The Stereoscopic Space-Image

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ABSTRACT: *In this paper is developed a new method of evaluating the amount of distortion an interpreter obtains in the space-image observed when he views aerial photographs stereoscopically. The method is based on the hypothesis that to produce a "scale model" space image, the apparent distance of a given frontal plane and the interpupillary distance must be in the same ratio as the actual distance of the given frontal plane from the camera base-line and the length of the base-line.*

INTRODUCTION

A PHOTO interpreter needs an evaluation of the amount of distortion in the space-image when aerial photographs are viewed stereoscopically. If known how the space-image compares with a correct scale-model of the scene, the heights of trees and hills, degree of slope, and topography can be better appraised. However, the evaluation of the space-image is subject to limitations because, as an optical instrument, the eye is defective. Each eye produces an image of the appropriate photograph upon its retina, and the optic nerve carries a corresponding report to the brain. Therefore, two images which may be defective go through an interpretation experience which may modify the final impression of the space-image. While this final impression may differ for different observers it is possible to evaluate the space-image in such a manner that these variations are fairly well taken into consideration. The method, presented in this paper, is somewhat different from those described in other papers on the subject in the past few years. It is logical, easy to understand, and easy to apply. While the method is applicable to all photographs taken in parallel and normal to the base-line, the emphasis is on vertical aerial photographs.

DEFINITION OF TERMS

To aid in the discussion several terms will be used. For convenience in explaining these, think of the human head as divided into two symmetrical halves by the *median plane*. Then when the head is held erect the planes passing through the head perpendicular to the median plane are called *horizontal planes*. Planes parallel to the median plane are *sagittal planes*. Consider these planes as extending forward into the space in front of the eyes. Then the planes which cut both the horizontal and sagittal planes at right-angles are *frontal planes*.

When stereoscopic photographs are viewed the space-image will be *orthoplastic, hyperplastic*, or *hypoplastic*. That is, the depth values will be correct, too large, or too small as compared with the apparent size of a stated area in a given frontal plane.

As defined, an orthoplastic space-image would be correct in the sense that it would be in agreement with the model of the object so far as the configuration of all points in a given frontal plane and the depth of all points were concerned. But it might exhibit differences from the model is regard to the angular intervals between points belonging in two different frontal planes. The effect of this would be to modify the direction of lines extending depthwise. Thus lines that were really perfectly sagittal would not appear so. Consequently, an orthoplastic space-image is not *orthomorphic*. In other words, it does not preserve the original shape of the infinitesimal parts. However, since we are hardly aware of these sagittal changes in form, they will not be discussed further. Instead there will be concentration on hyperplastic and hypoplastic variations which are very important.

BASIS FOR EVALUATING THE SPACE-IMAGE

The rule which gives the basis for evaluating the space-image was stated by Heine[1] in 1902. This is: To produce an orthoplastic

[1] Southall, James P. C. (Editor), 1925. *Helmholtz's Treatise on Physiological Optics*, ed. 1, 3 vols., New York, The Optical Society of America, p. 681, vol. 3.

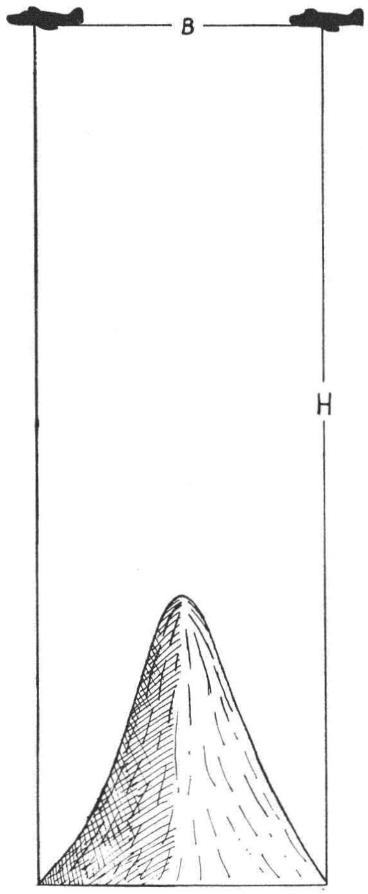

FIG. 1. View of hill photographed from two points.

