameras are discussed. The investigation is confined to errors affecting the measured values of focal length and distortion. Three photographic methods used in camera calibration are considered. Principal sources of error are errors in length measurements on the calibration negative, errors in angle separating targets in the object space, and errors arising out of departure from flatness in the photographic plates used in the calibration.*

1. INTRODUCTION

THE calibration of airplane mapping cameras (1, 2, 3) has occupied the attention of a number of organizations both here and abroad since the first development of aerial photography. As the airplanes and cameras have improved, the requirements placed on the cameras and the lenses with which they are equipped have become more stringent (4, 5, 6). To achieve the required accuracy of calibration, testing laboratories that are charged with the responsibility of certifying the accuracy of the camera calibrations have been steadily improving their techniques. The press of work and the need for speed have, in general, precluded close cooperation between the various laboratories. In consequence, many methods of camera calibration have been developed. In each instance, the method developed has been one that is capable of yielding the information required but these methods are not all equally simple or capable of handling the same volume of work.

In the United States and Canada, the emphasis has been on the photographic method of camera calibration while in Europe greater emphasis has been placed on visual processes. The photographic method has been employed in a variety of ways depending upon the location of the laboratory and the equipment readily available. Thus the National Bureau of Standards has developed the precision lens testing camera (7) and the camera calibrator (8). At Wright Field, the field method has been developed and perfected to a high degree by Sewell (9) and associates. Additional field methods that yield a high degree of accuracy and are suited to a variety of conditions have been developed by Merritt (10). Laboratory methods that yield the required information have been developed in Canada by Howlett (11) and Field (12).

Visual methods that employ a goniometer have been developed in Europe (13, 14, 15) and are described in the literature. In the United States, Merritt (16) has developed a successful goniometer method.

However divergent the approaches to the problem of precise camera calibration, the end results are necessarily the same because all are seeking the same type of information, namely accurate values of the scale factor to be used in map interpretation. Consequently, the factors affecting the final accuracy are much the same regardless of method. In the present paper, an analysis of several sources of error is given and the manner in which these errors affect the final accuracy is shown for several photographic methods.

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Particular emphasis is given to those errors that affect the determination of values of focal length and distortion. Detailed tabulations of actual data are shown in some instances in order to give a better understanding of the magnitudes involved.

2. Precision of Determination of Equivalent Focal Length and Distortion

The accuracy of the measured values of equivalent focal length and distortion for a given airplane camera lens is dependent upon several factors which may produce error. A prime source of variation in the measured value of the equivalent focal length itself is the actual selection of the preferred focal plane; this gives rise to definite measured differences in the equivalent focal length but has no appreciable effect on the distortion values. More serious and definite errors arise from inaccuracies present in the determination of the angles separating targets, and in the determination of distances on the negative from which the values of equivalent focal length and distortion are obtained. These errors may be systematic or accidental, but so far as this investigation is concerned only the accidental or random errors will be considered. Curvature or waviness of the registering surface also contributes to error in the values of equivalent focal length and distortion; this error is here treated as a random error although at times it may be regarded as a systematic error.

These factors affecting the variation of measured values of the equivalent focal length and the accuracy of determination of equivalent focal length and distortion are herein discussed as they apply to photographic methods of measurement. It is, however, apparent that these same factors may be present in visual methods and will affect the final accuracy of measurement in much the same manner.

2.1 Selection of the Focal Plane

When a lens is tested prior to mounting in a camera there is some uncertainty as to the location of the focal plane that will yield best average definition (17). Because of field curvature, the plane of best average definition seldom coincides with the plane of best axial focus. For a lens of aperture f/6.3, there exists a range of approximately 1.2 mm. along the axis wherein the resolving power will be not less than 20 lines per mm. For the same lens, the depth of focus along the axis may be as high as 1.8 mm. wherein the resolving power will not be less than 14 lines per mm. Consequently if the resolving power requirement is 20 lines per mm. on the axis and not less than 14 lines per mm. in other portions of the field, a range may exist of 0.6 to 0.8 mm. wherein the specified values of resolving power are obtained even if the curvature of field amounts to as much as 0.5 mm.

It is usually possible by careful examination of the test negative to locate the plane of best average definition within ± 0.1 mm. It must be remembered, however, that it is probable that the performance of the lens may satisfy the specified tolerances even if the selected focal plane may be separated as much as 0.4 mm. from the plane of best average definition.

It is this depth of focus that makes it possible for two laboratories to report values of the equivalent focal length for the same lens differing by as much as 0.2 to 0.3 mm. Both laboratories may be doing equally good work but may be using different criteria in locating the plane of best average definition. In order to judge whether or not the focal length determinations from two laboratories are consistent, values of both equivalent focal length and back focal length must be considered. The consistency may then be judged by comparing the arithmetic differences between corresponding equivalent and back focal lengths. This difference should show no appreciable variation.

This variation in the measured value of the equivalent focal length arising from choice of a focal plane cannot occur when the lens is mounted in an airplane camera having a fixed focal plane provided that the equivalent focal length is measured for the lens as mounted in the camera.

2.2 The Effect of Errors in R and β

a. Determination of the equivalent focal length.

