graph, is similar to the right triangle relating the focal-point to the ground datum of these points, the relationship between the scales of the x-coordinate and its rectification is similar to the ratio between the focal-length and the slant distance. Thus, the proportion between the two can be expressed as: $x_r = x/\cos t$.

It is the opinion of the author that the photo interpreter should be prepared to use any and all kinds of photography available over an area of interest. The apparent objection to the use of the panoramic aerial photograph is the difficulty of obtaining accurate measurements because of the constantly changing tilt angle over the photograph.

It is hoped that this report will create interest in developing methods of coping with this problem. Such methods would have to be a unified process for rectifying the entire photograph and would have to satisfy certain requirements of simplicity and accuracy.

To the author, it seems that the methods of rectification can be devised to meet these requirements. The concept of the perspective being the curved surface of a cylinder appears to be as usable as that of a tilted plane for the oblique photograph. The three suggested formulas should enable the photo interpreter to rectify a panoramic aerial photograph onto a horizontal plane using simple, straightforward procedures. The formulas are purposely stated so that calculations can be performed either on a slide-rule or by using a book of standard mathematical tables.

*Stereoscopic Profile-Scanning for Contour Line Information**†

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ABSTRACT: *Contours are used to express both the physiographic character of the terrain and to accurately locate the continuum of points of equal elevation. The use of the profile-scan technique to obtain contours in the latter sense is discussed. A method of converting the Stereoplanigraph into an interim scanning instrument for determining contour information is described. Applications of profile-scanning to mapping systems are briefly discussed.*

I. INTRODUCTION

PHOTOGRAMMETRY, by its very nature, is concerned with the cartographic representation of terrestrial features and their spatial relationships. With stereoscopic plotting instruments these spatial relationships are recreated, at a greatly reduced scale, in the form of the stereo-model. The major photogrammetric task generally is to transform this three dimensional stereo-model into the two dimensional map sheet.

Many recent technical developments in photogrammetry have been directed toward developing equipment and techniques to map the stereo-model more accurately and efficiently.

One of the techniques recently advocated has been that of obtaining contour line information from the stereo-model by profile-scanning(1).

Profile-scanning, that is moving the floating mark over the surface of the stereo-

* The opinions expressed herein do not necessarily represent those of the Navy Department.—*The Author.*

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model along previously selected parallel lines, has been used in the construction of terrain models, the orthographic restitution of aerial photographs and in some automatic contouring systems (2, 3, 4, 5).

From all of the above systems, either directly or as a by-product, it is possible to obtain information for the map portrayal of topography. The natural question to ask is "*How accurate are the contour lines developed as a result of profile scanning and how efficient is this technique?*"

To discuss this question properly we should be aware of some basic concepts concerning the portrayal of terrain features by contour lines.

II. Function of Contour Lines in Cartography

The contour line as a method for the portrayal of relief was proposed by the Englishman, Dr. Charles Hutton, in 1778 when he

> "fell upon a method of connecting together by a faint line all the points on the hill which were of the same relative altitude."

The first published map using the contour system appeared in this country in 1835. However, contour maps did not make their general appearance until much later because of the cost involved in obtaining the contour information (6). At present contour lines are the standard method for showing relief on large-scale maps.

At first the contours were largely sketched in the field rather than being completely surveyed. Contour sketching was considered to be an art as well as a surveying operation. The field sketcher had to rely upon his physiographic knowledge to create "topographic expression" in his work. He was allowed considerable "topographic license" in order to obtain the topographic expression needed. With the advent of stereoplotting equipment capable of plotting contours with engineering accuracy, the art of contour sketching is becoming almost a thing of the past.

Today there exist two schools on the use of contours. One group employes contours as a means of expressing the physiographic character of the terrain, while the other school utilizes the contours in the engineering sense, that is to accurately represent the continuum of points of equal elevation. Since most photogrammetrists fall in the latter group, the discussion will be based

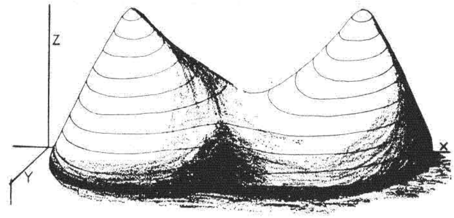


FIG. 1. Pictorial view of topographic feature represented by the surface $[(x-a)^2+y^2][(x+a)^2+y^2]=kz^2$.

upon the concept that the contour line is an accurate representation of points having equal elevations.

