Aerotriangulation Tests[*]

G. C. TEWINKEL,
Coast and Geodetic Survey
Washington, D. C.

ABSTRACT: *A test of accuracy of horizontal aerotriangulation was recently conducted by the Coast and Geodetic Survey. The purpose was to derive a basis for predicting the nature of errors to be expected from a flight strip of a given number of models, control points, etc. The results are expressed in the form of an equation for use in project planning.*

FOREWORD

A SERIES of tests was recently conducted to determine the magnitude and distribution of horizontal errors which arise in aerotriangulation. The purpose of the tests was to determine accuracy criteria for project planning. Answers were desired which would indicate such as how high to fly an aerial camera, and how much control to establish in order to meet specific accuracy requirements. For example, in a certain project it must be assured that an error of 40 feet will not occur with a probability greater than once in 100. Such information was not readily available from photogrammetric literature; moreover, it was fairly obvious that any such data would probably apply to the particular instruments, personnel, methods and idiosyncracies of only one organization.

A test area was selected northward from Miami, Florida, for taking a single strip of aerial photographs in which 12 control stations would be photographed. The strip was more than 100 miles long. The area was probably selected for convenience and accessibility inasmuch as a photographic airplane and field parties were already on the site. It seemed advisable to eliminate high relief from this particular study; thus the flat area was appropriate.

The photographs were taken with 60 per cent overlap with a 6-inch Wild Aviogon film camera from 20,000 feet. Fifty photographs were included in the strip of which 47 were aerotriangulated, and 39 models (89 miles) were actually utilized in the principal phase of the study.

The aerotriangulation was performed with a Zeiss C-8 Stereoplanigraph. The "free" method instrument operation was used wherein no photograph was absolutely controlled. On the other hand, the start of the triangulation was an assumed level orientation and a nominal scale for the first photograph, the other photographs being added in succession to give a connected but uncontrolled three-dimensional system of coordinates for image-points throughout the strip.

Relative orientation was performed in the customary visual routine, no analysis of residual parallaxes being made inasmuch as the maximum residual parallax was found in isolated instances to be in the order of 0.03 mm. Coordinates were recorded at control-points and pass-points alike, but no effort was made to fit control in any manner during the instrument operation. Later the coordinates were adjusted to fit control, using our adaptation of the general method developed at the Army Map Service [1, 2, 3, 4].

The strip was independently triangulated six different times: (1) by the first operator; (2) again by him; (3) by a second operator; (4) by the first operator using the "base-in" setting in the first model instead of "base-out"; (5) by the first operator beginning at the opposite end of the strip; (6) by the first operator with the diapositives rotated 180 degrees.

The Stereoplanigraph coordinates were transformed and adjusted: (1) using a combination of IBM and graphic spline techniques [3]; and (2) using a complete IBM procedure [4].

It is realized that the test was not exhaustive. It would have been informative to

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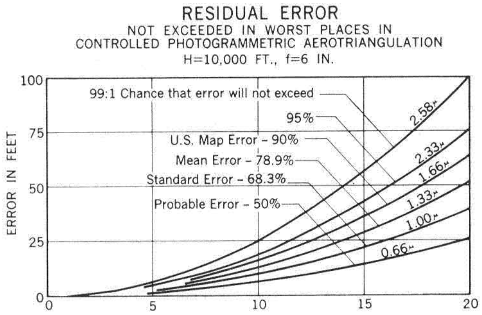

FIG. 1. Different classifications of error.

have had several flights over the area, or other areas, and photography with other cameras. Sets of diapositives printed from the film at different dates might have yielded helpful information. More operators, more instruments and more triangulations could have been employed. Obviously, economies prevented further tests but they resulted in data which were expected from nearly three years of aerotriangulation experience.

The primary interest in the test was the "huge" residual error (such as 2.58 times the root-mean-square error, having a frequency of one time in 100) rather than the usual 90% map error, although the two terms are related mathematically ([7, 8]. Consequently, the analysis considers only the errors in the worst places of the strip, namely, those farthest from the points that were used as control (Figure 1).

RESULTS

Several general findings may be of interest.

1. The curves of systematic errors cannot be predicted but must be determined from ground-control for each strip (Figure 2).

2. Three *valid* ground-control points are necessary in each strip; considerably better results can be obtained with four points; having five points is very desirable as a practical minimum.

3. The average position determined from two triangulations run in opposite directions between two control points, cannot be relied on to replace a third control point near the center of a strip. (See also Paragraph 12.)