The equivalent focal length is defined (18) by the equation

$$f = \lim_{\beta \to 0} r \cot \beta \tag{1}$$

where r is the transverse distance from the principal focus to the center of the image in the image-space focal plane of an infinitely distant object which lies in a direction making an angle β with the axis of the objective. The principal focus is defined as the axial image point of an object at infinity (2). For a lens free from distortion the value of f would be invariant with respect to the value of β. For many photographic purposes the distortion is negligible for points distant from the center of the useful field not more than one-fifth of its radius and within this region the above equation may, with sufficient accuracy, be written as

$$f = r \cot \beta. \tag{2}$$

It is evident that the accuracy of the value of f obtained from this equation is dependent on the accuracy of the measured angle, β. If the probable errors in r and β are known, the probable error in f can be determined (22). This can be done by differentiating the above equation which yields

$$df = dr \cot \beta - r \operatorname{cosec}^2 \beta d\beta. \tag{3}$$

It is accordingly clear that if Δf_r is the probable error* in f arising from the probable error Δr inherent in the measurement of r, then

$$\Delta f_r = \cot \beta \cdot \Delta r. \tag{4}$$

Similarly, if Δf_β is the probable error in f arising from the probable error $\Delta\beta$ inherent in the measurement of β, then

$$\Delta f_\beta = r \operatorname{cosec}^2{}_\beta \Delta\beta \tag{5}$$

which may be written

$$\Delta f_\beta = \frac{0.000004848}{\sin \beta \cos \beta} f\Delta_\beta = Bf\Delta\beta \tag{6}$$

for values of $\Delta\beta$ expressed in seconds. The total probable error in the value of f arising from probable errors in both r and β may be found from the relation

$$\Delta f = \pm \sqrt{(\Delta f_r)^2 + (\Delta f_\beta)^2}. \tag{7}$$

Values of the probable errors $\Delta\beta$ are usually obtained from the analysis of experimental data containing repeated measurements of r and β.

Values of $\cot \beta$ and B for those values of β frequently used in camera calibration are listed in the second columns of Tables 1 and 2 respectively. These values can be used to determine the probable errors Δf_r and Δf_β when the magnitudes of the probable errors Δr and $\Delta\beta$ are known. For convenience in computation, values of Δf_r are listed in Table 1 for a series of values of Δr. Similarly, values of Δf_β are shown in Table 2 for a series of values of $\Delta\beta$. The values of Δf_β in Table 2 are given for a lens having a focal length of 150 mm. since the value of Δf_β is also dependent on the focal length of the lens.

To illustrate the use of Tables 1 and 2 in evaluating the probable error in equivalent focal length, Table 3 shows the values of Δf arising from assumed specific values of Δr and $\Delta\beta$ for a lens having a focal length of 150 mm. It is clear from this table that for a distortion-free lens, the accuracy of an equivalent focal length determination increases with increasing angle β. The usual lens is, however, not free from distortion and consequently the magnitude of the angle β that may be used is limited. The presently accepted rule (18) is that the angle β, used in an equivalent focal length determination, should not be greater than the angle subtended at the lens by the axial image and a point one-fifth of the distance between the center and edge of the useful field. For a lens whose half-angular field is 45°, this limits the angle β to approximately 11.3°.

The foregoing discussion applies primarily to the equivalent focal length of a camera lens. In the case of precision cameras, the calibrated focal length is of greater importance to the user. Ordinarily, the calibrated focal length is thought of as adjusted value of the focal length calculated to minimize distortion over the entire field. It has been shown (8) that values closely approximating the calibrated focal length can be obtained by using a value of β in the determining equation of focal length that corresponds to a zero point of distortion computed with respect to the calibrated focal length. Such a zero point occurs when β has a value of approximately 40°. From Table 3, Δf is ± 0.007 mm. at $\beta = 40°$ as compared with $\Delta f = \pm 0.025$ mm. at $\beta = 10°$. It is clear, therefore, that the value assigned to the calibrated focal length for a given lens has appreciably less error than the value assigned to the equivalent focal length for

* Throughout this paper, the symbol Δ is used to designate probable error. Thus Δr is the probable error in the measurement of r; Δf_r is the probable error in focal length arising from a determination of f based on a value of r that has a probable error Δr. It is understood that all probable errors are plus or minus and that $\pm$ signs are often omitted herein.

TABLE 1

THE PROBABLE ERROR, Δf_r, IN FOCAL LENGTH ARISING FROM ERRORS IN r ALONE FOR VARIOUS ANGULAR SEPARATIONS (β) FROM THE AXIS FOR A SERIES OF VALUES OF Δr

These values are computed with the aid of eq. 2

β	cot β	Δf_r for values of Δr in millimeters of					
		0.001	0.002	0.003	0.004	0.005	0.006
degrees		mm.	mm.	mm.	mm.	mm.	mm.
5	11.430	0.011	0.023	0.034	0.046	0.057	0.069
7.5	7.596	.007	.015	.023	.030	.038	.046
10	5.671	.006	.011	.017	.023	.028	.034
12.5	4.511	.005	.009	.014	.018	.023	.027
15	3.732	.004	.007	.011	.015	.019	.022
20	2.747	.003	.005	.008	.011	.014	.016
22.5	2.414	.002	.005	.007	.010	.012	.014
25	2.145	.002	.004	.006	.009	.011	.013
30	1.732	.002	.003	.005	.007	.009	.010
35	1.428	.001	.003	.004	.006	.007	.009
37.5	1.303	.001	.003	.004	.005	.007	.008
40	1.192	.001	.002	.004	.005	.006	.007
45	1.000	.001	.002	.003	.004	.005	.006

the same conditions of test.