III. Mathematical Considerations

Mathematics furnishes one of the most powerful tools to approach the problem of evaluating contour line information. When a mathematical model of the contour system is formed, we can then subject our results and conclusions to rigorous analysis and obtain quantitative data on our systems.

A pictorial view of a hypothetical landscape is shown in Figure 1. The surface of the landform can be represented in the form of a mathematical model. It is the surface represented by the locus of a point in three dimensional space whose rectangular coordinates satisfy the equation

$$[(x-a)^2+y^2][(x+a)^2+y^2]-kz^2=0$$

A two dimensional contour line map of this landform is shown in Figure 2. The contour lines are the family of curves, in the x, y plane that satisfy the equation

$$[(x-a)^2+y^2][(x+a)^2+y^2]=0$$

where c is a constant for each contour line. This is the mathematical analogue of conventional stereoplotting of contour lines.

Vertical profiles of the landscape of Figure 1 are obtained by intersecting the surface with planes perpendicular to the

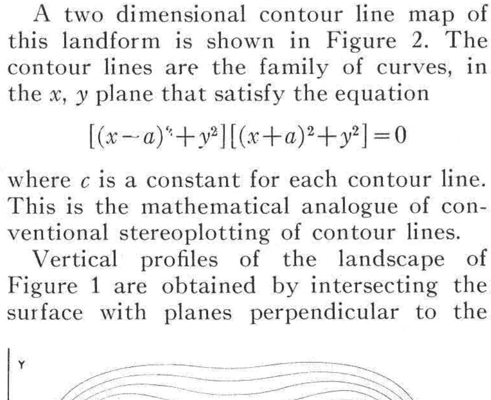


FIG. 2. Representation of the surface shown in Figure 1 by the use of contour lines.

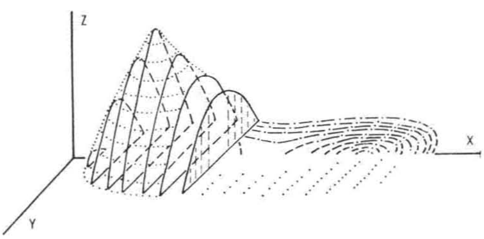


FIG. 3. Development of contour information by dropping dots during the profiling operation.

x, y plane and parallel to the y, z plane. This is illustrated in Figure 3. These profile curves when projected into the x, y (map) plane will be represented by a family of straight lines. Contained within these lines are points corresponding to contour line elevations. If only the points corresponding to contour line elevations are projected then a series of points in the x, y plane will be obtained. These points, separated by the profile-scan interval, represent the information available to determine the shape and position of the contour line curves. Connecting, with a smooth curve, points of the same elevation, an approximating curve is obtained. In effect the contour curve between the scan intervals must be interpolated. The degree to which the true contour line can be interpolated depends upon the shape of the curve and the number of points available (9).

As has been intuitively suspected, profile-scanning for contour information is an approximation process. Therefore we must quantitatively assign the degree of approximation desired.

The desired approximation is prescribed in the National Map Accuracy Standards (7). Basically it requires that 90 per cent of

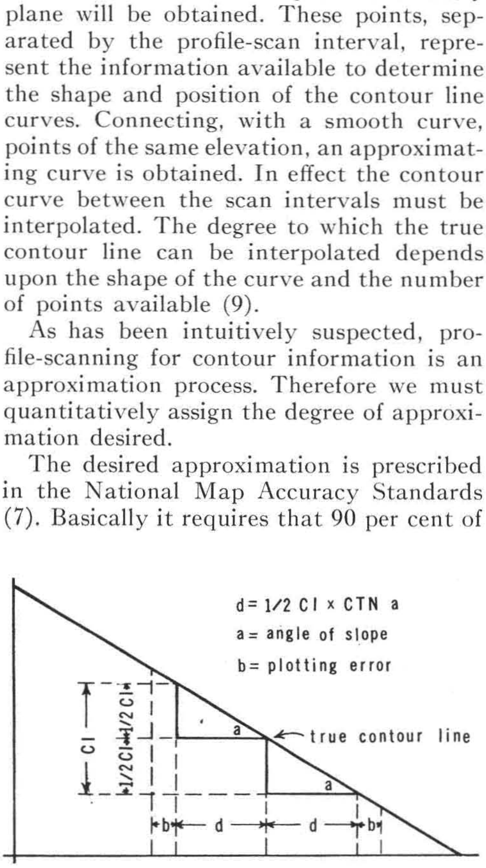


FIG. 4. Horizontal error allowable in location of contour line under National Map Accuracy Specifications.

