Reduction Process of Resection Problems by Photogrammetric Rectifiers

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ABSTRACT: *This paper deals with the application of the projective method to solve spatial resection problems by photogrammetric rectifiers; a simple set of rules is developed for the adjustment of aerial photographs by means of the above instrument type on a given set of control points. These rules can also be applied for the production of aerial mosaics. The theoretical results are verified by experimental checks. It is further shown that the projective method is especially suitable for the data reduction of high altitude and long distance flight runs; no special test areas would be required and be kept up. There are additional possibilities concerning the automation of the reduction process. All these points demonstrate the versatility and adaptability of the projective approach.*

1. REVIEW OF PROBLEM

SPATIAL Resection is used to determine the exterior orientation of aerial photographs by matching the surveyed configuration of control points on the ground with their corresponding positions on the photographic image. These problems can be solved with high accuracy and computational speed by projectively transforming the photographs to the ground control in available photogrammetric projectors. With regard to its simplicity this optical-mechanical approach compares favorably with the analytic method, as has been demonstrated in a previous paper (Ref. 1).

It will be shown in this paper how the photogrammetric rectifier, as a special type of photogrammetric instrument, can be exploited to full advantage for the adjustment process of the ray pencil and for the data reduction of spatial resection. Furthermore this adjustment procedure can also be applied efficiently in the production of aerial mosaics.

2. ADJUSTMENT RULES

The theory and operation of rectifiers is treated in detail in the handbooks of photogrammetry (Ref. 2) and the operational manuals. Therefore the deductions are abbreviated here to simple statements where the relations are already known or evident. The rules are set up for the more common types with non-tiltable lens where the lens axis is fixed with reference to the mechanical axis of the instrument, as with the Bausch & Lomb Autofocus Rectifier and the Zeiss SEG V; but those rules can be readily adapted also for any other types.

2.1 CONTROL CROSS

To put the projected control points into coincidence with their collateral configuration on the control chart, one does not operate best on all the details simultaneously, but rather on four selected control points at the outer parts of the frame, forming a pair of cross diagonals (A, B) approximately perpendicular to

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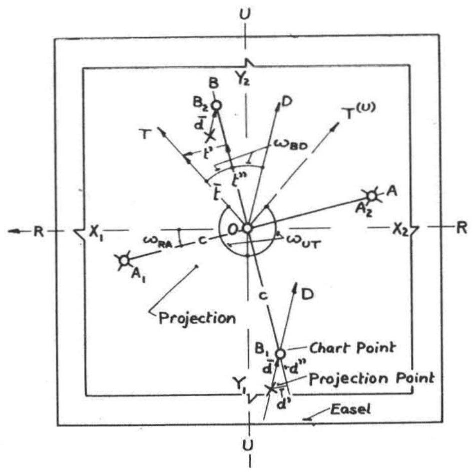

FIG. 1. Control configuration.

each other (Figure 1). It is convenient to have the diagonals A, B approximately falling into the easel axes, R $U(\omega_{RA} \rightarrow 0)$. Theoretically only three control points are required for the adjustment, if the interior orientation of the photograph is defined. But the fourth control point is welcome as an additional check.

2.2 HEIGHT ADJUSTMENT

The control points on either diagonal A or B are put to coincidence by operating the magnifying drive and by shifting the control chart appropriately. If A and B result in sensibly different magnifying settings, one may refine by meaning them. The advantage of using the diagonals is that the height settings obtained by them are fairly independent from the tilt, still undetermined.

2.3 TILT ADJUSTMENT

After the control points of the A-diagonal are put to coincidence, those of the B-diagonal show an identical displacement vector $\bar{d}$ of amount d and direction D; it is possible to derive the tilt angle t therefrom. Figure 2 shows a cross section of the original ray pencil. If the projection plane is tilted by t, the projection of B_1 (or B_2) is displaced by $d^{\cdot}$. From similar triangles

$$d^{\cdot} = c^{\cdot 2} t / H, \tag{1}$$

where all magnitudes related to the projection plane are marked by dot superscripts to distinguish them if necessary from those of the negative plane written without superscripts; if there is no danger of confusion the superscripts may even be dropped eventually. Eq. (1) is valid for any component plane or diagonal trace in the projection plane according to the vector character of $\bar{d}^{\cdot}$.