Recently mapping lenses having very low distortions are being supplied to various mapping organizations. The maximum distortion referred to the calibrated focal length frequently does not exceed ± 0.020 mm. For such lenses, the value of β used in computing the equivalent focal length may properly be as high as 20° with a resultant decrease in the value of Δf as compared with the value computed at $\beta = 10°$.

b. *Evaluation of the distortion*

Distortion D is defined by the equation (10)

$$D = r - f \tan \beta \tag{8}$$

where f is the equivalent focal length

TABLE 2

THE PROBABLE ERROR, Δf_β, IN FOCAL LENGTH ARISING FROM ERRORS IN β ALONE FOR VARIOUS ANGULAR SEPARATIONS (β) FROM THE AXIS FOR A SERIES OF VALUES OF Δ_β

These values are computed with the aid of eq. 6 and with the focal length, f, assumed to be 150 mm.

β	$B \times 10^6$	Δf_β for values of Δ_β in seconds of					
		1	2	3	4	5	6
degrees	1–sec	mm.	mm.	mm.	mm.	mm.	mm.
5	55.84	0.008	0.017	0.025	0.033	0.042	0.050
7.5	37.46	.006	.011	.017	.022	.028	.034
10	28.35	.004	.009	.013	.017	.022	.026
12.5	22.94	.003	.007	.010	.014	.017	.020
15	19.39	.003	.006	.009	.012	.014	.017
20	15.08	.002	.004	.007	.009	.011	.014
22.5	13.71	.002	.004	.006	.008	.010	.012
25	12.66	.002	.004	.006	.008	.010	.011
30	11.20	.002	.003	.005	.007	.008	.010
35	10.32	.002	.003	.005	.006	.008	.009
37.5	10.04	.002	.003	.004	.006	.008	.009
40	9.85	.001	.003	.004	.006	.007	.009
45	9.70	.001	.003	.004	.006	.007	.009

TABLE 3

THE PROBABLE ERROR, Δf, IN FOCAL LENGTH ARISING FROM ERRORS IN BOTH r AND β FOR VARIOUS ANGULAR SEPARATIONS FROM THE AXIS

These values of Δf are computed with the aid of eq. 7. For these computations, it is assumed that $\Delta r = \pm 0.002$ mm.; $\Delta\beta = \pm 5$ seconds; and $f = 150$ mm.

β	Δf_r	Δf_β	$\Delta f = \pm\sqrt{(\Delta f_r)^2 + (\Delta f_\beta)^2}$
degrees	mm.	mm.	mm.
5	±0.023	±0.042	±0.048
7.5	.015	.028	.032
10	.011	.022	.025
12.5	.009	.017	.019
15	.007	.014	.016
20	.005	.011	.012
22.5	.005	.010	.011
25	.004	.010	.011
30	.003	.008	.008
35	.003	.008	.008
37.5	.003	.008	.008
40	.002	.007	.007
45	.002	.007	.007

determined for a particular set of values of r and β. The probable error in distortion ΔD arising from the errors Δf. $\Delta\beta$, and Δf can be found by differentiating the above expression which yields

$$dD = dr - df \tan\beta - f \sec^2\beta\, d\beta. \qquad (9)$$

From which it is clear that if ΔD_r is the probable error in distortion arising from uncertainties in the measurement of r, then

$$\Delta D_r = \Delta r. \qquad (10)$$

Similarly, if ΔD_f is the error in distortion arising from errors in the value of f based on particular values of r and β, then

$$\Delta D_f = \tan\beta \Delta f. \qquad (11)$$

Likewise, if ΔD_β is the error in distortion dependent on uncertainties in the measurement of β, then

$$\Delta D_\beta = f \sec^2{}_\beta\, \Delta\beta \qquad (12)$$

$$= \frac{0.000004848}{\cos^2\beta} f\Delta\beta = Cf\Delta\beta. \qquad (13)$$

The probable error, ΔD, arising from all three sources is

$$\Delta D = \pm\sqrt{(\Delta D_r)^2 + (\Delta D_f)^2 + (\Delta D_\beta)^2}. \qquad (14)$$

Values of $\tan\beta$ and C for use in computing ΔD_f and ΔD_β are listed in Table 4. This table was used in the preparation of Tables 8, 10, 11, and 12.

TABLE 4

VALUES OF $\tan\beta$ AND C FOR VARIOUS ANGULAR SEPARATIONS (β) FROM THE AXIS

Values of $\tan\beta$ are for computing ΔD_f when using eq. 11 and the values of C are for computing ΔD_β when using eq. 13

β	$\tan\beta$	C
degrees	mm./mm.	mm./sec.
5	0.0875	0.000004894
7.5	.132	.000004933
10	.176	.000005000
12.5	.222	.000005086
15	.268	.000005194
20	.364	.000005489
22.5	.414	.000005678
25	.466	.000005906
30	.577	.000006461
35	.700	.000007222
37.5	.767	.000007706
40	.839	.000008261
45	1.000	.000009722

2.3 FLATNESS OF THE REGISTERING SURFACE

Appreciable errors in the values of the equivalent focal length and distortion can result if the glass negative upon which the images are registered is not truly flat (19, 20). Figure 1 illustrates one case of departure from flatness. The photographic plate is assumed to be concave toward the lens L with a radius of curvature R where R is very great compared with the focal

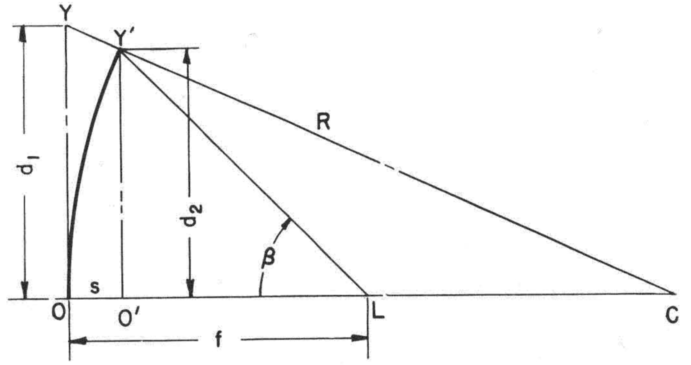

FIG. 1. Schematic drawing showing the effect of plate curvature on image position. OY is the focal plane, R is the radius of curvature of the plate. The distance d_2 which is the distance measured on the negative is less than d_1 because of the plate curvature.

