PANEL DISCUSSION
THE FUTURE OF ANALYTICAL AERIAL TRIANGULATION

MODERATOR

Edwin A. Roth	USAF Aeronautical Chart & Information Center, St. Louis, Mo.

PANEL DISCUSSANTS

Dr. Paul M. Pepper	Mapping & Charting Research Laboratory, Ohio State Univ.
Mr. G. C. Tewinkel	U. S. Coast & Geodetic Survey, Washington, D.C.
Dr. Helmut Schmid	Geodesist, Ballistics Research Laboratory, Aberdeen Proving Ground, Aberdeen, Md.
Mr. B. J. Bodnar	U. S. Army Engineer Research and Development Laboratories, Ft. Belvoir, Va.
Mr. V. A. van Praag	Bendix Computer Division of Bendix Aviation Corp., Los Angeles, Calif.
Mr. G. H. Schut	National Research Council of Canada, Ottawa, Canada

Introduction by the Moderator

WE ARE to discuss something in the future. The reason for talking about the future of analytical aerial triangulation is that quite a few problems have not yet been resolved, and there is not any definite method for use in accomplishing such triangulation. It is intended that the panel members will discuss some of the things that they have done. Then you can decide whether you have any questions on the attempt to bring this problem of the future closer to the present.

The first speaker is Dr. Paul M. Pepper. Without further introduction the other panel members will follow Dr. Pepper and in the order listed on the Convention Program.

Discussion by Dr. Paul M. Pepper

I HAVE given much thought to the future of photogrammetry as well as to its manifold past accomplishments.

Without any question, in your minds as well as in mine, there have been doubts about the existence of new directions in which photogrammetry could move to make possible the development of radical innovations in methods. Advances seem partially counterbalanced by disadvantages. Greater aircraft velocities and faster photographic emulsions which should allow more rapid gathering of data are offset to some extent by the degradation of the image through camera motion at these higher velocities, and in some cases shutter shock vibrations attendant to shorter exposure times. The ability to fly at high altitudes and thus take series of photographs with increased model strength, brings in the problem of earth curvature corrections. Each refinement of precision in instrumentation brings with it a great toll in cost of instrumentation.

Although each of these balances out with a net gain for photogrammetry, the "law of diminishing returns" seems to be coming ever more strongly into play. For example, it may be possible to offset the cost of a very expensive optical stereoscopic instrument, if the speed of operation of the instrument is sufficiently great to reduce the product cost per unit; but there seems to be a limit which the operator cannot surpass in reading data, or even recording data automatically, since the time for making the settings and the time for orienting the models have appreciably lower limits.

In order to secure the proper perspective view of the problems of analytic photogrammetry, it is desirable to take a backward look at the development of photogrammetry. Applied photogrammetry, that is, operational or production photogrammetry, today is divided roughly into two categories: *numerical* photogrammetry, more commonly called "analytical" photogrammetry, and *geometric* photogrammetry, more commonly called "instrumental photogrammetry."

Actually there is no such thing in applied photogrammetry as a branch that is independent of all instruments. At some stage or other a measuring instrument of some kind must be used to extract the data from the photographs. The primary difference between what is here classified as numerical photogrammetry and that termed geometric photogrammetry lies in the degree of complexity of the instrument used. In numerical photogrammetry the measuring instrument is usually some kind of graduated scale used either by itself or incorporated into a coordinate measuring machine. On the other hand the instrument employed in geometric photogrammetry is one which never does less than construct optically or mechanically a scale model of the original object photographed. Clearly numerical photogrammetry employs linear picture coordinates, whereas instrumental photogrammetry employs rectangular coordinates in the three-dimensional space of the optical or mechanical model.

Both of these branches are concerned with obtaining from photographs quantitative information for use in some applied field, be it mapping, civil engineering, highway engineering, physiology, dentistry, human engineering, or a multitude of others. Between these two and serving both is a branch of photogrammetry which few of us have thought to regard as a separate regimen, since its role is so closely interlocked with both of the others. This is *theoretical photogrammetry* which is even today largely algebraic and trigonometric in nature, but actually extends beyond algebra and trigonometry into methods of calculus. Although this branch might thus rightfully claim the name *analytical*, it will be termed *theoretical* in this paper since the word analytical is in current use to describe the branch of numerical photogrammetry.