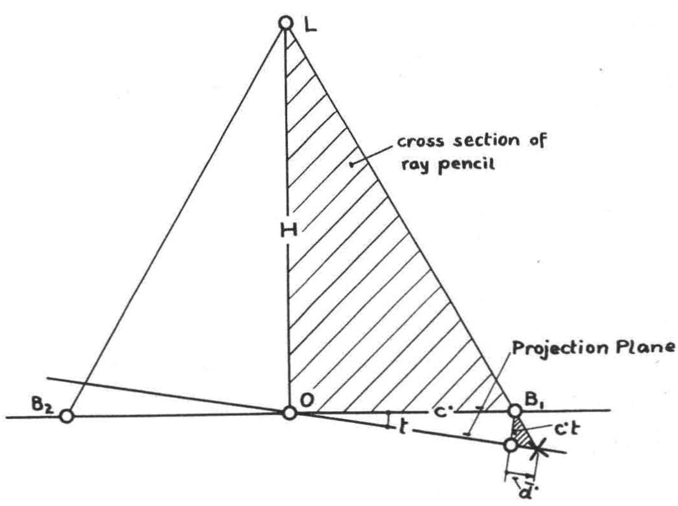

FIG. 2. Tilt—displacement.

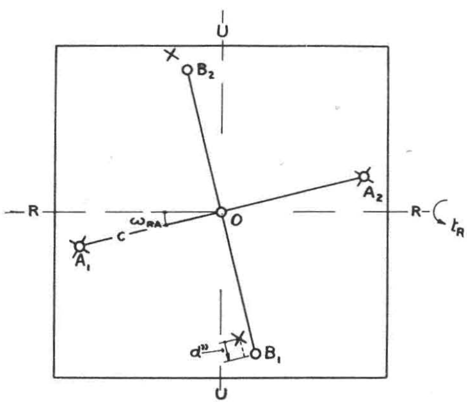

FIG. 3. Tilt reduction in components.

This vector quality of $\bar{d}^{\cdot}$ can be utilized in the well known tilt reduction with easels of two tilt components, as in the SEG V for example. After the one diagonal A (Figure 3) is put to coincidence according to §2.2, the displacement component d'' is removed by tilting the easel around its axis RR (nearest to AA) until the projector point of B_1 has moved closest to its chart point. As a rule the projections B_1 and B_2 move towards that edge of the easel which is lowered by tilting. Next the B-diagonal is put into coincidence, and the residual d'-component is removed correspondingly by tilting around axis UU. Any residuals can be removed by iteration of this highly converging process.

2.4 AZIMUTH ADJUSTMENT

In other easel systems (like the B. & L. Autofocus Rectifier) only one tilt movement is provided, e.g. around axis RR in Figure 1. Consequently the azimuth of the resultant tilt of a photograph (principal plane) has to be made coincident with the trace UU of the easel, i.e. the photograph has to be rotated azimuthally by ω_{UT}, yet to be determined, till the trace T of its principal plane is falling into UU.

The process is started with the easel in horizontal position (t or $\beta=0$) and azimuth $\omega_{RA}=0$, i.e. fiducial- and easel axes coinciding (see Figure 1). After coincidence adjustment of the A-diagonal according to §2.2, the displacement $\bar{d}$ at B_1 is resolved into the components d', d''. The corresponding tilt components t', t'' could be computed according to Eq. (1) and composed again to the resultant tilt $\bar{t}$ along trace T: t', t'' are proportional to d', d'' respectively; the signs of t'' and d'' are equal, those of t' and d' are reversed, as d' had first to be transferred to points A_1A_2 by bringing the B-diagonal to coincidence. In plotting the components t' t'' in the respective lines of fall, it follows that the direction T is found by mirroring line D at diagonal B. Now T should be marked on the negative and be rotated with it into the line U. It is more convenient however to mirror T at UU into $T^{(U)}$, mark it on the projection table (easel) and rotate the negative till the projected fiducial mark Y_1 is falling on $T^{(U)}$. By considering the geometry it is quite obvious that the azimuth rotation is ω_{UT} in both cases. For the special case of the control diagonals coinciding approximately with the easel axes ($\omega_{RA}\approx 0$), the line $T^{(U)}$ is simply identical with D: here the fiducial mark Y_1 has just to be rotated into the displacement direction D (see Figure 4).