length f, and O is the focal point of the lens L. Normally, Y is the point where a ray at angle β would be imaged but because of plate curvature the image appears at Y'. The plane $Y'O'$ is nearer to the lens than YO by the magnitude of the sagitta S. From the figure it is clear that

$$d_1 = f \tan \beta \tag{15}$$

and

$$d_2 = (f - S) \tan \beta. \tag{16}$$

For very large R and small S

$$d_2 = \sqrt{2RS} \tag{17}$$

from which it can be shown that

$$S = \frac{f^2 \tan^2 \beta}{2(R + f \tan^2 \beta)}. \tag{18}$$

For $\beta = 45°$, this equation becomes

$$S_{45°} = \frac{f^2}{2(R + f)}. \tag{19}$$

Then for S very small compared to R, values of S for other values of β can be computed from the approximate formula

$$S_\beta = S_{45°} \cdot \tan^2 \beta. \tag{20}$$

Values of the distortion, D, are obtained from the relation

$$D = d_2 - d_1 \tag{21}$$

or

$$D = (f - S) \tan \beta - f \tan \beta \tag{22}$$

whence

$$D = - S_\beta \tan \beta \tag{23}$$

which yields the values of distortion computed with respect to the equivalent focal length.

It is not unusual to find magnitudes of the sagitta, S, as large as 0.200 mm. for glass plates that have not been selected for flatness. This is equivalent to a radius of curvature of $R = 56.1$ M for a chord of approximately 300 mm. in length. Table 5 shows the values of the distortion, induced in the image of a distortion-free lens, having a focal length of 150 mm. when a curved plate having a sagitta S of 0.200 mm. is used to register the image. The second column shows the values of S

TABLE 5

DISTORTION INTRODUCED BY PLATE CURVATURE FOR A DISTORTION FREE LENS HAVING A FOCAL LENGTH OF 150 MM.

The plate is assumed to be concave toward the lens with a maximum departure from flatness of 0.200 mm. at $\beta = 45°$, which is equivalent to $R = 56.1$ M. Values of the distortion based upon the equivalent focal length *EFL* and two values of the calibrated focal length *CFL* are shown for selected values of β.

β	S	Distortion for		
		EFL of 150.000 mm.	*CFL* of 149.848 mm.	*CFL* of 149.899 mm.
degrees	mm.	mm.	mm.	mm.
0	0.000	0.000	0.000	0.000
7.5	.0035	−.0005	.020	.013
15	.014	−.004	.037	.024
22.5	.034	−.014	.049	.030
30	.067	−.039	.048	.022
37.5	.118	−.091	.025	−.010
40	.141	−.118	.009	−.030
45	.200	−.200	−.048	−.095

computed for each angle β. The third column, headed *EFL* of 150.000 mm. shows the magnitude of the distortion when it is computed with respect to the equivalent focal length. The fourth column, headed *CFL* of 149.848 mm., shows the extent by which this type of distortion can be minimized over the entire image plane by proper selection of a calibrated focal length. The fifth column, headed *CFL* of 149.899 mm., shows the magnitude of the distortion when a calibrated focal length is selected to minimize distortion over that portion of the image plane that lies within 40 degrees from the axis.

Table 6 gives the same type of information as Table 5 except that a value of the sagitta $S=0.0250$ mm., is used in the computation. This is an especially interesting case because the values shown in this table are those which can be expected when the plate is plane to within ± 0.0005 inch which is the tolerance usually specified for camera platens. The values of $+0.006$ mm. at $\beta=22.5°$ and 30° and -0.006 mm. at $\beta=45°$ are particularly significant when it is recalled that the distortion requirements on some wide-angle lenses are set at ± 0.020 mm. referred to the calibrated focal length.

Unfortunately, actual photographic plates are neither truly flat nor do they usually exhibit a uniform curvature. All measurements made on a glass negative are affected to some extent by the varying curvature or waviness of the surface. Consequently it is probable that the values of focal length and distortion derived from such measurements are affected by the departures from flatness of the photographic plate used in the calibration. While no method existed at the time* this study was made for yielding a quick and accurate contour map of the surface of the photographic plate as it exists at the moment of exposure, it is possible from the measurements on pairs of similar negatives to deduce the probable magnitude of the effect of plate curvature on the values of focal length and distortion. This is done by comparing the distances between the same two corresponding points on successive negatives made under the same conditions. The differences found should either be zero or proportional to tangent Δ, if the two plates are truly flat or have identical figures. It so happens that the measurements

* It is worthy of mention that Carman (21) has recently devised an interferometric method employing infrared light that enables one to bend an emulsion coated plate to minimize the departures from flatness. The photographic plate with the constraints still in place is then used in that condition to make the calibration negative. By this procedure, one is assured that the departures from flatness of the photographic plate are held to a few microns during exposure. It seems likely that this process should appreciably reduce the errors arising from plate curvature.

TABLE 6

DISTORTION INTRODUCED BY PLATE CURVATURE FOR A DISTORTION-FREE LENS HAVING A FOCAL LENGTH OF 150 MM.

The plate is assumed to be concave toward the lens with a maximum departure from flatness of 0.0250 mm. at $\beta = \pm 45°$, which is equivalent to $R=449.85$ M. Values of the distortion based upon the equivalent focal length, *EFL*, and two values of the calibrated focal length are shown for selected values of β.