The backward look will include considerations about all three branches and will provide a vantage point from which new vistas can be viewed.

RETROSPECT

There is little doubt that applied photogrammetry had its origin in numerical photogrammetry. Measurements made, usually on single photographs, were reduced by means of algebraic and trigonometric equations to quantitative information—sometimes to distances on the earth's surface, sometimes to dimensions of terrestial objects, sometimes to the determination of date and time of day when a photograph was taken (as evidenced by dimensions and location of the shadows seen in the photograph), etc.

At some stage within the development of photogrammetry it was discovered that by relating information in two photographs having a common area of coverage, but taken from different exposure stations, considerable additional refinement could be made. It matters little whether this was first discovered through the equations of theoretical photogrammetry or was conceived geometrically and carried into practice in geometric photogrammetry, with the later development of the mathematical equations which parallel the geometry. What is important, however, is that because the complexity of the corresponding mathematical equations which would have to be used in numerical photogrammetry was so great, as contrasted with the simplicity of the instrumental reconstruction of the model, the pendulum swung toward geometric photogrammetry, which assumed the dominant role in applications, with numerical photogrammetry being relegated to a subservient position wherein it was used for checking and adjustment processes more frequently than for obtaining original solutions.

MUSINGS

But most pendulums swing two ways. The equations of theoretical photogrammetry, which early became too complex to enable numerical photogrammetry to be competitive with the instrumental methods of geometric photogrammetry, had actually grown much more complex during the passing years. On the other hand the advent of extremely high-speed electronic computing machines created a new numerical tool which showed promise not only of putting numerical photogrammetry back "into the running," but hinted that, suitably reinforced by automation, numerical photogrammetry might even eventually outstrip geometric photogrammetry in speed and efficiency.

It is not quite so simple, though, for although applied photogrammetry is rather clearly separated into two branches, numerical and geometric, this is not true of the applied photogrammetrists. Each is free to use what he wishes from either branch, singly or in combination. Not only are the reading instruments and mathematical equations of numerical photogrammetry susceptible of acceleration by electric and electronic means, but so also are the instruments and adjustment equations of instrumental photogrammetry.

Thus it might appear reasonable to conjecture that any advantage of numerical photogrammetry over geometric photogrammetry could materialize only from the fact that there is a considerable time required to establish each stereoscopic model in the instrument. Even this is somewhat offset, however, by the fact that at present in the numerical method each model, or its numerical equivalent, must be established by a computing machine computation; a resultant advantage can accrue to the numerical method only if the computing program is sufficiently brief and the computer fast enough to permit the over-all numerical method to take less time than the over-all geometric method.

Wherein, then, does the advantage of numerical photogrammetry lie? Will there actually be any such advantage if it should happen that geometric photogrammetry can keep pace with numerical photogrammetry in the matter of over-all time reduction through automation and the application of electronics? It is not in the difference of accuracy of end results which can be obtained from the data handled by the two methods, for experiments at The Ohio State University show that there is probably no significant difference in the accuracies of model connections—that long cantilever extensions using data from the same photographs yield errors of essentially the same magnitude in the locations of points in the terminus of the strip.

UNIFICATION IN PHOTOGRAMMETRY

If one were required to use a single word to describe the advantage of numerical photogrammetry over geometric photogrammetry, that word probably would be *unification.* The capability of making a simultaneous triangulation of all the data contained in a large block is a prime example of such unification. This one added capability of numerical photogrammetry, when fully implemented (after all the attendant problems of numerical analysis have been solved), will be of great benefit to photogrammetry.

