# *A Study of the Effect of Differential Film Shrinkage on the Space Resection and Orientation of an Aerial Photograph**

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ABSTRACT: *The effect of differential shrinkage on space resection and orientation of aerial photography has long been clouded with speculation. This work represents an attempt to gain some insight as to exactly what happens to the space geometry when some reasonable values of shrinkage are applied to the film.*

## PROCEDURE

FROM a given set of fictitious photography two characteristically common configurations of pass points were selected. (See diagram.) On the basis of values obtained from "Kodak Materials for Aerial Photography," Kodak Publication No. M-4 Fourth Edition, a range of shrinkages was applied to the film coordinates of the aforementioned control points. The fictitious photo and control points selected are given in Table 1. To the film coordinates in Table 1 shrinkages in the $x$ direction of 0.00%; 0.04%; 0.08%; 0.12% were applied; the same values were applied in the $y$ direction. Two configurations were then selected:

configuration #1—pts. $C_3\ E_1\ E_5$
configuration #2—pts. $C_1\ C_5\ E_3$

For each configuration 16 cases were considered featuring the above values of shrinkage applied to the film coordinates.

TABLE 1

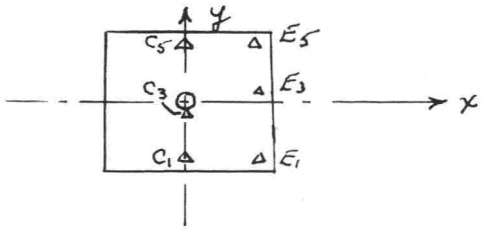
GROUND CONTROL DATA

| *Pt. No.* | *Lat.* | *Long.* | $X_0$ *ft.* | $Y_0$ *ft.* | $Z_0$ *ft.* |
|---|---|---|---|---|---|
| $C_1$ | 39 52′00″ | 76 37′30″ | 2,315,798.79 | 196,294.48 | 800.00 |
| $C_3$ | 39 56′00″ | 76 37′30″ | 2,315,489.43 | 220,577.52 | 3,000.00 |
| $C_5$ | 40 00′00″ | 76 37′30″ | 2,315,180.07 | 244,860.60 | 1,000.00 |
| $E_1$ | 39 52′00″ | 76 32′30″ | 2,339,189.88 | 196,603.52 | 1,000.00 |
| $E_3$ | 39 56′00″ | 76 32′30″ | 2,338,857.61 | 220,886.25 | 600.00 |
| $E_5$ | 40 00′00″ | 76 32′30″ | 2,338,525.34 | 245,169.03 | 1,800.00 |

PHOTO COORDINATES (*measured*)

| *Pt. No.* | $x$ *mm.* | $y$ *mm.* | $z$ *mm.* |
|---|---|---|---|
| $C_1$ | 0.000 | −95.600 | 150.00 |
| $C_3$ | 0.000 | − 2.168 | 150.00 |
| $C_5$ | 0.000 | 88.854 | 150.00 |
| $E_1$ | 88.983 | −96.004 | 150.00 |
| $E_3$ | 88.065 | − 2.577 | 150.00 |
| $E_5$ | 89.702 | 90.742 | 150.00 |

* This paper was awarded First Prize in the 1957 competition for the Bausch & Lomb Photogrammetric Award.




Case 1 would be 0.00% in $x$ and 0.00% in $y$; case 2 0.00% $x$, and 0.04% $y$; case 3 0.00% $x$, 0.08% $y$; case 4 0.00% $x$, 0.12% $y$; case 5 0.04% $x$, 0.00% $y$; case 6 0.04% $x$, 0.04% $y$; etc.