4. The maximum error occurs midway between adjacent control points.

5. The magnitude of error varies essentially as the square of the number of models between adjacent control points.

6. Although not a product of this test,

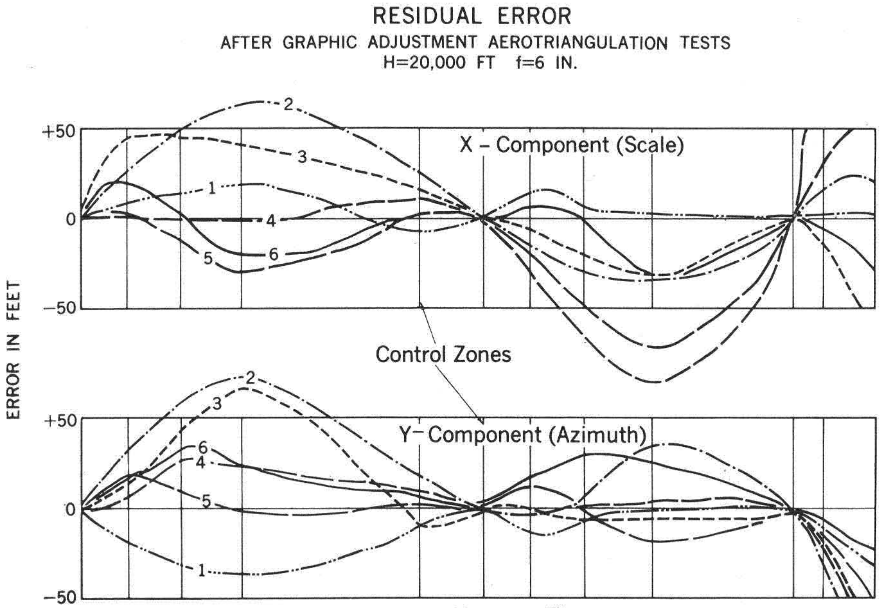

FIG. 2

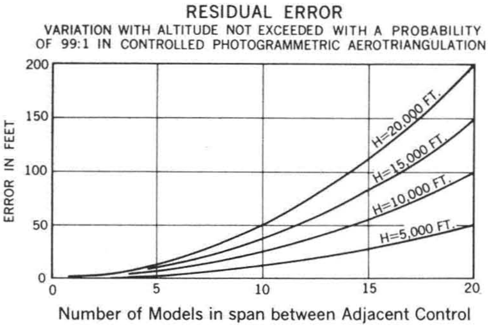

FIG. 3. Error variation with camera altitude.

it appears logical to assume that the error varies almost directly with the flight altitude where the number of models between control is considered constant (Figure 3).

7. An effective formula for determining the error to be expected at a specific image is

$$\text{Error} = k(m_t m_i - m_i^2)$$

where m_t is the number of models between adjacent control points in the span where the image lies, considering that the strip is controlled by three or more points, m_i is the number of models from the image to the nearest control point, and k is an appropriate error factor. For example, the tests indicated that $k=1$ is an appropriate value for a flight altitude of 10,000 feet where the huge error in feet will have a chance of only 1 to 100 of being exceeded. If an image is in the third model in a span of 10 models between control, the corresponding huge error is $1(10\times3-3\times3)=21$ feet.

8. For consistent results, the control should be about equally distributed throughout the strip: two control points near each other constitute essentially only one point. The method of adjustment is not harmed if all the points lie on a line, or on the flight line. It is not essential that two or more points lie in the first nor any other model. In other words, a great deal of latitude exists relative to the amount and placement of the control.

9. Only five minutes are required by the IBM 650 to make the complete adjustment from instrument coordinates to adjusted ground rectangular coordinates for all the observed points in the strip.

10. The residual maximum error after adjustment varied from 5% to 10% of the unadjusted deviation based on two end control points (Table 1). Thus it may be reasoned that the residual errors can be reduced by adopting any techniques which will reduce the unadjusted deviations (sys-

TABLE 1

HORIZONTAL DEVIATIONS RESULTING FROM AN AEROTRIANGULATION OF 39 MODELS (89 MILES) AT A NEGATIVE SCALE OF 1:40,000 BASED ON THREE HORIZONTAL CONTROL POINTS

Run No.	*Unadjusted deviations based on two end control points*				*Residual errors at two worst places after graphic adjustment*		
	x (scale)	*y (azimuth)*	*Radial*	*Radial*	*x (scale)*	*y (azimuth)*	*Radial*
	mm. at approx. twice negative scale			*Feet*	*Feet*	*Feet*	*Feet*
1	−8.70	+11.06	14.07	935	+ 7	−59	59
					+44	− 6	44
2	−7.58	+ 8.62	11.48	778	+52	+48	71
					+12	+29	31
3	−7.44	+ 8.25	11.11	868	+22	+41	47
					+12	−11	16
4	−4.78	+ 5.67	7.42	586	−12	+ 1	12
					−47	− 1	47
5	−7.57	+ 7.16	10.42	834	−40	−26	48
					−31	−26	40
6	−7.84	− 0.21	7.84	535	−32	− 2	32
					+ 9	+17	19

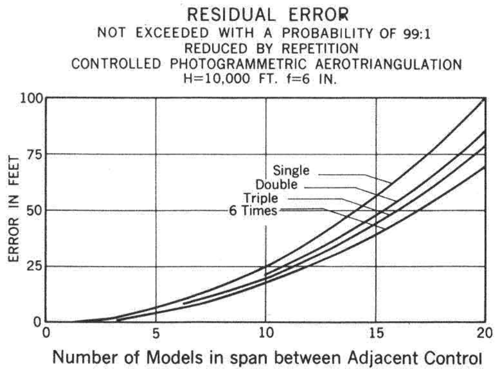

FIG. 4. Expected residual error reduced by repetition.

tematic errors) [5, 6].