Great as this benefit will be, however, there is another potential unification much more far-reaching. This is *the capability of using simultaneously and jointly large amounts of quantitative information, of like or different kinds, from many sources, both photographic and instrumental, weighting each item according to its estimated precision.*

Up-to-now attempts to use external data have been essentially on an *exchange* basis rather than in combination. The shoran coordinates of the camera station may be taken as gospel, and what would have been the computed coordinates are then condemned to the discard by forcing the triangulation to make the camera station fit the shoran determinations. Profile recorder elevations may be given preference over otherwise computed elevations. Ground-control point coordinates may be forced to fit what was identified on the plates as the image of the ground-control point, whereas experiments actually reveal that the ground-to-photograph identification is in general of poorer accuracy than photograph-to-photograph identification; thus the *substitution* of ground-control coordinates for pass-point coordinates may actually *worsen* the triangulation.

SUBSTITUTION VERSUS UNIFICATION

Geometric instruments to perform true unifications in cases of this sort do not seem capable of physical realization. The present instruments of geometric photogrammetry are designed to use *only the information explicitly contained within the photographic record*. A degree of success has been achieved recently by researchers who have devised new techniques to use external data in conjunction with the existing geometrical instruments. However, such external data can be used in these instruments only if these data are reasonably closely related to the quantities which are usually to be set into the instrument. Thus, information on terrain clearance at the time of exposure could be used since it is related to one of the settings of the instrument. However, in this connection it must be remarked that in relatively orienting two photographs, *any alteration of instrumental settings from those obtained by using only photographic information is equivalent to discarding the corresponding information as contained explicitly or implicitly in the photograph and substituting external data for it.* This is justifiable in case it is certain that the external data are far more reliable than the data from the photograph, or if the discarded photographic information is implicit in such a way that it is theoretically only very weakly determinable from the photographs. For example, there are circumstances under which the coordinates of one exposure station are ill-determinable in relation to the preceding one from the data contained in the two photographs. In this case it would be justifiable to use external information for the location of the second exposure station if the reliability of the external data were far superior to that of the photographic data.

In particular, Dr. Blachut and others in Canada and the Engineer Research and Development Laboratories in the United States, using what seems to amount to a *substitution* of airborne profile recorder data for optically reconstructed ground clearance measurements, have found considerable improvement in the strip triangulations. It would seem probable that much greater improvement would result by the joint use of all reliable data.

However, if the information from the photograph is of equal reliability with corresponding information from external sources, there is no justification for discarding that from the photographs in favor of that from the external sources. Thus the photographic information is used and the *external information discarded.*

Obviously, then, the term unification as used here does not mean substitution of one set of data for another, but a system of employing all reasonable data jointly, each with its proper influence.

THE INSTRUMENT OF UNIFICATION—THE HERGET METHOD

That this true unification can be done on high-speed digital computers by suitable modifications of the present numerical methods is of extreme importance. One example in point is the case where cantilever extension is to be used to establish point locations in uncontrolled areas. Every bit of significant available data should be applied in the extension if maximum accuracy is to be obtained.

Machine methods now in use employ the vector equations of the "Herget Method" developed by Dr. Paul Herget in conjunction with Air Force sponsored research conducted in the Mapping and Charting Research Laboratory at Ohio State University. This system of equations is susceptible of being enlarged to include many other items (the added equations being of essentially the same form) and of being modified to account for the individual limits of accuracy of the various items of information. The system will then effect a true unification of the data.

SCOPE OF THE UNIFICATION

Some reflection and a bit of mathematical equation construction shows that it is possible to include in the unification at least those items of data shown in Table I. To be sure, there will be required some modifications of the Herget program to make the proper application of some of the items, such as the degree of camera stabilization, but these modifications take a rather simple form.

In most cases the cost in terms of money, or time required to produce the end information, is of considerable importance. Thus, consideration should also be given to the additional cost as weighed against the gain in accuracy, for each item added increases by some amount both the time and cost of the information put out by the system.