β	S	Distortion for		
		EFL of 150.0000 mm.	*CFL* of 149.9811 mm.	*CFL* of 149.9868 mm.
	mm.	mm.	mm.	mm.
0	0.0000	0.0000	0.0000	0.0000
7.5	.0004	−.0001	.0024	.0016
15	.0018	−.0005	.0046	.0030
22.5	.0043	−.0018	.0060	.0037
30	.0083	−.0048	.0061	.0028
37.5	.0147	−.0113	.0032	−.0012
40	.0176	−.0148	.0010	−.0037
45	.0250	−.0250	−.0061	−.0118

used in determining the prism effect in lenses can be used for this purpose.

The results of measurement have been analysed for 12 cameras using 1/16 inch glass plates, 7 cameras using $\frac{1}{8}$ inch glass plates, and 7 cameras using $\frac{1}{4}$ inch glass plates. All cameras were equipped with lenses having a focal length of six inches. The probable errors of calibrated focal length for a single camera were found to be ± 0.020 mm. for the 1/16 inch plates; ± 0.008 mm. for the $\frac{1}{8}$ inch glass plates; and ± 0.004 mm. for the $\frac{1}{4}$ inch glass plates. The values of the probable errors in distortion for a single camera are shown in Figure 2. It is interesting to note the striking reduction in the error with increasing thickness of the photographic plates. This is undoubtedly a consequence of the departures from planeness becoming progressively less as the thickness of the plates increase. It is clear from these curves that the contribution to the error in distortion is appreciable for both the 1/16 and $\frac{1}{8}$ inch glass plates, and that the $\frac{1}{4}$ inch plates are the best of the three for use in camera calibration. All three varieties of plates were initially selected for planeness so the analysis indicates that the thicker plates are less likely to warp or depart from their initial state of planeness than the thinner plates. Consequently in this type of work, it is usually advantageous to use thick plates.

3. Analysis of Errors in Focal Length and Distortion for Several Methods of Calibration

In section 2.2, it was shown that, for the case of lens, focal plane, and targets properly positioned with respect to one another, the errors in focal length and distortion arise from errors in the measurement of the test negative and errors in the measured value of the angles separating the the targets.

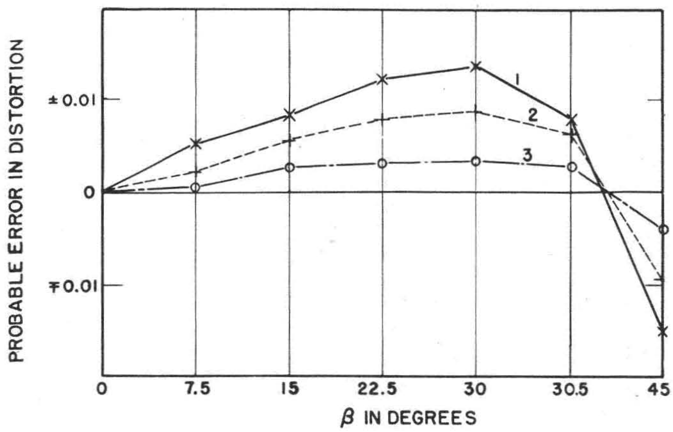

FIG. 2. Probable error in distortion versus angle for three plate thicknesses. Curve 1 shows the variations in the probable error in distortion of 1/16″ plates; curve 2 shows results for 18″ plates; and curve 3 shows the results for 1/4″ plates.

In this section the magnitudes of the probable errors that arise from these sources are computed for several methods of lens and camera calibration. The methods are (a) precision lens testing camera. (b) camera calibrator, and (c) field calibration method. In making these computations, the magnitudes assigned to the errors are those which are reported by the various operators. It is further assumed that the errors arising from plate curvature of the type described in section 2.3 can be neglected. This assumption is justified as the primary aim of these computations is for the purpose of comparing the reliability of these methods. The presence of appreciable plate curvature would impair the accuracy to approximately the same extent in all three methods.

3.1 PRECISION LENS TESTING CAMERA

This is the instrument used by the National Bureau of Standards for

TABLE 7

THE PROBABLE ERROR IN AN EQUIVALENT FOCAL LENGTH DETERMINED WITH THE PRECISION LENS TESTING CAMERA

In the computation of Δf, it is assumed that $\Delta r = \pm 0.002$ mm.; $\Delta\beta = \pm 3$ seconds per 5° angle; and $f = 150$ mm.

β	Δr	$\Delta\beta$	Δf_r	Δf_β	Δf
degrees	mm.	seconds	mm.	mm.	mm.
5	±0.002	±3.0	±0.023	±0.025	±0.033
10	±0.002	±4.2	±0.011	±0.018	±0.021

photographically determining the focal length and distortion of lenses intended for use in airplane mapping cameras (7). Two negatives are usually made which together cover a single diameter. Measurements are made along each of the two radii and the results are averaged. The probable error in the determination of angle between adjacent collimators is believed to not exceed ±2 seconds but for the purpose of the following calculations, a more conservative estimate of ±3 seconds is used. Because the angles between adjacent collimators are measured separately, the probable error in the angle between the first and third collimator is $\pm 3\sqrt{2}$ seconds; between the first and fourth, it is $\pm 3\sqrt{3}$ seconds, and so on. The probable error of measurement of a distance on the negative does not exceed ±0.002 mm. for moderately good images.