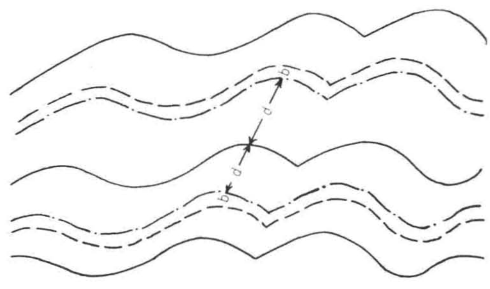


FIG. 5. Error envelope of allowable horizontal displacement from the true contour as determined from National Map Accuracy Standards.

the contours tested shall not be in error by more than one-half the contour interval. In addition the apparent vertical error may be decreased by assuming a horizontal shift of the contour by 1/50th of an inch at publication scale.

Figure 4 illustrates the relationships between the allowable error in horizontal position as specified by the standards.

Figure 5 shows three contour lines and the envelope within which 90 per cent of the length of the contour line must fall to meet specifications. It is based upon the assumption that the position and shape of the true contour lines are known.

Since the slope of the ground is determined in nature, and the contour interval is usually a function of the map scale, these two factors are considered as constants (8).

The factor of the scan interval is a variable which can be assigned at our discretion. It is the most crucial factor, affecting both the accuracy and the efficiency of the contour line scanning system.

The terms "line scan" and "points" have been thus far used without definition; however, difficulties will arise if precise definitions are not now given. Point will be defined as the area under the floating dot of the stereoplotting instrument. Line will be defined as the area traced as the floating dot is moved.

The maximum accuracy in the profile-scan system will occur when the scan width interval is equal to the diameter of the floating dot. This certainly will not be the most efficient scan interval when compared with conventional photogrammetric contour plotting. Efficiency greater than that obtained by conventional photogrammetric contour plotting will occur when the total lengths of the several profile-scans are less than the total lengths of the true contour

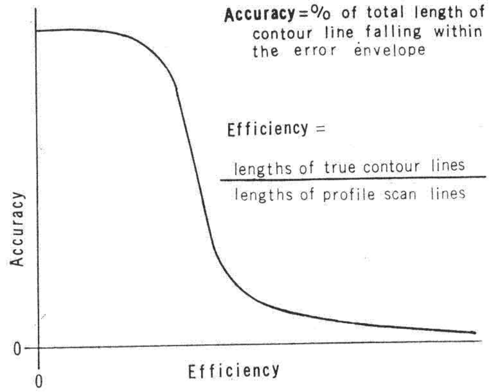


FIG. 6. Conjectured relationship between accuracy and efficiency.

lines. This can be illustrated by the graph of Figure 6.

A general theoretical development of the relationships between efficiency, accuracy and scan interval is not possible at this time because the shapes and positions of the true contour lines on the ground are generally random variables in nature. For specific stereomodels an empirical relationship is always possible.

IV. OPERATIONAL CONSIDERATIONS

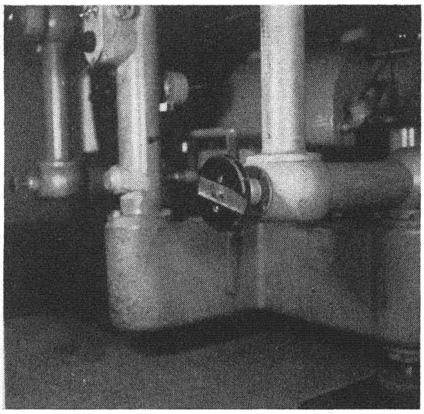
It was noted earlier that profile-scanning is used in the production of restituted orthophotographs. Orthophotographs have been used as planimetric map substitutes (4). Since contour information can be developed by the same profiling operation, it should be possible to produce both orthophotographs and contour information simultaneously during the profile-scan operation.

The U. S. Navy Hydrographic Office is currently conducting a preliminary study on the feasibility of such a mapping system. Our present efforts are directed toward establishing empirical information on the accuracy and efficiency of the profile-scan contour technique. If acceptable contours can be constructed from the profile information, then future studies will be in the direction of designing specific "hardware" for this mapping system.