space-image, the apparent distance of a given frontal plane and the interpupillary distance must be in the same ratio as the actual distance of the given frontal plane from the camera base-line and the length of the base-line. In formula form this relationship may be written

$$S/e = H/B,$$

or, since H/B equals F/P, as

$$S/e = F/P,$$

where S is apparent distance from the eyes of a given frontal plane in the space image; e is interpupillary distance or eye base; H is actual distance of the given frontal plane from the camera base-line; B is length of camera base-line, or distance between camera stations; F is focal-length of camera lens; P is absolute parallax of a point at distance H.

Figures 1 and 2 illustrate the significance of this relationship. Figure 1 represents a view of a hill photographed from two points. If the photographs are so developed and viewed that the space-image is seen at the orthoplastic distance of S, i.e., S/e equals H/B, the image will be orthoplastic. This is as shown in Figure 2. But if the image is seen at a greater distance, i.e., S/e is greater than H/B, the image will be hyperplastic; if at a shorter distance, i.e., S/e is less than H/B, the image will be hypoplastic.

It will be evident that the above relationship clearly explains the remarkable hyperplastic distortion which occurs when viewing aerial photographs. Viz., one result of the general habits of vision is that objects are seldom viewed at a distance of less than 10 inches. Consequently, if the ratio H/B is such that the distance of S for an orthoplastic space image is less than 10 inches—as is usual for aerial photographs—the image will not be seen at the orthoplastic distance, but at a distance of over 10 inches. Exaggerated relief is the result.

For landscape photographs taken with a

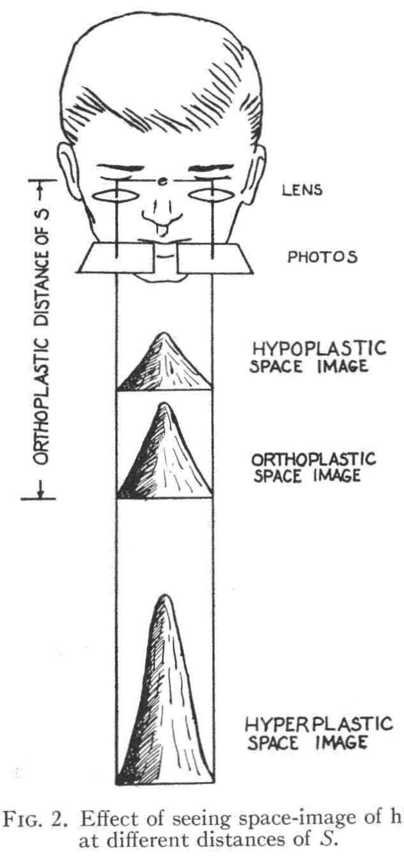

FIG. 2. Effect of seeing space-image of hill at different distances of S.

stereo camera with B equal to e, the distance S for an orthoplastic image must equal H. However, the image will be located much closer to the eyes. Thus the space-image will be hypoplastic. Because of insufficient difference in convergence, such a view will give the notion in the background as if it were painted on a screen.

Effect of Absolute Convergence and Difference in Convergence

Many people accept the premise that the apparent location of the stereoscopic image of a point is obtained by projected rays from the eyes through the respective stereoscopic views of the point, the image being located at the intersection of the rays. Figure 3 illustrates this. This is a very satisfactory assumption for describing the mathematical model, and is the basis for the development of numerous photogrammetric formulae. However, this assumption does not hold for the space-image. If true, the lines would normally intersect at some distance beyond the picture to form the image, and the spacing of the photographs would affect the position and shape of the image. Actually, when photographs are fused stereoscopically, the space-image forms at a point near the plane of the pictures, normally a little to the rear. Increasing the spacing of pictures gives at most a barely perceptible change in the position or shape of the image.