To compute the focal length, the distance between the 0° and 5° images is used for one determination, and the distance between the 0° and 10° images is used for the second determination. The final value of f is found by taking a weighted average of the values obtained for $\beta = 5°$ and $\beta = 10°$. It is customary to assign weights to the separate determinations that are inversely proportional to their probable errors. The probable errors of $f_{5°}$ and $f_{10°}$ are determined from Tables 1 and 2 for the given values of β, Δr, and $\Delta\beta$. The computed values of Δf are shown in Table 7.

The probable error, Δf, of the weighted average of $f_{5°}$ and $f_{10°}$ is given by the relation

$$\Delta f = \pm \frac{1}{\sqrt{\left(\frac{1}{\Delta f_{5°}}\right)^2 + \left(\frac{1}{\Delta f_{10°}}\right)^2}} \tag{24}$$

Using the values in Table 7 in this equation, the probable error in f for a single negative is

$$\Delta f = \pm\ 0.018 \text{ mm.} \tag{25}$$

As two negatives are usually made, the above value is divided by $\sqrt{2}$, so that the final value of the probable error for the average of the two negatives is

$$\Delta f = \pm\ 0.013 \text{ mm.} \tag{26}$$

The probable errors in the distortion for each value of β are computed in accordance with the procedure shown in section 2.2 (b). Table 8 shows the value of ΔD_r, the probable error arising from errors in the measurement of r; ΔD_f, the probable error arising from errors in the determination of f; and ΔD_β, the probable error arising from errors in β. Table 4 is used in determining the values of ΔD_f and ΔD_β for the given conditions, ΔD_r remaining constant. The final value of ΔD for a single negative is obtained by finding the square root of the sum of the squares of each error as shown in eq. 14. The final value for two negatives is shown in the column headed ΔD for $n = 2$.

Because of the manner in which the angles are measured, there is little gain in using a calibrated focal length in evaluating the probable errors in distortion. In this particular case, when the approximate calibrated focal length is based on $D = 0$ for $\beta = 40°$, the probable error Δf becomes ±0.009 instead of ±0.013 mm. In turn this reduces ΔD to ±0.009 mm. and ±0.011 mm. at $\beta = 40°$ and $\beta = 45°$ respectively.

In the past few years precision theodolites have become readily available that permit the measurement of β within 2 seconds of arc. With an instrument of this type, the angle separating the first collimator and any of the other collimators can be measured directly. Consequently there is no cumulative error such as occurs when each 5° interval is measured separately and $\Delta\beta$ is the same for each

TABLE 8

THE PROBABLE ERROR, ΔD, IN DISTORTION FOR MEASUREMENTS MADE WITH THE PRECISION LENS TESTING CAMERA FOR VARIOUS ANGULAR SEPARATIONS (β) FROM THE AXIS

These values of the probable error are for a lens having an equivalent focal length of 150 mm. It is estimated that $\Delta f = \pm 0.013$ mm.; $\Delta r = \pm 0.002$ mm.; and $\Delta\beta = \pm 3$ seconds for each 5° interval.

β	$\Delta\beta$	ΔD_r	ΔD_f	ΔD_β	ΔD $n=1$	ΔD $n=2$
degrees	sec.	mm.	mm.	mm.	mm.	mm.
5	3.0	0.002	0.001	0.002	0.003	0.002
10	4.2	.002	.002	.003	.004	.003
15	5.2	.002	.003	.004	.005	.004
20	6.0	.002	.005	.005	.007	.005
25	6.7	.002	.006	.006	.009	.006
30	7.3	.002	.008	.007	.011	.008
35	7.9	.002	.009	.008	.012	.008
40	8.5	.002	.011	.010	.015	.011
45	9.0	.002	.013	.013	.018	.013

value of β. If the error $\Delta\beta$ is assumed to be ± 4 seconds, no reduction in the error in equivalent focal length is found but the error in distortion is reduced, the maximum error falling to ± 0.010 mm. at $\beta = 45°$ for the average obtained from determinations for two negatives. If the calibrated focal length for $\beta = 40°$ is used, the error in *CFL* falls to ± 0.006 mm. as does the error in distortion for $\beta = 45°$ for $n = 2$.

3.2 CAMERA CALIBRATOR

The camera calibrator (8) was developed at the National Bureau of Standards and has been in use since the latter part of 1949. It was designed especially to simplify calibration of precision airplane mapping cameras; to increase the accuracy of measurement of focal length, radial distortion, tangential distortion, and the location of the principal point; and to reduce markedly the time required for calibration. It has performed in accordance with the expectations. Moreover, it is a compact piece of equipment so designed that all work is performed in the laboratory under controlled conditions. The cost of construction is relatively low. The calibration is sufficiently simple that the instrument can be recalibrated in the course of a day. Because of its rugged and compact nature and the simplicity of recalibration, it could easily be transported from one place to another and put into operating condition following the move within a very few days.

In the course of calibration, it is customary to determine the equivalent focal length and distortion of the lens as mounted in the camera. A single negative, made with a camera on the calibrator, contains images at 7.5° intervals along four radii from the centers to the four corners of the square image area. Measurements are made along each of the four radii and the results averaged. While all of the necessary information can be obtained from a single negative, a second negative is usually taken particularly when measurements on tangential distortion and prism effect are required.

There are now two methods available for determining the angle β. In the first method, angles separating adjacent collimators are measured one at a time using a calibrated reflecting prism. The angles separating the central collimator and any of the other collimators are obtained with consequent increase in the error $\Delta\beta$ with increasing β. In the second method, the total angle β is measured in one step with the aid of a precision theodolite.