TABLE I

EXAMPLES OF TYPES OF DATA CONCEIVABLY USEFUL FOR IMPROVING PHOTOGRAMMETRIC MODEL CONNECTIONS

Type of Information	Elements Most Directly Influenced*
A. *Aircraft Positional Data*	
1. Ground control point data.	1. Absolute ground coordinates.
2. Loran, hiran, or shoran aircraft coordinate data.	2. Absolute camera station horizontal coordinates and, indirectly, absolute ground coordinates.
3. Aircraft coordinate data obtainable from inertial systems.	3. Absolute camera station horizontal coordinates and, indirectly, absolute ground coordinates.
4. Integrating accelerometer data giving vector distance between successive aircraft stations.	4. Relative camera station coordinates.
5. Profile-recorder relative elevation data in successive photographs.	5. Relative camera station vertical coordinates.
6. Pressure altimetry data for differential elevation between camera stations.	6. Relative camera station vertical coordinates.
7. Doppler radar data on distance between successive aircraft station nadir points.	7. Relative camera station horizontal coordinates.
B. *Directional Data*	
1. Aircraft attitude data (when camera mount is unstabilized).	1. Camera orientation elements and, indirectly, horizontal camera station coordinates.
2. Ground-track direction information. a. Unstabilized camera mount. b. Stabilized camera mount.	2. a. Camera orientation and relative camera station coordinates. b. Relative camera station coordinates.
3. Knowledge of limits of accuracy for vertical stabilization of camera mount. a. Absolute limits of deflection. b. Limits relative to preceding station deflections.	3. Camera orientation and, indirectly, horizontal camera station.

* Since all the data selected for use in the aerial triangulation will be used in each model connection, all the camera elements, both station coordinates and orientation elements, will reflect the influence of each piece of data. Clearly, changes in the camera elements will also induce changes in the ground coordinates. This column lists only some of those which will be *most* affected by the item of data.

AREAS OF RESEARCH

The unification as herein visualized offers great promise, but it also brings many procedures to be developed and questions to be answered.

First, the complete and detailed equations to be added to the Herget set must be developed for each item.

Second, although the Herget equations are susceptible of carrying out the triangulations with different levels of influence, it is necessary to determine what weight should be given each type of data, and to develop the details for securing the weight. That is, there is a calibration problem.

Third, the pattern of mathematical solution must be examined for complications which might result from each enlargement of scope, and a complete computing program must be written for the set of equations corresponding to a maximum feasible scope.

Fourth, an operations research study must be conducted weighing time or cost against gain in product value in order to determine the optimal scope. The optimal scope may differ according to the situation. Thus, in one case time to produce might be important but with cost no object, whereas in another instance cost would be all-important and time relatively unimportant.

Fifth, there should be a final checkout of the method and computing program on fictitious and actual data.

CONCLUSION

The cost of such a program of study is not small, but the potential pay-off in precision of cantilever determination of point location might be extremely great. If substitution methods can sometimes be very advantageous, we may expect to achieve far greater success through joint and unified use of all available significant data.

Discussion by G. C. Tewinkel

IN THE Coast and Geodetic Survey efforts are being devoted to the development of a practical procedure of analytic aerotriangulation.

Presumably you are curious as to the reasons behind the interest of the Bureau in this development. The answer lies in the fairly logical expectation that the analytic solution ought to be preferable to the instrumental methods on the basis of accuracy and economy. These goals cannot be defended conclusively at the present stage of development, but several of us are convinced and encouraged that these ideals can eventually be achieved.

The use of the word "preferable" may indicate to you either that (1) accuracies obtained with the present instrumental methods are not entirely satisfactory, or (2) that the analytic methods are expected to produce accuracies which are not ordinarily obtainable with the instrumental methods. Actually, both implications are correct. However, we are cautioned by some of our colleagues that we should not necessarily expect any great increase in accuracy, but that we should prefer the analytic approach because it employs classical, sound, systematic methods which allow automation to be applied at a saving in time, personnel and funds.

Although the present aerotriangulation instruments are constructed with the most highly developed techniques and skills, systematic mechanical and optical discrepancies are propagated at a rather alarming rate even where the errors are essentially insignificant in a single model. Aerial camera lenses, shutters, and film emulsions have now been developed to the point where image resolution may be o higher quality than one can utilize with the instruments. (It is noted, however, that new developments in instruments, such as auxiliary electronic analogue devices, may overcome present instrumental inaccuracies.) The application of corrections for measured systematic instrumental deviations is cumbersome and is unstable in practice, and some types of known corrections nearly defy a just method of compensation. The operation of the instrument is subject to human skill, and the application of statistical analysis is restricted. The operator is unable to consider more images than those from two photographs, although corresponding images may appear on from three to nine photographs.

