errors in planimetry proved to agree very well with the practical results.

In summary, the results of the tests are very promising, and it seems desirable that the method be tested under varying circumstances, also with such simple instruments as mirror stereoscopes and single image comparators (coordinatographs etc.)

Some examples of the theoretical distribution of the standard errors of the final x-and y-coordinates are demonstrated in the accompanying diagrams.

The error propagation formulae were derived in accordance with the well known laws of the method of least squares and for two different cases of the control point location.

In particular, the principles of the numerical corrections and of compensation in the control points were applied. In a similar way the error distribution in a triangulation strip can be derived and demonstrated.

Two Sketching Devices for Oblique Aerial Photographs

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ABSTRACT: *The Mechanical Oblique Sketcher and the Optical Oblique Sketcher are two novel answers to the problem of rectifying aerial photos. The Mechanical Oblique Sketcher is a mechanization of a point by point geometric rectification procedure. The Oblique Photo Sketcher is a projection device for diapositives which utilizes an intense point source of light focused on a small aperture, simple lens to produce an unusual depth of field, permitting imaging the diapositive on a grossly tilted surface.*

AERIAL photographs, taken at an oblique angle to the earth, provide useful photogrammetric information only if the normal, vertical view can be reproduced. Rectification of aerial photos is a complicated procedure since it involves the translation of a perspective view into orthographic projection. Rectification is generally accomplished by either special drafting instruments or by rectifying projection machines. Two characteristics of these devices have been:

1. the rectifying projectors were too bulky to be conveniently transported;
2. the drafting machines required tedious, eye-straining plotting.

In view of these objections, one can see that there existed a need for a simple, reasonably accurate, transportable rectification device. One use for such a device would be the prompt bringing up-to-date of maps and charts from daily reconnaissance information. The need for a devise was expressed by the Photo Reconnaissance Laboratory, Wright-Patterson Air Force Base. In answer thereto Mast Development Company, Inc., Davenport, Iowa, designed two such instruments—first the Mechanical Oblique Sketcher and second, the Optical Oblique Sketcher.

THEORY UNDERLYING THE MECHANICAL OBLIQUE SKETCHER (M.O.S.)

The Mechanical Oblique Sketcher is based on an application of Synthetic Projective Geometry suggested by Prof. A. C. Bendixen of Ohio State University Research Foundation. His suggestion applies a characteristic of homologous quadrangles namely, homologous figures are projections of the same plane figure on different planes.

In Figure 1, $ABCD$ and $A'B'C'D'$ are

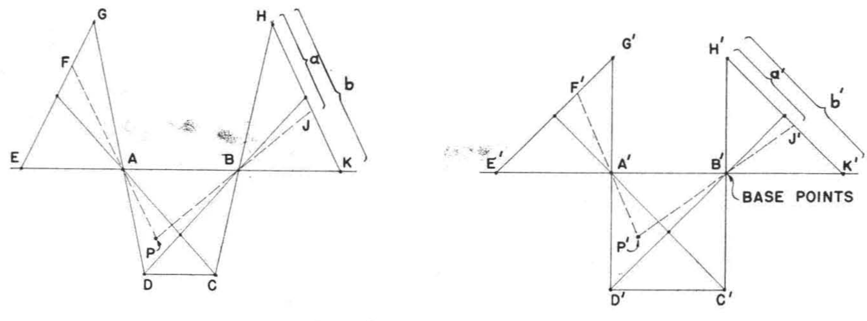


FIG. 1. Geometry of mechanical oblique sketcher.

two homologous quadrangles. $A'B'C'D'$ form a square in a map; $ABCD$ are the same four points in an oblique photograph of an essentially plane surface, and are obviously projections of the same points on a different plane. Extending three of the sides of the square and its diagonals we can find two lines such as $E'G'$ and $H'K'$. These lines serve a similar function to measuring lines in the measure point perspective drawing system. Similarly, extending the sides of the quadrangle $ABCD$, lines EG and HK are constructed so that the diagonals of the quadrangles intersect the lines at their mid-point.

Within this geometric configuration any point in the plane of figure $ABCD$ can be transferred to the plane of $A'B'C'D'$. For example, let P be any point in the plane $ABCD$. Lines from it may be drawn through the base points A and B and extended until they cross EG and HK at F and J. Reversing the procedure, point J' is located on line $H'K'$ so that a' and b' are in the same proportion as a and b in Figure $ABCD$. Point F' is located in the same manner. From points F' and J' lines are extended through A' and B'; their intersection is the rectified position of point P.