If the photographs are viewed without the aid of some form of viewing instrument, the plane of the pictures is at the actual distance of the prints from the eyes. For a mirror stereoscope, assuming no magnification, the distance will be the total distance that the light travels from the prints to the eyes. With the lens-type stereoscope the lenses form virtual images of the photographs at a distance dependent upon the distance between the lenses and the photographs, and the focal-length of the lenses. The position of the virtual images is the plane of the pictures in this case. Since in a lens-type stereoscope the pictures are ordinarily placed at the focal-point of the lenses, the eyes will be accommodated for distant vision, and the apparent location of the picture plane will vary for different observers due to individual idiosyncrasies.

In Figure 3 the convergence angles for points A and B are angles a and b. The difference in convergence angle between these points is angle a minus angle b or angle d. While the absolute convergence of a point

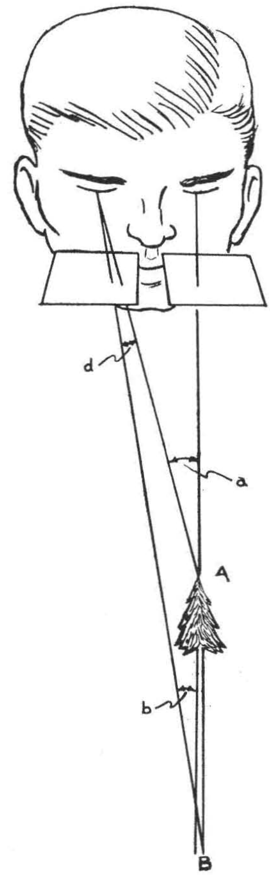

FIG. 3. Drawing showing convergence angles a and b for points A and B, and difference in convergence angle, d, between these points.

does not determine its apparent depth, the difference in convergence angle with each of the other points is used by the brain to locate a point with regard to other points after the position of the image is fixed. A modification of an experiment attributed to Wheatstone[2] neatly illustrates this concept.

Over the end of each tube of two mailing tubes about fifteen inches long, fasten two threads with different spacing (Figure 4). Place the tubes before the eyes in a horizontal plane with the threads at the far ends. Fuse the threads stereoscopically so that only two threads are seen. The left thread will then appear to be at a greater distance than the right thread. Now alter the angle of convergence by an angular movement of

[2] Trump, R. J., 1924. "Binocular Vision and the Stereoscopic Sense," *Transactions of the Optical Society* (*London*) 25: 261–272.

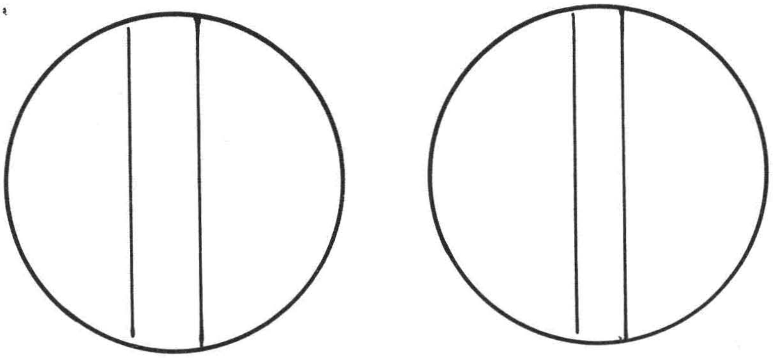

FIG. 4. Threads as seen at far ends of mailing tubes when looking into tubes.

one of the tubes in the horizontal plane. It will be found that a considerable angular displacement is necessary to produce the impression of a very slight change in the distance of the threads from the eyes, or in the distance between the threads along the line-of-sight.