It seems that analytic aerotriangulation should eliminate several of the deficiencies encountered in the instrumental work. The systematic effects of earth curvature, normal atmospheric refraction, lens distortion and rectangular film shrinkage can be corrected to the full extent that one is able to determine them. For example, it is possible to measure coordinates of images to their limit of resolution. In a recent isolated test involving 43 marked geodetic stations on one photograph, the mean-square deviation from a geometric perspective was ± 4.2 microns, with a maximum of 12 microns. The inaccuracies of scales, guide rails, bearings and load deflections in a plotting instrument are not involved in an analytic solution, whereas the accuracy problems in the design and construction of a stereocomparator are greatly simplified. In the analytic system, there does not exist the problem of producing two distortion compensation systems exactly alike, and opposite from a given aerial camera lens. Operator skill is simplified in the analytic system, and corrections to relative orientation are not applied by means of a doubtful mechanical device. Instead, relative orientation is achieved by not only considering the images on *all* the overlapping photographs, but also all of the 10 or 12 points in a model instead of six.

A most attractive feature of the analytic method is the simultaneous solution and adjustment of a complete strip or block to fit all the control in the area exactly, and relegating all discrepancies into residual errors of observation of image coordinates according to least-squares principles. The computational method is the classical system used to adjust geodetic triangulation, but applied to a vast interlocking system of three-dimensional triangular figures. If the images of a pass-point appear on five photographs of adjoining strips, the image rays are caused to intersect in a single point at the cost of residual coordinate discrepancies (where they should be, according to least-squares), which are to be printed out in the computation. It is obvious that the geometric figure is a very rigid one; the degree of rigidity remains to be determined. There is some justification for the hope that a block adjustment may result in errors not much larger than those which now occur in a single instrumental model.

The mathematics is long but not overpowering. Although higher mathematics is used for convenience in many publications, all the principles can actually be demonstrated to the complete satisfaction of a sophomore engineering student. Basic algebra, trigonometry, analytic geometry and differential calculus constitute the extent of the mathematical theory involved.

The problem resolves itself into the equation of a line connecting three points: a ground object, the camera lens and the photographic image. The entire problem is based on this idea. It may seem evident that the two lines from two camera stations intersecting in a common ground point satisfy the condition of zero parallax, a condition which is equally valid in both the analytic and instrumental methods.

A series of significant steps or operations are conceivably necessary, namely:

Coordinate measurement using a comparator
Observing each image several times and using the average
Observing extra images per model, such as 12
Film distortion compensation
Translation of the origin to the perspective center
Lens distortion correction
Atmospheric refraction correction
Earth curvature correction using geocentric transformations
Designation of eight or ten types of image points according to the type of condition to be applied, such as pass-point, horizontal-control, etc.
Test of each model for mistaken observations by analytic relative orientation
Estimate of values of six unknown parameters per photograph
Computation of coefficients of the observation equations
Forming and solving the normal equations applying least-squares principles, including proper weighting
Solution of the normal equations for corrections to the six parameters per photograph
Adjustment of the parameters and repetition of the solution (iteration)
Tabulating the two residual coordinate adjustments per image
Computation of the space coordinates for each object
Transformation from the geocentric system into the desired coordinate system

The Coast and Geodetic Survey is particularly fortunate in already having developed methods and procedures in geodesy, the application of least squares, the solution of systems of simultaneous equations, programming and coding for an electronic computing device.

The developments are based on the use of an IBM 650 computer. This apparatus may not be ideally suited for this problem, but its availability within the Bureau precludes the use of other devices at the present time. The most difficult part of the numerical problem involves the solution of a large set of simultaneous equations. For example, 100 photographs result in the formation and solution of 600 normal equations. The complete solution may require 20 to 30 hours on the IBM 650, or one to two hours on a high speed computer.