The above procedure positions the azimuth rather accurately in a first step. Further refinement is achieved by the iteration process, according to Figure 5: After having performed the routine coincidence adjustment on the A-diagonal and the tilt correction around RR, the residual displacements at B_1 and B_2 are set to the minimum $d^{\cdot\prime}$. It is assumed that B_1 is lying on the lower side of the easel. The residual $d^{\cdot\prime}$ can be removed preliminarily by the small azimuth rotation

$$\Delta\omega = d^{\cdot\prime}/c^{\cdot}$$

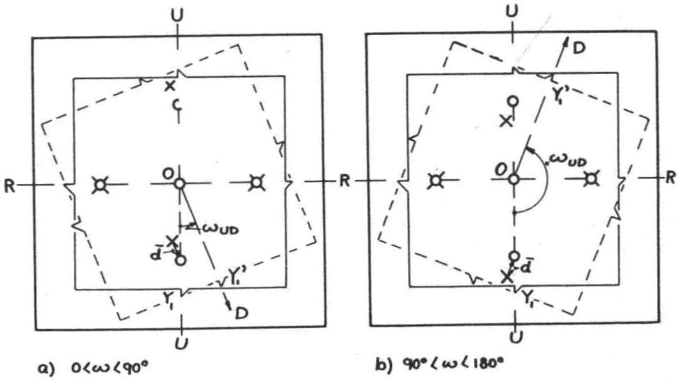

FIG. 4. Special aximuth setting ω_{UD}, for $\omega_{RA} \approx .0$.

which can be directly read at the angular scale of the negative carrier. Then the azimuth correction $\Delta\Omega$ is set by overcorrecting $\Delta\omega$ in the same direction

$$\Delta\Omega = k\Delta\omega, \tag{2}$$

where the overcorrection factor k has yet to be deduced. From the vector diagram of Figure 5, it follows:

$$\Delta\Omega \approx t'/t'',$$

where according to Eq. (1)

$$t' \approx Hd'/c^{\cdot 2},$$

and t'' is known by the tilt angle β of the easel, according to the well known relation of §4.1 below

$$t'' \approx \beta f_c/f_R,$$

with f_c and f_R as focal distances of the camera and the rectifier respectively. By substitution and the geometrical relation

$$H/c^{\cdot} = f_c/c,$$

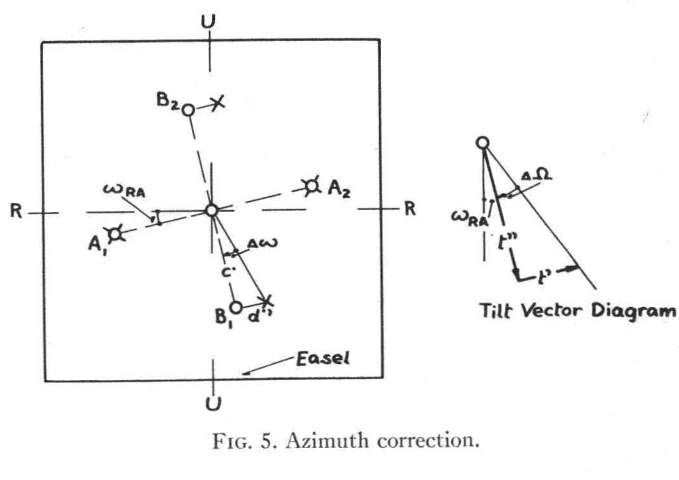

FIG. 5. Azimuth correction.

it results

$$k \approx f_R/c\beta = \beta_0/\beta, \qquad (3)$$

where

$$\beta_0 \equiv f_R/c.$$

The angle factor β_0 depends on the geometrical configuration of the control points. Numerical values are given in Table 1. A singular case arises for $\beta \rightarrow 0$ with $k \rightarrow \infty$; this is quite natural as the azimuth of the tilt is indeterminate for a non tilted photograph.

If necessary the process of azimuth adjustment may be iterated. Practical application shows that the process converges very rapidly.