Using the Church Method of Space Orientation and Resection, and the I.B.M. "650" electronic calculator, values were computed in all 32 cases for the following space coordinates of the exposure station:

| | |
|---|---|
| $X_0$ | $x$ coordinate of exposure station |
| $Y_0$ | $y$ coordinate of exposure station |
| $Z_0$ | $z$ coordinate of exposure station |
| $T$ | tilt |
| $S$ | swing |
| $A$ | azimuth |

From these, the changes in these quantities due to shrinkage were calculated; these are given in Table 2. The incremental changes in the tabulated quantities as functions of shrinkage in the $x$ and $y$ direction are shown in graphs 1 to 10. Graphs 1 through 5 cover

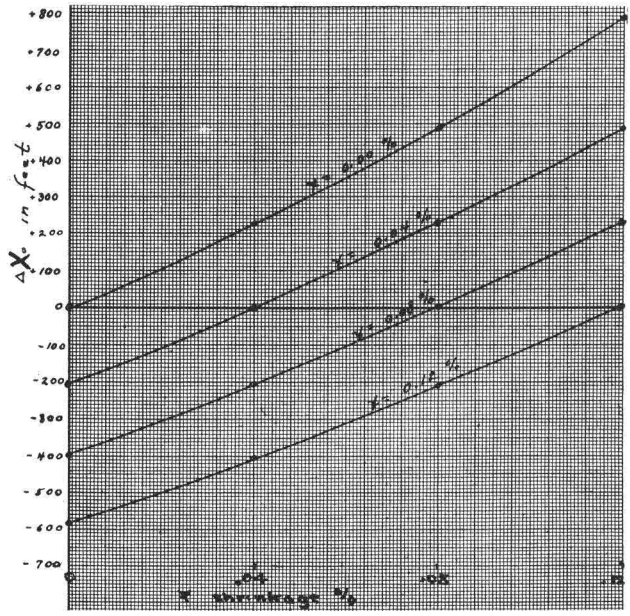
TABLE 2

CHANGES IN EXPOSURE STATION COORDINATES DUE TO FILM SHRINKAGE

| *Configuration No. 1* | | | | *Configuration No. 2* | | | |
|---|---|---|---|---|---|---|---|
| *Case No.* | $\Delta X_0$ *ft.* | $\Delta Y_0$ *ft.* | $\Delta Z_0$ *ft.* | *Case No.* | $\Delta X_0$ *ft.* | $\Delta Y_0$ *ft.* | $\Delta Z_0$ *ft.* |
| 1 | 0.00 | 0.00 | 0.00 | 1 | 0.00 | 0.000 | 0.00 |
| 2 | 229.39 | 5.78 | 157.55 | 2 | 26.18 | −0.380 | 15.81 |
| 3 | 486.22 | 12.32 | 329.14 | 3 | 52.41 | −0.770 | 31.64 |
| 4 | 781.84 | 19.97 | 520.51 | 4 | 78.77 | −1.150 | 47.37 |
| 5 | −209.10 | − 5.50 | −131.60 | 5 | −26.19 | −0.110 | 0.00 |
| 6 | −000.24 | − 0.27 | 15.45 | 6 | − 0.03 | −0.500 | 15.81 |
| 7 | 229.37 | 5.49 | 173.12 | 7 | 26.18 | −0.880 | 31.64 |
| 8 | 486.22 | 12.04 | 344.65 | 8 | 52.45 | −1.260 | 47.47 |
| 9 | −402.38 | −10.48 | −255.03 | 9 | −52.40 | −0.220 | 0.00 |
| 10 | −209.57 | − 5.75 | −116.23 | 10 | −26.26 | −0.600 | 15.81 |
| 11 | −000.62 | − 0.56 | 30.83 | 11 | − 0.06 | −0.990 | 31.64 |
| 12 | 229.18 | 5.22 | 188.56 | 12 | 26.20 | −1.380 | 47.47 |
| 13 | −582.31 | −15.12 | −371.59 | 13 | −79.27 | −0.340 | 0.08 |
| 14 | −402.94 | −10.76 | −239.70 | 14 | −53.23 | −0.720 | 15.81 |
| 15 | −210.09 | − 6.04 | −100.88 | 15 | −27.05 | −1.100 | 31.64 |
| 16 | −000.96 | − 0.85 | 46.25 | 16 | − 0.83 | −1.490 | 47.47 |