11. The residual error after adjustment seemed to be composed of two independent parts, both of which may be accidental but having different periodicity. One part is clearly of random or accidental nature; the other was fixed more or less throughout the test except that it was different in the two "worst" places, possibly indicating that it varies from job to job also. The limited extent of the test was insufficient to determine the nature of the recond part.

12. Results can be improved by running the strip through the Stereoplanigraph more than once independently, making separate independent adjustments, and adopting the average positions of points (Figure 4). The likelihood of obtaining a given "huge" error is thus reduced; or else, with the same likelihood, the corresponding expected error is smaller. For example the factor k in Paragraph 7 may be reduced possibly by such a formula as

$$k' = a + (k - a)n^{-1/2}$$

where a is the non-random residual mentioned in Paragraph 11, and n is the number of repetitions. Thus if a is equal to $\frac{1}{2}k$ (which was approximately true), k' may be reduced by two operations to 85% of the value expected from a single operation; 79% by three operations; 75% by four operations, etc.

13. The graphs of the residual errors indicate that the x-curve (scale curve) is of a higher degree than the y-curve (azimuth curve), as has been suggested by operators from time to time [9]. Therefore, Harris [4] applied a cubic correction to the x and quadratic to the y curves in the computational program.

14. No data have been accumulated as yet relative to vertical bridging and elevations, but to answer questioners, a logical conclusion seems to be: First, the vertical residual error factor is expected to be 1.67 times the horizontal error factor from geometric considerations. Secondly, vertical control points are required in pairs on opposite edges of a strip, a pair constituting a single control in the sense where it is compared to a horizontal point. Thirdly, vertical accuracy specifications are frequently more stringent than horizontal. Lastly, vertical discrepancies and curves are usually more erratic and less predictable than horizontal. Consequently, a considerably larger number of vertical control points than horizontal are usually required.

15. The purely computational method of adjustment did not alter the general error conclusions based on the graphic adjustment, except that running the strip in opposite directions gave a consistently different direction of residual error for the x (scale) component only. The advantage of the computational technique lies in the speed and the corresponding release of manpower for other essential work.

16. The use of four control points located at the ends, and 20% in from the ends of the strip, resulted in essentially the same residual error in the worst place as was encountered when three evenly distributed points were used where the number of models between control was equal.

THE COMPUTATIONAL ADJUSTMENT METHOD

The test was first adjusted several months ago using the graphic-numerical method [3]. Later, the same instrument data were adjusted by means of the Harris method [4] using both three and four control points. This method is described in a pamphlet of the Coast and Geodetic Survey. Some features of this method will be pointed out.

The adjustment equations are:

$$Ax^3 + Bx^2 + Cx - 2Dxy - Ey + F - c_x = 0$$

$$3Ax^2y + Dx^2 + Cy + 2Bxy + Ex + G - c_y = 0.$$

The first is cubic in x and the second is quadratic. Seven unknown constants $A \cdots G$ exist which are found from four or more control points, by applying the principle of least squares. If only three control points are available the A-terms are suppressed, both curves are then quadratic, and least squares is invalid.

The equations are significant inasmuch as they are interrelated rather than independent [1, 2], thus not only allowing the use of a minimum number of control points, but placing a minimum restriction on their location.

The least squares idea is considered important since smooth curves most nearly agree with all the conditions, and residual values are displayed which indicate the presence of mistakes and the magnitude of variance.

The program incorporates three coordinate transformations: (1) the instrument coordinates into an "axis-of-flight" system; (2) the coordinates of control stations into the same axis-of-flight system, in which the adjustment is applied; and (3) the adjusted coordinates in the axis-of-flight system back into the system of the control data. These transformations allow any flight line direction to be designed to connect control points, and at the same time recognize that the c_x and c_y deviations are propagated in different manners related to the flight line.

The program is completely self contained: all transformation and adjustment coefficients are automatically derived and applied.

Now that the system has been programmed, it forms a relatively inexpensive method of testing with one set of instrument data such ideas as the effect of the number and spacing of control and cantilever extension. Moreover, any future production job which may have more control than necessary can be used by the research personnel afterwards to add more statistical data to the files.

ANALYTIC AEROTRIANGULATION

Studies continue on analytic aerotriangulation techniques which some day may replace the instrumental-computational method used in these tests. The advantage which might be gained from the analytic approach is in effect to decrease the maximum uncorrected deviation (Table 1) by applying corrections for all known sources of systematic error as well as to measure coordinates with greater accuracy. For example, it is possible to correct for film shrinkage in the analytic approach, whereas it is not possible with the instrumental methods. Also, the analytic solution allows a complete coordination between adjacent flight strips as well as between the vertical and horizontal dimensions.

CONCLUSIONS

The tests have given a few criteria for use in project planning where aerotriangulation might be appropriate. These criteria are recognized as being based on far too few tests. Moreover, they apply to one organization and may be quite different for another.

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