Table 9 shows the resultant error in focal length for a lens having a focal length of 150 mm. for $\Delta r = \pm 0.002$ mm. and $\Delta\beta = \pm 4$ seconds for each value of β for both methods of determining β. The individual value of Δf_r and Δf_β are determined with the aid of Tables 1 and 2. The column, headed $\Delta f(n=1)$, gives the magnitude of the probable error in focal length for a determination based on measurements along a single radial bank. The column headed $\Delta f(n=4)$, gives the magnitude of the probable error in focal length based on the average obtained from measurements along

TABLE 9

PROBABLE ERROR, Δf, IN THE FOCAL LENGTH DETERMINATIONS FOR THE CAMERA CALIBRATOR

For the computations, it is assumed that $f=150$ mm.; that $\Delta r=\pm 0.002$ mm.; that $\Delta\beta=\pm 4$ seconds per 7.5° interval in the first part of the table and that $\Delta\beta=\pm 4$ seconds per angle in the second part of the table. The calculations are for a single negative. If two negatives are used, the error of the average is 0.7 of the value listed for $r=4$.

β	$\Delta\beta$	Δf_r	Δf_β	Δf	Δf
	$\Delta\beta=\pm 4$ seconds per 7.5° angle			$n=1$	$n=4$
degrees	seconds	mm.	mm.	mm.	mm.
7.5	4	±0.015	±0.022	±0.207	±0.014
15	5.6	.007	.016	.017	.009
22.5	6.9	.005	.014	.015	.008
30	8.0	.003	.013	.013	.007
37.5	8.9	.003	.013	.013	.007
45	9.8	.002	.014	.014	.007
			$\Delta\beta=\pm 4$ seconds per angle		
7.5	4	±0.015	±0.022	±0.027	±0.014
15	4	.007	.012	.014	.007
22.5	4	.005	.008	.009	.005
30	4	.003	.007	.008	.004
37.5	4	.003	.006	.007	.003
45	4	.002	.006	.006	.003

the four radii. If the results from two negatives are averaged, the resultant error is reduced by the ratio of the square root of two.

For an equivalent focal length determination for a lens having appreciable distortion, the values of Δf for $\beta=7.5°$ is assigned. For lenses having near zero distortion, a larger value of β can be used with a resultant decrease in Δf. In assigning the error in a calibrated focal length, the value of Δf nearest to the outermost zero point of distortion may be properly used. It is clear from a consideration of these tables that the error in calibrated focal length arising from these magnitudes of error in r and β is almost negligible.

The probable errors in the values of distortion, based on the assumed errors in r and β, are listed in Table 10. The error in equivalent focal length used in the computation is that associated with $\beta=7.5°$. The tabulation is based on the first method of measurement of β with the consequent increase in $\Delta\beta$ with β. The values of ΔD are given for a single radial bank under the column headed, $n=1$; and the average for the four radial banks is given under the column, $n=4$. The table lists the error for the least favorable conditions of the given error in r and β. If the distortion is based on the calibrated focal length, there is a reduction in the maximum value of ΔD. Also, if $\Delta\beta$ is assumed constant, the error is still further decreased as shown in Table 11.

3.3 FIELD CALIBRATION METHOD

The field calibration method (9) employs a calibration range consisting of a camera station and a number of targets spaced along a line at a moderately great distance therefrom. This distance is usually so great that it is believed that the imagery is comparable to that which usually obtains for infinitely distant targets. When targets at a finite distance are used there is always the possibility of an error arising because the camera does not occupy the correct position with respect to that occupied by the central axis of the theodolite when the angle was measured. This is the supreme advantage of the infinitely distant target. Neglecting spherical aberration of collimator, the value obtained is independent of placement of cameras so long as the aperture is filled.

In performing a test, the camera is so placed that the image of the line of targets falls along one of the diagonals of the focal plane frame. The camera is aimed at the central target by approximate methods.

TABLE 10

THE PROBABLE ERROR, ΔD, IN DISTORTION FOR MEASUREMENTS MADE WITH THE CAMERA CALIBRATOR FOR VARIOUS ANGULAR SEPARATIONS FROM THE AXIS

These values of the probable error are for a lens having an equivalent focal length of 150 mm. It is estimated that $\Delta f = \pm 0.014$ mm.; $\Delta r = \pm 0.002$ mm.; and $\Delta\beta = \pm 4$ seconds for each 7.5° interval.

β	$\Delta\beta$	ΔD_r	ΔD_f	ΔD_β	ΔD $n=1$	ΔD $n=4$
degrees	sec.	mm.	mm.	mm.	mm.	mm.
7.5	4.0	0.002	0.003	0.002	0.004	0.002
15	5.6	.002	.004	.004	.006	.003
22.5	6.9	.002	.006	.006	.009	.004
30	8.0	.002	.008	.008	.012	.006
37.5	8.9	.002	.010	.011	.015	.008
45	9.8	.002	.014	.014	.020	.010

These errors are further reduced if an average is made for two negatives.