| *Configuration No. 1* | | | *Configuration No. 2* | | |
|---|---|---|---|---|---|
| *Case No.* | $\Delta T$ *seconds* | $\Delta S$* *degrees* | *Case No.* | $AT$ *seconds* | $AS$* *degrees* |
| 1 | 0.0 | 0.00 | 1 | 0.0 | 0.000 |
| 2 | 202.1 | 19.38 | 2 | 3.5 | 2.175 |
| 3 | 852.8 | 36.70 | 3 | 10.1 | 4.34 |
| 4 | 1,943.1 | 50.15 | 4 | 29.5 | 6.51 |
| 5 | 192.6 | −17.79 | 5 | 1.2 | −2.178 |
| 6 | 0.6 | − 0.02 | 6 | 1.2 | −0.003 |
| 7 | 201.0 | 19.37 | 7 | 3.5 | 2.173 |
| 8 | 852.8 | 36.69 | 8 | 9.5 | 4.34 |
| 9 | 656.6 | −31.71 | 9 | 5.9 | −4.35 |
| 10 | 193.8 | −17.82 | 10 | 1.2 | −2.183 |
| 11 | 00.0 | − 0.06 | 11 | 0.6 | −0.006 |
| 12 | 200.5 | 19.35 | 12 | 2.4 | 2.172 |
| 13 | 1,271.6 | −41.81 | 13 | 21.3 | −6.572 |
| 14 | 658.1 | −31.73 | 14 | 6.5 | −4.42 |
| 15 | 193.8 | −17.86 | 15 | 1.8 | −2.25 |
| 16 | 00.0 | − 0.09 | 16 | 0.0 | 0.0 |

* Note: values for $\Delta S$ and $\Delta A$ are the same.

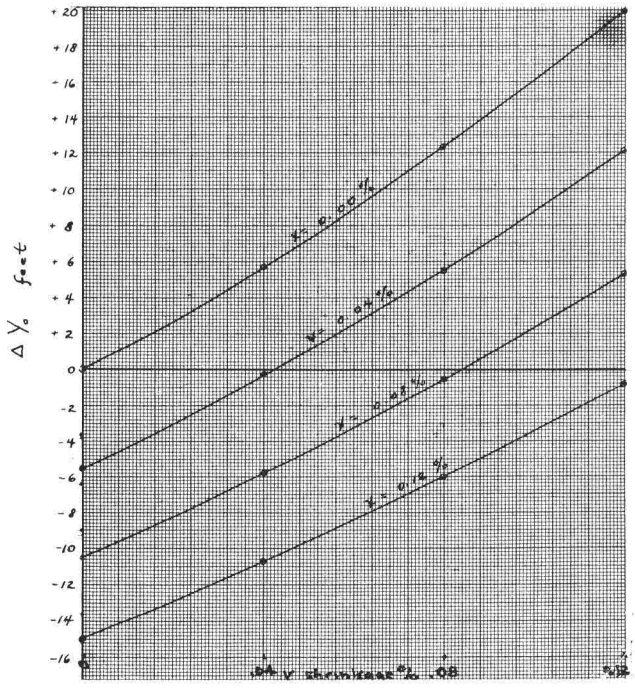



GRAPH 1. Ordinates $-\Delta X_0$ in feet; abscissae $-y$ shrinkage in per cent.

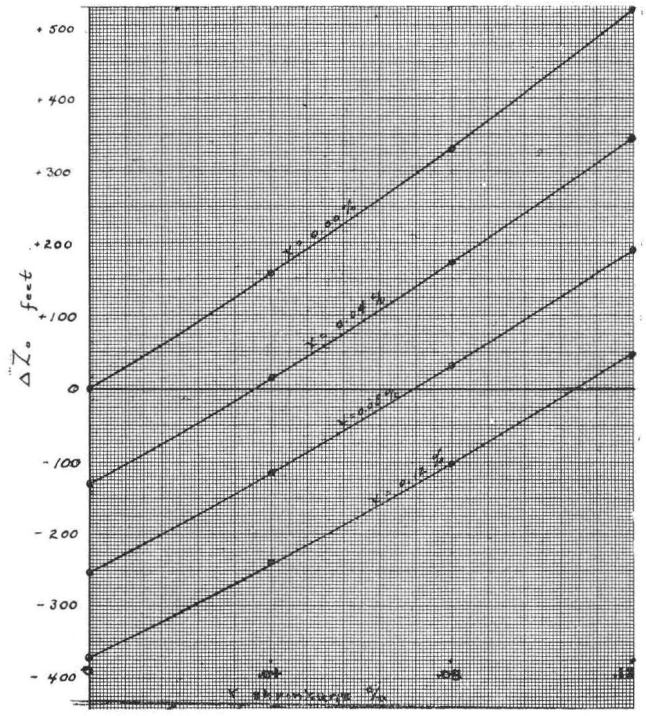
configuration No. 1. 6 through 10 cover configuration No. 2.