The development of the system is based on the eventual purchase of a stereocomparator. In the meantime, provisional coordinate measurements may be made with a stereoplanograph, or with a coordinatograph for nine-lens photography. It is noteworthy that a *simple* comparator is applicable if all the control and pass points are marked on the plates before measurement, either by premarking on the ground, or by means of an identifying stereoscope of sufficient accuracy. Although it is fairly evident that maximum accuracy can only be obtained from aerial photography by premarking ground points, this practice is not considered to be practicable at this time. The extent to which the results will deteriorate with unmarked points has yet to be shown.

At least two sources of inaccuracy—anomalous film distortion and atmospheric refraction—tend to reduce the quality of any type of aerotriangulation, and apply particularly to the analytic method whose accuracy is not limited by any mechanical device.

Irregular film shrinkage can be overcome completely with either glass plate cameras or cameras containing a reseau. But it seems that American photogrammetrists in general are unwilling to accept either of these solutions. Glass plates of sufficient rigidity to provide the necessary flatness are objectionably heavy, bulky and costly; the reseau idea implies the purchase of a new aerial camera. It is hoped that:

First, new film bases will provide satisfactory distortion characteristics, and

Second, the anomalies will be sufficiently small in magnitude and frequency as to be of relatively little significance.

It seems to be nearly impossible to cope with anomalies in atmospheric refraction. You are undoubtedly familiar with the dance of the image in a telescope, and you are aware that the aerial camera catches a candid image of the object in some unknown instantaneous phase of its path of oscillation. The normal refraction correction at an altitude of 20,000 feet for an image 40 degrees off axis is more than three times the magnitude of the maximum lens distortion correction in the present day distortion-free lenses. But anomalous atmospheric conditions may conceivably be double or half the normal amount. Consequently, this effect may place an ultimate limit on the accuracy obtainable from analytic aerotriangulation. Seemingly, the only counteracting measure is the plan to use many extra observation points, and discard those observations which do not agree fairly well with the others. Examples are observations which indicate a residual error of more than four times the standard error.

In conclusion, the Coast and Geodetic Survey is in the process of developing analytic aerotriangulation on the basis of improving accuracy over present methods and in the practical economies that can result from automation.

Discussion by Dr. Helmut Schmid

FROM a general viewpoint the meaning of "aerial triangulation" includes all measuring problems resulting from the application of airborne metric photography. Such an interpretation includes the reconstruction and orientation of a single bundle of rays, as well as the problems of combining two or more bundles for the purpose of triangulating the spatial coordinates of certain target points. Furthermore aerial triangulation must deal with problems concerning the metric qualities of the photogrammetric camera. These problems include the determination of the metric parameters of the lens cone and the control of its dimensional stability, the determination of the amount of distortion and its distribution with respect to a central perspective. Equally important are the problems concerned with the metric control of flatness and dimensional stability of the emulsion carrier. Furthermore, the general concept of aerial triangulation must allow the use of any additional information which may be available from independent measurements of either the camera orientation elements or of certain geometric parameters of the ground-control data. The problem of aerial triangulation is comparable to the general problem of photogrammetry and therefore in this particular discussion the following statements will be made with respect to the "Future of Analytical Photogrammetry."

A discussion of analytical photogrammetry can only mean a discussion of problems concerned with the analytical evaluation of photogrammetric records. In order to express an opinion on the future of analytical photogrammetry it seems necessary to define first the general potentiality of such a method and its specific limitations. Generally speaking, analytical photogrammetry, based on the concept of the central perspective, allows us to express in rigorous mathematical terms the functional relation existing between

(a) the information stored on the photogrammetric record
(b) the given control data, and
(c) the unknowns of the specific triangulation problem.

Any number of photogrammetric records, obtained from any type of photogrammetric camera, in any arbitrary orientation can thus be combined with any type and quantity of given control data in a set of formulas expressing the geometry which existed during the collection of such records. The mathematical model so obtained serves as the frame for the statistical treatment of the redundant information usually present in the form of both plate measurements and control data. The rigorousness of a completely unconditioned triangulation figuration, and the ready inclusion of any number and type of redundant information, are outstanding features of the analytical evaluation method.