The angles separating the central target from the other targets are measured with an accurate theodolite. The probable error in the determination of the angle between selected targets is believed to be not in excess of ± 5 seconds. The probable error of measurement of distance on a test negative does not exceed ± 0.003 mm. Targets separated by 12.5° from the central target are used in determining the equivalent focal length. So for a single determination given that

$$\Delta\beta = 5 \text{ seconds} \tag{27}$$

$$\Delta r = 0.003 \text{ mm.} \tag{28}$$

the values of the probable error from Tables 1 and 2 for $\beta = 12.5°$ are

$$\Delta f_r = 0.014 \text{ mm.} \tag{29}$$

$$\Delta f_\beta = 0.017 \text{ mm.} \tag{30}$$

whence

$$\Delta f = \pm\, 0.022 \text{ mm.} \tag{31}$$

Two determinations of the equivalent focal length are made from a single negative, so the probable error for one diagonal is

$$\Delta f = \frac{\pm\, 0.022}{2} = \pm\, 0.016 \text{ mm.} \tag{32}$$

The probable errors in the distortion have been computed on the basis of the error in r and β, reported for the field calibration method, and the results are shown in Table 12. The values are slightly higher than those present in the camera calibrator method. Some reduction in the error is possible by computing with respect to a calibrated focal length but not as much as for the two previous methods because the equivalent focal length is

TABLE 11

THE PROBABLE ERROR, ΔD, IN DISTORTION FOR MEASUREMENTS MADE WITH THE CAMERA CALIBRATOR FOR VARIOUS ANGULAR SEPARATIONS (β) FROM THE AXIS

The distortion values are based on a calibrated focal length of 150 mm. calculated to yield zero distortion for $\beta = 40°$. It is estimated that $\Delta f = \pm 0.003$ mm.; $\Delta r = \pm 0.002$ mm.; and $\Delta\beta = \pm 4$ seconds.

β	$\Delta\beta$	ΔD_r	ΔD_f	ΔD_β	ΔD $n=1$	ΔD $n=4$
degrees	sec.	mm.	mm.	mm.	mm.	mm.
7.5	±4.0	±0.002	±0.000	±0.003	±0.004	±0.002
15	4.0	.002	.001	.003	.004	.002
22.5	4.0	.002	.001	.003	.004	.002
30	4.0	.002	.002	.004	.005	.002
37.5	4.0	.002	.002	.005	.006	.003
45	4.0	.002	.003	.006	.007	.004

TABLE 12

THE PROBABLE ERROR, ΔD, IN DISTORTION FOR MEASUREMENTS MADE WITH THE FIELD CALIBRATION METHOD FOR VARIOUS ANGULAR SEPARATIONS (β) FROM THE AXIS

These values for the probable error are for a lens having an equivalent focal length of 150 mm. It is estimated that $\Delta f = \pm 0.016$ mm.; $\Delta r = \pm 0.003$ mm.; and $\Delta\beta = \pm 5$ seconds.

β	ΔD_r	ΔD_f	ΔD_β	ΔD $n=1$	ΔD $n=2$
degrees	mm.	mm.	mm.	mm.	mm.
5	0.003	0.001	0.004	0.005	0.004
10	.003	.003	.004	.006	.004
15	.003	.004	.004	.006	.004
20	.003	.006	.004	.008	.006
25	.003	.007	.004	.009	.006
30	.003	.009	.005	.011	.008
35	.003	.011	.005	.012	.008
40	.003	.013	.006	.015	.011
45	.003	.016	.007	.016	.011

initially based on a large value of β.

The field method of camera calibration when performed carefully should yield reliable values of the calibrated focal length and distortion. It can give reliable information on the location of the focal point in the absence of prism effect. It is, however, deficient in some respects, although in most instances these defects do not seriously impair the accuracy of the calibration. The features most subject to criticism are finite distance of targets and alignment of camera.

a. Finite distances of targets

The fact that the targets are located approximately 400 ft. from the camera station places the film plane of the camera 0.188 mm. nearer to the lens than the true image plane, assuming the camera is set at infinite focus. It is probable that this will not materially affect the focal length and distortion determinations for the film plane of the camera. However, it is likely to produce some changes in the values of resolving power. This effect will be increased with increasing focal length of the lens, and for 36 inch lenses it will mean evaluating the resolving power for a plane 1 mm. inside the best image plane.

b. Alignment of camera

An auto-collimating telescope should be used to make the focal plane of the camera normal to the line of sight between telescope and central target. When this is not done, the camera is almost certain to be tipped with respect to the line of sight and asymmetrical distortions are sure to result. This has been compensated in the field method by operating on the data to find a point of symmetry. These calculations, in the absence of prism effect, serve to locate the focal point of the camera (referred to as the center cross by the present writer and also referred to as the principal point of auto-collimation in recent literature (3). Since only one diagonal of the camera is used at a time, it is probable that the camera is also tipped about a horizontal axis as well as the vertical. It is possible that this tipping of unknown magnitude may sometimes affect the final results, although such effects will generally be very small. For example, a one degree tip about the horizontal axis will produce an increase of 0.02 mm. in the measured value of the focal length of a 150 mm. lens.

While the location of the focal point, in the absence of prism effect, is sufficiently accurate for the purpose by this method, it is accomplished by difficult and time consuming computation. The greater part of this computation can be avoided by using the auto-collimating telescope in aiming the camera.

When prism effect is present, this method locates a point about which the distortion values at a specified angle are equalized. Asymmetries will still exist at all other angles although they are reduced from the values originally found. Fortunately the prism effect in modern cameras is so small that the remaining assymmetries are usually small.

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PSC Announces Venezuela Survey

A Canadian company has been chosen to help carry out a vital 225,000 acre aerial survey by the Venezuelan Government along the South American country's Caroni River. The survey is intended to help develop electric power for the rapidly growing industrial area in the vicinity of Ciudad Bolivar.

A contract has been given The Photographic Survey Corporation, providing for topographical mapping of the area at the scale of 1:10,000 with five metre contours.

PSC's interest in developing the rich resources of Venezuela goes back ten years when PSC first began operating. Hardly a year has passed that PSC has not been in Venezuela on flying operations. In 1953 Aeromapas was established by PSC in Caracas, patterned after the Toronto company. Technical supervision and training were supplied by PSC in the initial stages of the new company and PSC has remained as a shareholder.