## DISCUSSION OF RESULTS

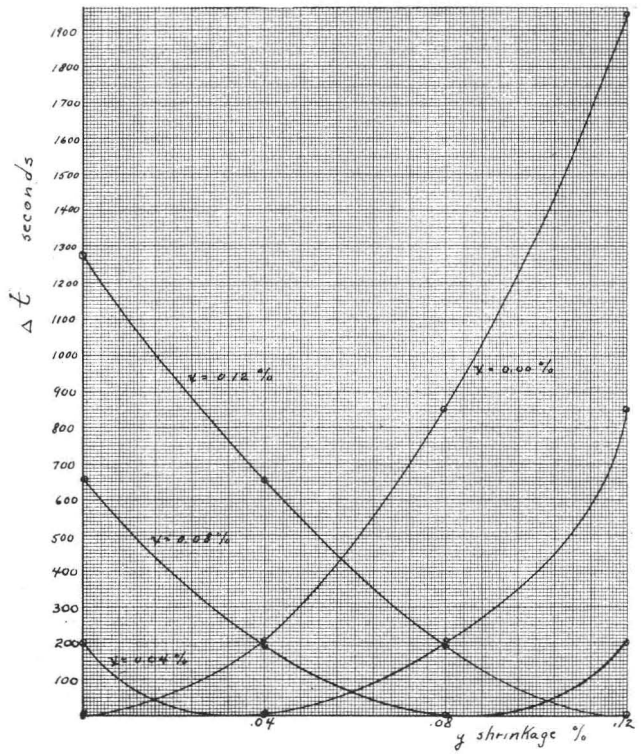
It must be borne in mind that the work involved only two particular configurations and it by no means approaches a general analysis of the problem of differential shrinkage. Values for shrinkage were based on expected maximum average values which



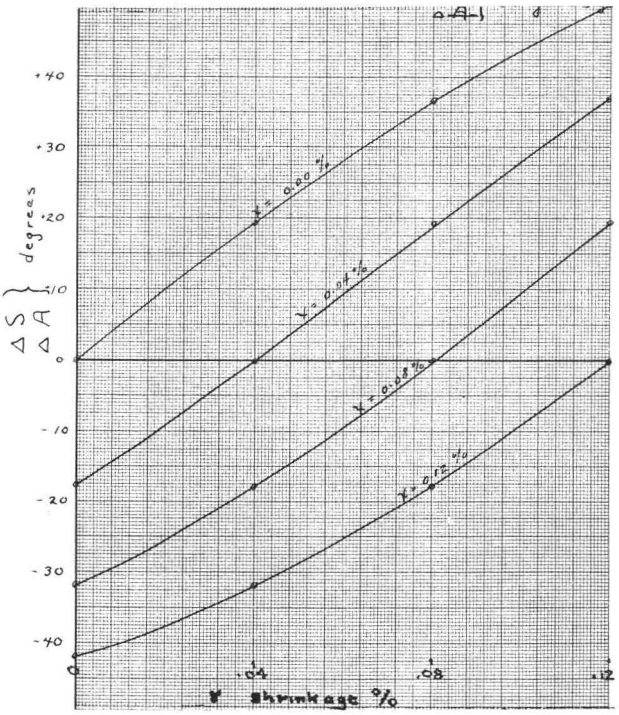

GRAPH 2. Ordinates $-\Delta y_0$ in feet; abscissae $-y$ shrinkage in per cent.