For the purposes of this study, the Zeiss C-5 Stereoplanigraph was converted into an interim scanning instrument. However, before it could be used for contour-line profile-scanning it was necessary to develop a system to automatically position a dot on the map manuscript each time the floating mark passed over the various contour line elevations.

The system adopted for the preliminary investigations consisted of (a) transference of the z pedal motion to the y handwheel, (b) attachment to the y reversal shaft of an electrical commutator to activate the dot positioning device (Figures 7a and 7b), (c) disengagement of the coordinatograph y movement, and (d) mounting of the dot positioning device on the coordinatograph movement—Figures 8a and 8b.

For profile scanning the photographs are oriented in the normal manner. The commutator and the dot positioning device are then attached to the instrument. The floating mark is then moved to the beginning of the first scan line. The y and z motions are transposed by the y and z reversal lever. The y coordinatograph movement is disengaged by means of the clutch. The stereo-



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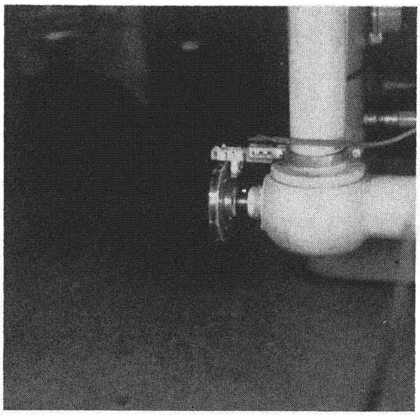

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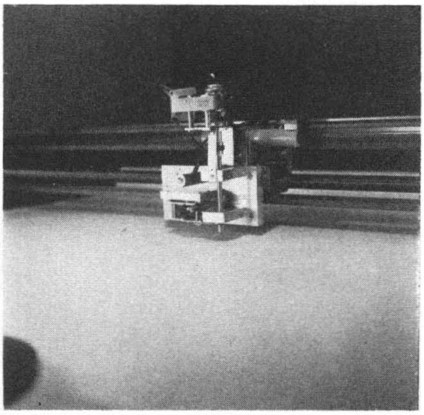
FIG. 7. Electrical commutator attached to the Y reversal shaft of the C-5 Stereoplanigraph.



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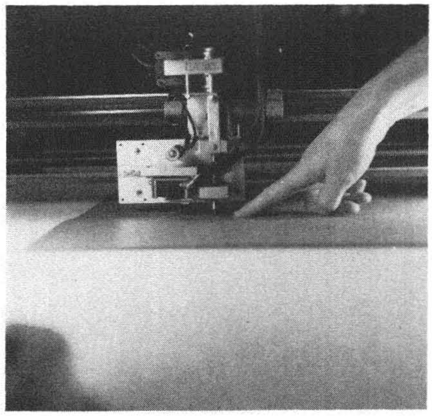

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FIG. 8. Dot carving device attached to the coordinatograph of the C-5 Stereoplanigraph.

operator keeps the floating mark on the surface of the stereo-model with the y handwheel and moves along the scan line with the x handwheel. The x movement is transferred to the dot positioning device mounted on the coordinatograph. Each time the floating mark rises or falls through a z increment, corresponding to the contour interval, the electrical commutator causes the continually rotating drill, in the dot positioning device, to cut a small dot in the coating of the plastic. When the scan line has been completed the y and z reversal lever is disengaged, and the y coordinatograph movement is engaged. The floating mark is moved to the starting point for the next scan line and the procedure repeated.

Operational tests of this system are presently being conducted using aerial photography flown over the Ft. Sill, Oklahoma test area. These tests will provide the basic empirical information needed to answer the question, "How accurate and efficient is the line-scan contour technique?"

V. SUMMARY

There are many possible variations in the methods used to obtain mapping information from the aerial photograph. Profile-scanning required to produce orthophotographs and terrain models can furnish, without additional effort, valuable contour line information for the preparation of contour maps. The relationships between the accuracy and efficiency of the contours produced by profile scanning cannot be presently determined. From theoretical considerations we can get a "feeling" of the accuracy possible. As operational test results become available and are analyzed, it may be possible to answer basic questions on the accuracy and efficiency of the line-scan contour technique.

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