Mechanization of the Geometry

The *Mechanical Oblique Sketcher* consists of three units, the transmitter, or "master," the receiver, or "slave," and a four-channel amplifier (see Figures 2 and 3). The master and slave units are basically alike. Each unit consists of two torque units, and a base bar, on which the base points are positioned. One end of each torque unit base slides and pivots on the base bar. The torque unit bases are provided with three reticles for aligning the three points on the metrical line. The torque unit carriages ride along the bases on stainless steel rods. Each torque unit of the slave is equipped with a servo motor. The scribe arms pivot about a common point, the scribe. Each arm extends through a pivot mounting above the base point and then through a similar mounting on the torque unit.

In use, the *M.O.S.* master unit is placed over the photo; the *M.O.S.* slave unit is

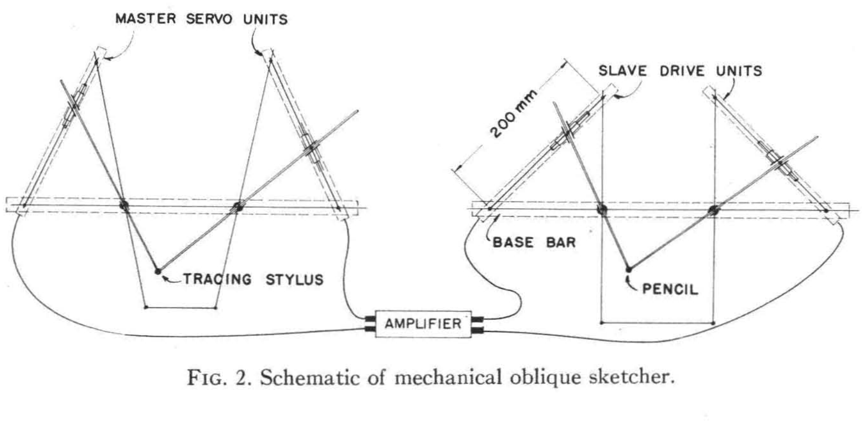


FIG. 2. Schematic of mechanical oblique sketcher.

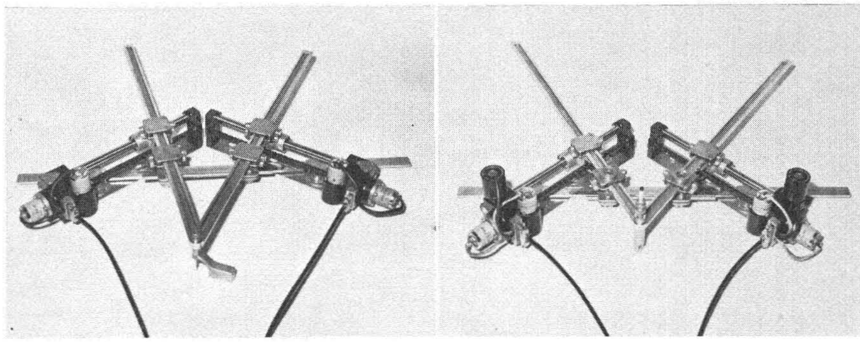


FIG. 3. Mechanical oblique sketcher.

similarly aligned over the map. The torque unit bases are made to coincide with the metrical lines, while the base points are located on the base bar. The scribe arms correspond, Figure 1, to the extensions of the lines through *P* and the base points. As the scribe moves, the angle between the scribe arm and the torque unit base changes. To compensate for this change in angle, the torque unit is forced to move, rotating a ball-bearing lead screw. The degree of rotation of the lead screw is measured by 60 cycle coarse and vernier synchros. The signal of these synchros is amplified, combined, and compared with the signal from like synchros on the slave unit. The unbalance of the synchro signals actuates the servo motors on the slave unit. The servo motor on the slave drives a similar lead screw, moving the scribe arm of the slave.

The amplifier consists of two separate, twin units, one for the right servo and one for the left.

Although the *M.O.S.* is simple to manipulate, its use is restricted. A minimum error of about plus or minus 1/32" is the greatest accuracy which this instrument can achieve. The imperfections in the *M.O.S.* stem from the fact that the geometrical construction upon which it is based is not readily adaptable to mechanization, e.g., in some positions backlash of the slide bars tends to be critical, and great freedom of movement is required to avoid binding.