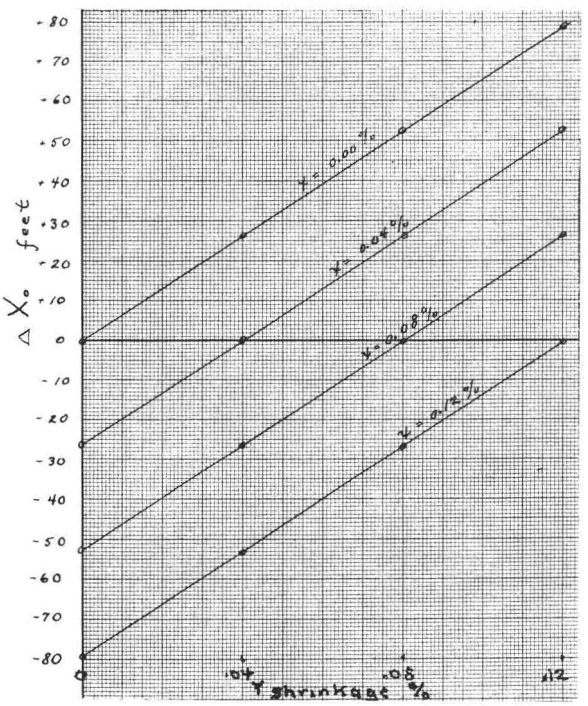

GRAPH 3. Ordinates $-\Delta Z_0$ in feet; abscissae $-y$ shrinkage in per cent.



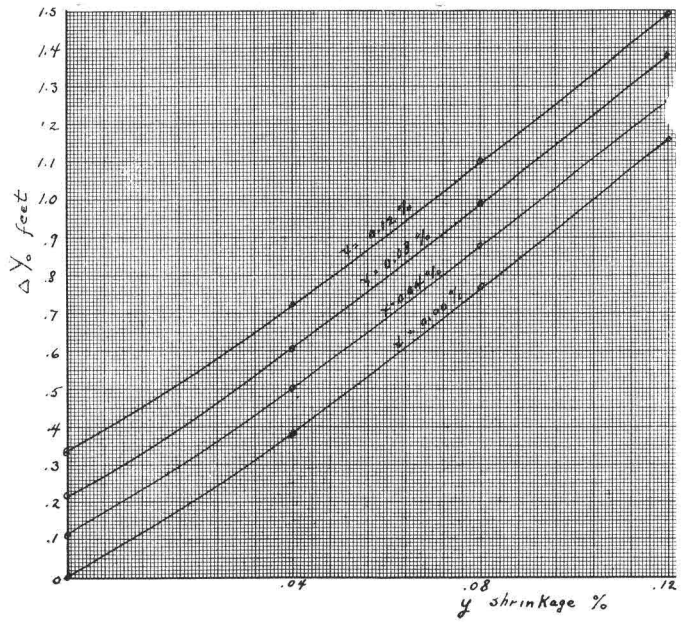

GRAPH 4. Ordinates $-\Delta t$ in seconds; abscissae $-y$ shrinkage in per cent.




GRAPH 5. Ordinates $-\Delta S$ and $\Delta A$ in degrees; abscissae $-y$ shrinkage in per cent.




GRAPH 6. Ordinates $-\Delta x_0$ in feet; abscissae $-y$ shrinkage in per cent.




GRAPH 7. Ordinates $-\Delta y_0$ in feet; abscissae $-y$ shrinkage in per cent.

are integrated values over the length of the film strip used in testing for dimensional stability. Values of maximum local distortion may prove more critical. As yet no data on such values are available. A detailed analysis of the reasons behind the occurrences noted from the graphs is beyond the scope of this work. Certain pertinent occurrences should be considered:

In both configurations, the values of $\Delta Y_0$ are considerably less than corresponding values of $\Delta X_0$.

In configuration 1, $\Delta Z_0$ is affected not only by differential shrinkage but also by equal values of shrinkage in both $x$ and $y$ directions.

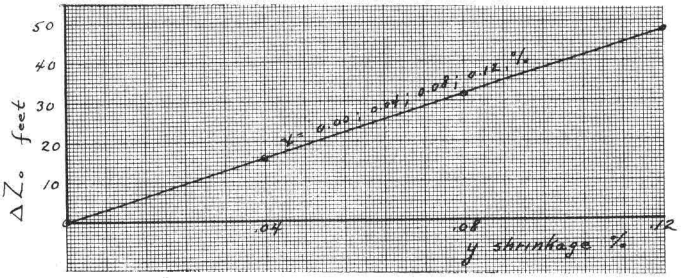
In configuration 2, $\Delta Z_0$ is not affected by shrinkage in the $x$ direction.