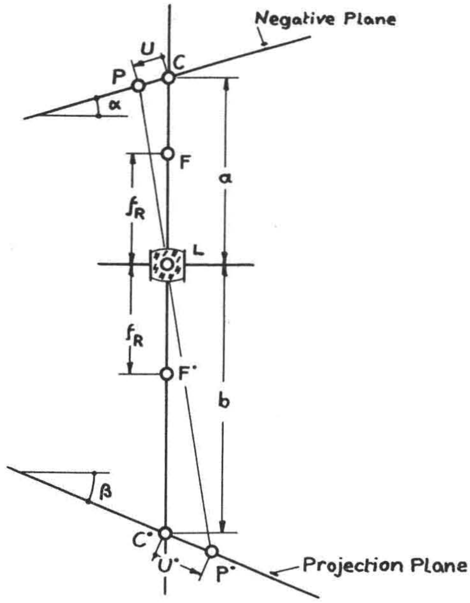

FIG. 6. Rectifier settings.

3. Perspectivity Displacement

To complete the adjustment process one has to check to determine that the interior orientation of the original ray pencil is maintained also for the transformed pencil of the rectifier. This condition is fulfilled if the negative carrier is displaced by a certain amount U (Figure 6), deduced in previous paper (Ref. 3) as

$$U \approx \frac{f_R}{2}\left(g^2 - 1 + \frac{1}{n^2}\right)\beta, \qquad (4)$$

where

$$g \equiv f_c/f_R, \qquad n \equiv b/a = (b - f_R)/f_R = f_R/(a - f_R), \qquad \beta \approx n\alpha,$$

f_R, f_c focal distance of rectifier and camera respectively. In practice U may be tabulated for the numerical constants of the specific camera rectifier combination. The rectifier parameters a, α or b, β may be taken as entrance variables, depending on which of them are more conveniently readable at the scales.

The operational speed can further be increased by an automatic U-control instead of manually setting it. This is made for example in the rectifier SEG V (Ref. 4). But here a disturbing interference with the tilt adjustment of §2.3 has to be considered. The movement d'' of the tilt adjustment in Figure 3 is now superimposed by the movement U, projected to the easel. Numerical analysis shows that the component U has a critical and even overpowering influence over component d, depending on the magnification n; the convergence of the t-adjustment may be even reversed to divergence. However this difficulty could be

TABLE 1

ADJUSTMENT FACTORS, $f_R = 139$ MM., 9×9″ NEGATIVE

Control Array		Axial	Diagonal
ω_{RA}	[°]	0	45
c	[mm]	100	140
β_0	[°]	80	60
k, for β=3°		27	20

quickly remedied by equipping the tilt handwheels with electric switches for cutting out the U-control till d is removed by tilting, then letting in the U-movement afterwards.

4. Reduction Formulae

After the projection of the photograph is adjusted to the control chart, its data of exterior orientation are determined explicitly. They may be evaluated either by measuring the projected fiducial marks, as demonstrated in a previous paper (Ref. 1), or they may be reduced from the data of the rectifier by the well known perspective relations (Ref. 2). Each approach has its merit, the former by its accuracy circumventing the error influences of structure deformation, the latter by its speed.

4.1 Tilt angle t

The tilt angle t can be deduced correctly from the tilt angles α or β of the rectifier (Figure 6):

$$\sin t/\sin\beta = g, \quad \text{where} \quad g \equiv f_c/f_R.$$

For small tilt angles this equation can be simplified by development in series:

$$t/\beta = g + \frac{g(g^2 - 1)}{6}\beta^2 + \cdots \tag{5}$$

The convergence is very rapid. Even if only the first term g is taken, the error Δt remains within the Limit of Camera Accuracy (Ref. 1) for a relatively wide range of t. Numerical values are for example

$$f_c = 154 \text{ mm.}, \qquad f_R = 139 \text{ mm.}, \qquad \Delta t \leqq 4\cdot 10^{-4} \approx 1\tfrac{1}{2} \text{ min. of arc},$$

resulting in:

$$t \lessapprox 13.5°.$$

4.2 Height H and scale n

The flying height H is given as model height, corresponding to the scale n_0 of the control chart:

$$n_0 \equiv H/f_c. \tag{6}$$

The rectifier scale is defined however as $n \equiv b/a$. This value follows directly from the a- and b-scales of the rectifier, but only for $t=0$ is it identical with n_0. The general relation is:

$$(n/n_0)^2 = \left(1 - \frac{1}{n_0{}^2}\sin^2\beta\right) \bigg/ (1 - \sin^2\beta),$$

and after development in series

$$n_0/n = 1 - \tfrac{1}{2}(1 - 1/n^2)\beta^2 \cdots \tag{7}$$

By means of these relations n_0 and H can be readily computed from the known n and β. The second term has to be taken into account only if it surpasses the admissible error limit $\Delta n/n = \Delta H/H$. For a numerical example with $\Delta n/n \lessapprox 2\cdot 10^{-4}$ (Limit of Camera Accuracy), $n \approx 1.2$, it is: $t \lessapprox 2.3^{\beta}$.

4.3 PLOTTING OF NADIR POINT N

After the control chart is adjusted on the easel, the projected fiducial axes X, Y and the principal point P may be directly pricked on it (Figure 7). In the case of a one component easel, the nadir point N is then fixed on the chart by means of a T-square held against the easel edge and by plotting

$$P^{\cdot}N^{\cdot} = n^{\cdot} = H\,tg\,t \approx Ht, \qquad (8)$$

and analogously in the case of the two component table, by plotting the corresponding X- and Y-components.

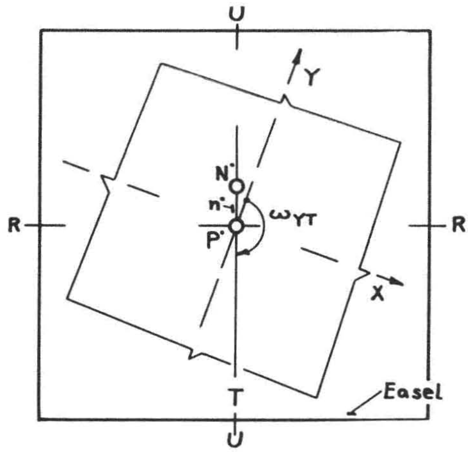

FIG. 7. Plotting of nadir point.

5. CONTROL CHARTS

The control charts have to be constructed to an appropriate scale. There are some advantages in making it as small as possible, as in this case there is a reserve in using the control chart also for flight runs at higher altitudes. The rectifier with its big range of magnification alleviates additionally the task of matching the scale of the photograph with that of the chart.

For moderate altitudes, say 10,000 ft., sometimes artificially marked control points are used, laid out and accurately surveyed in special test areas with sufficient density. But for higher altitudes (30,000 to 100,000 ft. and higher) these structures become too big and impractical. It is better then to use the natural features and details of the ground. For medium flying heights these may be road systems, contours of field lots and buildings, as sketched in Figure 8. The intersections of any well defined lines serve as control points. The lines are not drawn in full through the intersections but are interrupted: the projected lines can now be matched to their charted counterparts with a high accuracy, comparable with

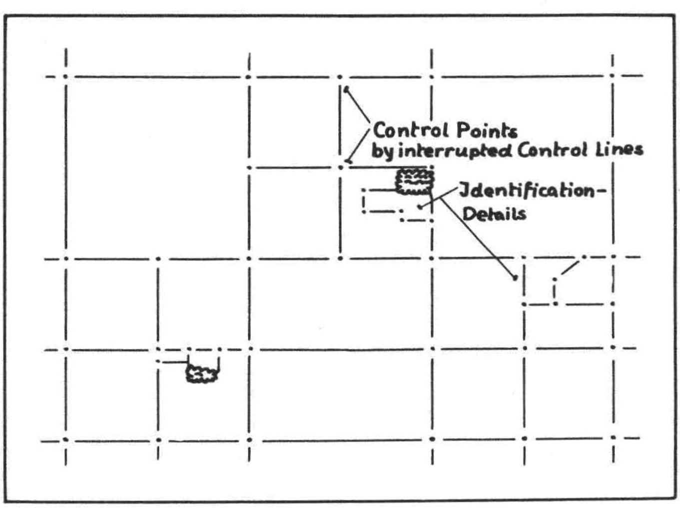

FIG. 8. Control chart.

a vernier setting. Some characteristic field and forest patches may be included for quick identification and orientation of the individual photographs. Those photographic features should be mapped which offer the sharpest and most distinct details. For high flight altitudes these might be rivers, mountain ridges, shore lines. The ground relief can also be taken into account by appropriate corrections (Ref. 1); but for high flight altitudes this influence tends to become more and more negligible.