The specific limitation of this obviously powerful approach lies in the fact that analytical photogrammetry, because of its nature must limit itself to a sequence of individual points and corresponding rays, selected from the total photogrammetric information stored on a specific photograph. Analytical photogrammetry therefore can deal only with the reconstruction of a finite number of rays based on corresponding image positions of the photographs, and usually expressed for any one ray by two coordinates measured in reference to an arbitrarily established rectangular plate coordinate system. This situation indicates the restriction of analytical photogrammetry to the evaluation of the photogrammetric model by a point-by-point method.

A typical example for such an application is the use of precision photogrammetry for cadastral purposes where the problem is to determine the spatial coordinates of numerous marked property monuments. The future of analytical photogrammetry for this type of work is strictly a question of economy. During the last few years, governmental agencies and private companies have proved that model coordinates obtained from the evaluation of single models using first-order restitution equipment, are so accurate that it is unlikely that the over-all accuracy of these results can be improved significantly by an analytical reduction method.

In my opinion it is a fallacy to expect any stereocomparator to be much less expensive than a first-order plotter of comparable accuracy. It is true that designwise the stereocomparator is by far a simpler instrument. But the ingenious idea underlying the design and use of the three dimensional restitution equipment requires an absolute measuring accuracy only for the model coordinates, that is *after* the optical or mechanical magnification stage, while with the application of a stereocomparator the absolute measuring accuracy must be assured in terms of the original plate coordinates. Furthermore the stereocomparator measurements, lacking the benefit of a preceding relative orientation, will be no less time-consuming than the corresponding settings on a three-dimensional restitution instrument after a relative orientation has been accomplished.

Qualities favorable to the analytical reduction method are that (1) the calibration, control and operation of a stereocomparator require, in comparison with the operation of a first-order plotter, less skilled manpower and (2) in addition to the coordinates to be determined, the analytical method provides the corresponding residuals, making possible locating and analyzing identification errors more easily and to derive expressions of precision in a more objective manner.

Plotting topographical information from single stereoscopic models is today's most important application of photogrammetry. It is the common opinion that analytical photogrammetry at the present time has no promise for this type of work. This attitude is based mainly on the fact that even the most accurate universal plotting machines do not respond in their orientation movements with sufficient geometrical truthfulness, making it impossible to set precalculated orientation parameters.

Despite the correctness of the above statement I feel that analytical photogrammetry can make a considerable contribution to making precision plotting procedures more economical. The missing link in this approach is a precision rectifier which would allow the transformation in scale 1:1 of the original more-or-less oblique photogram into a photogram normal with respect to the coordinate system of the control data. The settings for the rectifier would be available from the result of the preceding analytically performed relative and absolute orientation. The actual plotting would now be possible with a restitution instrument of simple design and consequently of high precision. Such a plotter could be designed, for example, similar to the classic Stereoautograph by Orel-Zeiss.

The manufacturers of the most precise photogrammetric evaluation equipment are equally well versed in the manufacturing of the most accurate geodetic and microscopic equipment. Therefore it appears quite plausible that a rectifier could be built, which would feature the automatically controlled conditions for optical projection, and provide means to set and measure the necessary rotational and translational movements with a precision corresponding to the metric qualities of the most precise photogrammetric records.

Since the early days of aerial photogrammetry there has been the challenge to replace geodetic triangulation procedures by establishing ground-control over areas larger than a single model, thus reducing the need for given geodetic reference data to a minimum. These problems, commonly referred to as strip and block triangulations, are the fields in which analytical photogrammetry appears to have its brightest future.

For these applications the analytical approach is not limited to a numerical analogue of the optical-mechanical restitution procedure, but the analytical method has, besides the economy feature, a theoretically sounder foundation. The almost exclusively practiced method of strip triangulation is today the method known as "Folgebildanschluss," which is a procedure whereby each consecutive photogram is being oriented relatively and absolutely against a preceding bundle of rays which is kept fixed in its position and whereby the three spatial coordinates of at least one point of the preceding model are enforced. This method is dictated by the performance characteristics of the three-dimensional plotters. These instruments originally were not built for this type of work, and it is therefore not surprising that they do not allow an optimum result.