In graph 9, configuration 2, it should be obvious that the ordinate for $x$ of 0.04% and for $y$ of 0.04% as well as $x$ of 0.08% and $y$ of 0.08% should be 0.00. The observed ordinates are due to residual error in the computations.

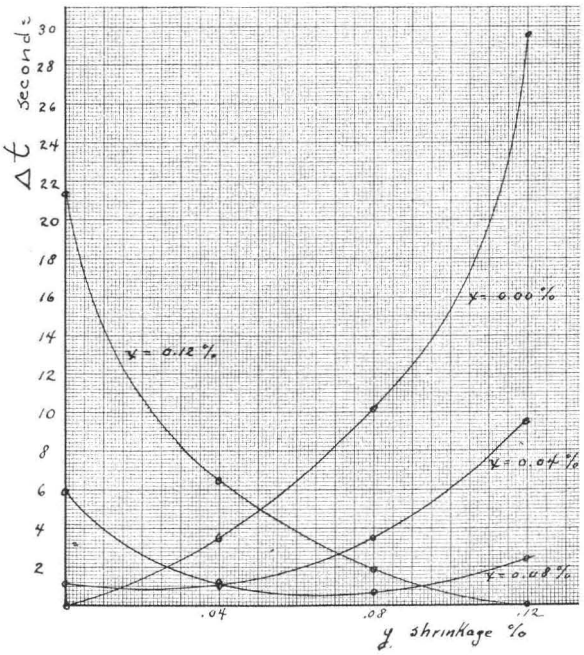
In configuration 1, the $Y_0$ displacement is in both directions while in configuration 2, the $Y_0$ displacement occurs in one direction only, meaning that in the latter instance $Y_0$ is affected by absolute as well as differential shrinkage. It is suggested that the ordinates in graph 2 for equal shrinkage in $x$ and $y$ are also probably attributable to residual error in the computations.

Swing and Azimuth appear to be affected only by differential shrinkage.

It is obvious that much of the shrinkage effects depend on the configuration. Many aspects of the results are seemingly per-



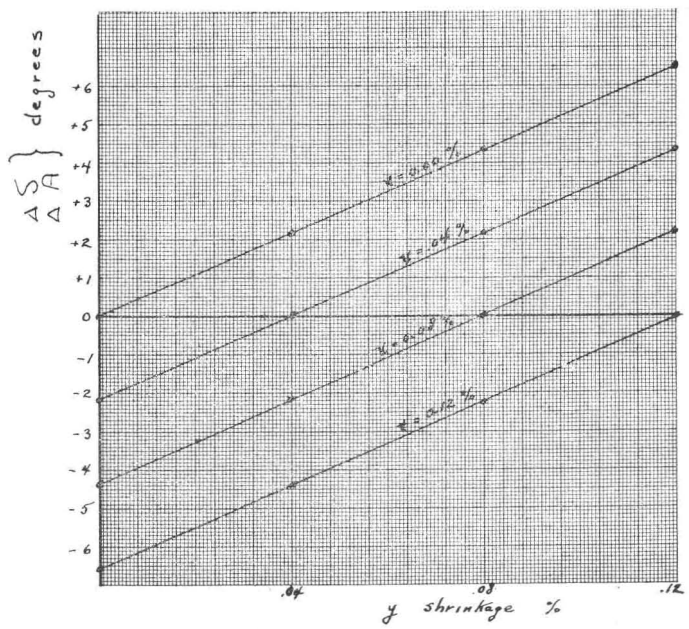

GRAPH 8. Ordinates $-\Delta Z_0$ in feet; abscissae $-y$ shrinkage in per cent.




GRAPH 9. Ordinates $-\Delta_t$ in seconds; abscissae $-y$ shrinkage in per cent.

plexing. There appears to be a need for more intensive research to be done in this field. To date very little concern has been expressed over the effects of shrinkage on space geometry. The effect on the stereo model will be different but it is of course directly related to the change in exposure station coordinates.

It is hoped that the qualitative results of this work will serve to stimulate further discussion and research in this aspect of Photogrammetry about which comparatively little is known.




GRAPH 10. Ordinates $-\Delta S$ and $\Delta A$ in degrees; abscissae $-y$ shrinkage in per cent.