The inadequacies of the *Mechanical Oblique Sketcher* led Mast Development Company engineers into a search for a simpler, more accurate method of rectifying aerial photos. This search resulted in proposing to the Air Force the development of the *Optical Oblique Photo Sketcher.*

Optical Principles Involved in Oblique Projection

Theoretically, projection of a positive transparency of an aerial photo on an oblique surface is an accurate and simple means of rectification. However, in practice, accuracy must be sacrificed to retain simplicity. In Figure 4 the conventional systems of pinhole, point source, and stereoptic projection are compared. The pinhole method and point source method have the advantage that no projection distance is defined. This unrestricted projection distance is limited by the fact that the circles of confusion which accompany pinhole projection, increase as the image is magnified. Stereoptic projection, on the other hand, forms a sharp image at some definite distance from the projector. This characteristic is objectionable in oblique projection for two reasons: first, focus can be maintained on a tilted screen only by tilting the lens to meet the Scheimpflug condition, i.e. that the plane of lens, film, and screen intersect in one line.

Second, a condition which a rectification projector must satisfy in order to reproduce the actual view, is that proportionality exist between the projector and the photographing camera. That is, the ratio of the diapositive size to the distance between the diapositive and the lens, must equal the ratio of the negative size to the distance between the negative and the camera lens.

The *Optical Oblique Sketcher* combines some of the advantages and eliminates some of the disadvantages of conventional pinhole and point source and stereoptic projection. The ingenuity of the instrument is in its size relationships. By using an intense point source of light focused on a very small lens, the effective aperture

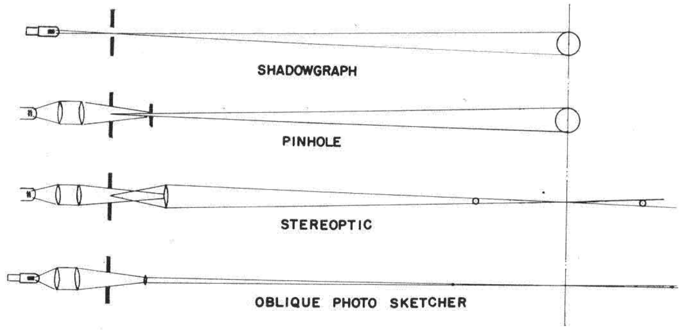


FIG. 4. Comparison of methods.

becomes on the order of f/80. One can see in Figure 4 that the effect produced by such a system is similar to pinhole projection in that there are no distortion problems. On the other hand, the fact that a tiny lens is used provides a sharp image at one distance from the lens, and a circle of confusion that is very small for a substantial distance either side of the plane of sharp focus. Thus a reasonably clear image will still be projected if the projection surface is in front of or behind the exact focal-length. This great depth of field allows the screen to be tilted at a large angle with respect to the axis of the projector, permitting accurate alignment of the photo projection and the map.

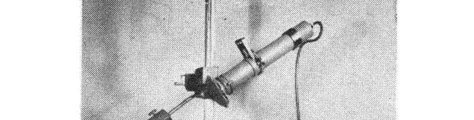
OPTICAL OBLIQUE SKETCHER

The *O.O.S.* optical system consists of a zirconium arc lamp, a camera quality condenser lens system, the diapositive holder, and a turret of projection lenses. The turret of projection lenses, each of slightly different focal-length is provided to give greater localized definition for tracing small areas of the map. Altogether the lenses for each camera focal length provide a sharp focus (zero circle of confusion) range of approximately 3 feet at an average projection distance of 3 feet. The diapositive holder may be rotated plus or minus 15 degrees about the optical axis. A removable condenser system and spacer are furnished with the *Optical Oblique Sketcher* to allow projection of photos from cameras of several focal lengths. The housing of the optical system tilts abouts its transverse axis. It may also be moved along the vertical stem on which it is mounted (see Figure 5).

The lenses used are simple biconvex lenses. Since the lenses operate at such small aperture, no definition is gained by the use of multiple element lenses. The circle of the arc itself forms the iris of the lens. Tilting the lenses, in accordance with the Scheimpflug condition, was demonstrated to give no perceptible improvement in definition.

Patent No. 2,792,747 was issued in May 1957 to Gifford Mast on the *Optical Oblique Sketcher*.

The major limitation of the O.O.S. comes from the limited amount of light available from the zirconium arc, high though its intensity is compared to an incandescent lamp. A substantially dark room is required to use the *O.O.S.* More recent work has been done with a high pressure Xenon lamp, but its present production status is experimental and its useful life uncertain.



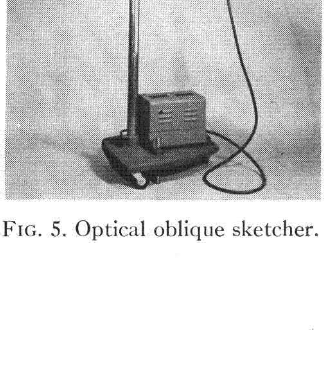


FIG. 5. Optical oblique sketcher.