Next move the tubes away from the eyes along the line-of-sight keeping the threads fused. It will be observed that as the threads recede, the apparent distance between them will increase. As a general rule, the change in apparent depth is directly proportional to the change in the apparent distance of the image. Since there is a minimum difference in convergence-angle that can be resolved—it will vary from ten to sixty seconds of arc for different individuals—beyond a certain point very little change in distance between the threads can be noted.

A similar experiment may be conducted using stereo photographs and viewing them without a stereoscope.

THE FORMULA FOR EVALUATING THE DEGREE OF DISTORTION OF THE SPACE-IMAGE

Since the degree of distortion of the space-image varies directly with the change in the ratio S/e relative to the ratio H/B or F/P, there can be derived a simple formula to express the degree of exaggeration depth-wise (E):

$$E = \frac{S/e}{H/B} = \frac{SB}{eH} \quad \text{or} \quad E = \frac{S/e}{F/P} = \frac{SP}{eF}.$$

In viewing stereoscopic photographs with most stereoscopes a magnification is introduced. However, this does not affect the results obtained with this formula. For any given distance S, the effect of introducing a magnification is simply to multiply the frontal and depth values all by the amount of the magnification. However, for this to attain the interval between the optical centers of the lenses of the stereoscope should be the same as the interpupillary distance.

When the formula is solved, if E is equal to one, the effect is orthoplastic, if greater than one hyperplastic, and if less than one hypoplastic. A value of two indicates that a hill with an actual slope of 50 per cent will have an apparent slope of 100 per cent in the space-image. The appearance of the roofs of buildings is changed in the same manner, i.e., the apparent slope per cent is increased E times. Also, the apparent vertical dimensions of buildings are increased E times relative to the horizontal dimensions, and, comparatively, a tree will appear E times its height in a correct scale-model.

In computing E it is most convenient to use the formula in the form $E = SP/eF$. The values to insert in the formula are then easily determined.

DETERMINING THE VALUES TO USE IN THE FORMULA

The interpupillary distance (e) is equal to the distance between the right, or left borders of the two pupils. By looking into a mirror with a ruler before the eyes, this distance may be accurately measured.

Theoretically, the absolute parallax (P) should be measured for a point at H distance. However, since the determination of P for a given distance can be made only with some difficulty, it is much easier to measure the distance between the principal and conjugate principal-points on one of the photographs, and to use this value for P. Any introduced error will be negligible.

The focal-length of the camera lens (F) can be obtained from the agency which supplies the photographs. In some cases it will be printed on the photographs.

To determine the apparent distance S, one should place the photographs on the edge of a table, and carefully align them for stereoscopic viewing. Then put the stereoscope in position and place a measuring stick which just reaches from floor to eyes vertically three to four inches to the rear of the instrument, on a line between the eyepieces. Next look first through the stereoscope and then to its rear, alternately, and compare the position of a pencil held against the stick with that of the stereoscopic image in space. When the location of the image has been determined, measure the distance S from the top of the stick to the pencil. Theoretically, the distance S should be to the reference frontal plane which was at H distance at the time of photography. However, it is sufficient to note the general position of the space-image.

Application of the Formula

The accompanying table gives results obtained by the author for different aerial photographs and three different stereoscopes. An interpupillary distance of 2.5 inches has been used for all calculations.

The results obtained by a number of other observers have been similar to those given in the table. From a comparison of their results with his, the author has drawn up a few simple rules as an aid in using the formula. The rules are:

1. For any lens stereoscope (magnification two to three times) with photographs placed at focal-point of lenses:
   a. S will vary from 14 to 18 inches for different observers.
   b. S will not vary more than $1\frac{1}{2}$ inches for a given observer.
2. For a double reflection mirror stereoscope without supplementary lenses or binoculars, S will vary from 13 to 16 inches for different observers when the plane of the pictures is at a distance of about 12 inches.