If no other means are available the control charts can even be mapped from the photographs of the flight run by photogrammetric triangulation methods. Only some few control points are required: for this purpose some sharply defined points on the photographs are selected and each of them is surveyed with reference to the nearest geodetic point, simply by compass-transit and tape line. The projective method is versatile and adaptable, and there is no restriction to test areas which have to be kept up and are limited in size.

6. Operational and Accuracy Tests

The above reduction procedure was checked in practical operation by means of:

1. A computed control configuration.
2. An aerial photograph from an actual flight run.

The projections were repeatedly adjusted to their corresponding chart points by different operators using a B. & L. Autofocus Rectifier. The evaluation of all the tests showed the following RMS-errors of the components of a single adjustment:

$$\Delta\Omega = \pm 1°, \qquad \Delta\beta = \pm 0.05°,$$
$$\Delta t = g\Delta\beta = \pm 9\cdot 10^{-4} \text{ rad} \approx 3 \text{ min. of arc},$$
$$\Delta n/n = \Delta b/b = \Delta H/H \approx \pm 3\cdot 10^{-4}.$$

An analysis of these errors shows that they are determined by and are identical with the accuracy limit of the rectifier scales: $\Delta\beta$ corresponds to the vernier estimation interval of $\approx 0.05°$, $\Delta n/n$ to the vernier interval of $\Delta b \approx 0.1$ mm.; $\Delta\Omega$ is determined by Eq. (2) with $k=20$ and the error of the azimuth setting of $\Delta\omega \approx 1/20°$. Compared with the limit of camera accuracy (Ref. 1), the above errors are just double. The manufacturer of the rectifier had no reason of course to raise the accuracy of these scales above the direct purpose of this instrument of making mosaics for which it is quite adequate. It would be possible however to raise the accuracy of these scales for this new field of application.

The rectifier offers the additional advantage of compensating affine deformations which may eventually drop in by non uniform shrinkage of the negative and the control chart. For this purpose the negative carrier has to be displaced appropriately, analogous to U in §3; the procedure is described in a special paper of Ref. 5. All these refinements contribute to raise the accuracy as far as possible.

The computational speed of a full adjustment on the rectifier (height, tilt and azimuth) was checked by different operators as 2 to 3 minutes per frame under favorable conditions of routine reduction: in flight runs the orientation data vary only slowly from frame to frame. The adjustment rules are so simple that they can be handled even by inexperienced people after a short training period. No identification errors are possible as the photograph is processed as an over-all entity. Finally it seems possible even to automatize the whole adjustment process by an appropriate electronic scanner. After the adjustment is finished the orientation data can be picked up at the rectifier by reading off its scales. In a more advanced stage the instrument could be equipped with digital converters which by pushing a button, would automatically transfer the results to an automatic typewriter, or feed them into an automatic electronic computer.

It is interesting to compare this projective solution of the resection problem with the analytic approach. Extensive efforts have been made from different sides to modernize the latter by the application of automatic electronic computers; see e.g. Ref. 6. As to the accuracy of both methods one has to consider that both of them are dependent lastly on the camera accuracy as a limit. The reduction equipment is in the one case an optical comparator and automatic computer, in the other case a photogrammetric restitutor. Both reduction equipments can be so designed as not to deteriorate this basic limit of camera accuracy. How high this limit and its stability of calibration can be taken is lastly dependent on the ever developing state of the art. With respect to computational speed however, the analytic method has the drawback on closer examination of the photographic coordinates of a large number of control points having to be measured individually by means of an optical comparator, before the automatic process can be started: this is a time consuming task, subject to personal error and misidentification of points. The projective approach on the other hand avoids this vulnerable work phase completely: the projected control points are just matched with their chart counter parts.

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