The above rules in conjunction with the other information given should enable one to calculate the degree of exaggeration depthwise (E) with reasonable accuracy. However, until one gains confidence in the determination of S, he can rewrite the formula with S equal to 16. Then,

$$E = \frac{16P}{eF} \cdot$$

In this form the formula will give good results when using a lens stereoscope. How-

Table 1

Values of E for Four Different Photographs and Three Stereoscopes

Photo No.	*Film and paper*	*RF*	*F*	*P*	*Values of S for Stereoscope No.*			*Values of E for Stereoscope No.*		
					1	2	3	1	2	3
			(*Inches*)	(*Inches*)	(*Inches*)	(*Inches*)	(*Inches*)			
1	Pan. Glossy	1:12,000	6	2.1	17	$17\frac{1}{2}$	15	2.4	2.4	2.1
2	Infra-red Semi-matte	1:15,840	$8\frac{1}{4}$	3.4	$16\frac{1}{2}$	17	14	2.7	2.8	2.3
3	Pan. Semi-matte	1:11,300	6	3.6	$16\frac{1}{2}$	$16\frac{1}{2}$	$14\frac{1}{2}$	4.0	4.0	3.5
4	Pan. Semi-matte	1:20,000	$8\frac{1}{4}$	3.2	16	15	15	2.5	1.5	2.3

Stereoscope No. 1 was lens type with focal-length of 3.5 inches. Pictures placed at focal-point of lens. Magnification approximately 2.8 times.

Stereoscope No. 2 was lens type with focal-length of 4.75 inches. Pictures placed at focal-point of lens. Magnification approximately 2.1 times.

Stereoscope No. 3 was double reflection mirror type. Total distance light travels from print to eyes is 12 inches. This corresponds to focal-length of instrument. Essentially no magnification.

ever, it may result in slight overestimates if a mirror stereoscope is used.

SELECTED REFERENCES

If the reader desires to obtain a rather complete picture of the thinking on the subject of the stereoscopic space image, reference to the following sources should be helpful.

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*Mapping of Glaciers in Alaska**

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ABSTRACT: *As a part of its contribution to the International Geophysical Year program the American Geographical Society is mapping a number of glaciers in Alaska. The author is the photogrammetrist for this project. A pilot project was first undertaken using available ground-control and photography of the Lemon Creek Glacier in Alaska. In preparing the map of that glacier, the author was able to develop the procedures used in mapping several more glaciers during the summer of 1957. These procedures included both terrestrial and aerial photogrammetric methods. A triangulation network of high accuracy was put in on each glacier, and a sufficient number of points was determined so as to insure a check on the accuracy of the mapping procedures used. A comparison of these mapping methods is now being made.*

INTRODUCTION

INCLUDED among the many activities of the International Geophysical Year is the preparation of accurate topographic maps of a number of selected glaciers. As its part in the IGY program, the American Geographical Society is sponsuring IGY Project Y/4.11, the mapping of glaciers in Alaska. The author is photogrammetrist for this project. The maps are being plotted using the Wild A-7 Autograph at the Institute of Geodesy, Photogrammetry, and Cartography of The Ohio State University. The map manuscripts are being prepared at scales of 1:5,000 and 1:10,000 with a contour interval of 5 meters.

These maps will be used primarily for correlating long-range changes in the weather and corresponding changes in the glaciers. A method for determining such changes in glaciers as advance or retreat, and annual accumulation (snowfall) and ablation (snow and ice melt and evaporation) have been thoroughly described by Dr. Richard Finsterwalder.[1] In his method a comparison is made between maps pre-

[1] Finsterwalder, R., "Photogrammetry and Glacier Research with Special Reference to Glacier Retreat in the Eastern Alps," *The Journal of Glaciology*, Vol. II, No. 15, pp. 306–315, April, 1954.

* Presented at the Society's 24th Annual Meeting, Hotel Shoreham, Washington, D. C., March 28, 1958.