Contrary to the just-mentioned method the correct solution for the control extension in a strip is based on the sole condition that for at least one point in the triple-overlap area three corresponding rays originating from three consecutive photographs intersect. The deviation of today's extension procedure from this concept causes two significant shortcomings.

(1) The favorable geometry existing between the first and third photograph with a base-height ratio comparable with or even better than covergent cases is not used, because the first photograph must be taken out of the restitution instrument, when the third photograph is being introduced.

(2) The relative and absolute orientation of the connecting bundle is being biased by enforcing both the orientation of the preceding bundle and the spatial coordinates of the carry-over point. Actually there is no difference in the physical or geometrical significance of any one bundle relative to any other one. Still today's extension method gives more weight to the two bundles from the preceding model than to the bundle to be annexed. The continuously performed repetition of this process along a strip must cause an unfavorable propagation of errors.

Other advantages of the analytical method for a strip or block triangulation are the possibility of incorporating any type of control data without any restriction to its location. There are no difficulties to enforce any independent measured orientation elements like statoscope registration, sun camera data, etc., or certain parameters as, for example, the length of a side between two relative control points, even though these points do not belong to the same model, an important problem with the advances made in the application of optical and electronical distance measuring devices. It should be mentioned that corrections for distortion and refraction can readily be incorporated.

It is the practically unlimited flexibility of the method combined with the theoretically rigorous approach and the statistically correct treatment of the redundant information, which makes analytical photogrammetry especially suited for the treatment of problems concerned with control extension stripwise or blockwise.

In order to utilize fully the analytical reduction potential it will be necessary to identify and measure the image coordinates of corresponding points on several photographs. To minimize identification errors a three-plate comparator with a stereoscopic viewing device has been occasionally proposed. Such an instrument should be of value for strip triangulations, provided it will function with the necessary absolute accuracy. However the strip is at best a poor second as compared to the block. This situation leads us to believe that it might be better to encourage instrument manufacturers to develop an instrument which will enable an operator using stereoscopic observational techniques, to identify corresponding image points on several plates, preferably more than three, and provide means to mark these points for measuring by pricking the emulsion. The actual measuring could then be done on each photograph individually with a simple comparator featuring only x and y movements. Because the pricked points to be measured will be of uniform shape and high contrast, a technique for scanning the plate could be combined with a fully automatic setting and measuring procedure. Such a solution would be applicable for all photogrammetric measuring problems suited for the analytical reduction method, and would help solve the problems of reducing identification errors and of increasing the over-all economy of photogrammetry.

Discussion by Mr. B. J. Bodnar

The attitude of the average photogrammetrist toward analytical triangulation is probably one of considerable curiosity, mixed with a little apprehension and confusion. He may have shuddered at the seemingly endless pages of equations involved in the computations. By now, he is duly impressed with the ability of the giant electronic computer to conquer this high mathematics.

Unfortunately, the more glamorous features of analytical triangulation that have been publicized have overshadowed one of its most important, yet weakest, aspects. This is the fundamental operation of obtaining the necessary pass-point data. It should be stressed that the key to successful results is in good data. No amount of algebra can make up for poor data. Therefore, the relatively simple but tedious operations of selecting, marking, measuring, and correcting pass-points are all important. Lack of good instrumentation constitutes the most serious bottleneck in the test and development of analytical triangulation.

As my contribution to this discussion, I would like to enlarge upon the subject of procedures and instrumentation involved in gathering pass-point data; then I will briefly outline the current instrumentation tests at U. S. Army Engineer Research and Development Laboratories.

Before discussing instrumentation, however, it would be useful to summarize the potential advantages of analytical triangulation. These advantages must serve as the guide-posts and goals in designing the most effective procedures and instrumentation. The potential advantages of the analytical triangulation system are:

First, increased accuracy over triangulation techniques using map plotting instruments. This is possible through more precise compensation for systematic errors, mathematical adjustment of random errors, and less reliance on the skill of the operator.

The second advantage is more effective military use. This is possible through simple, lighter, and less temperamental equipment, use of less skilled operators, and speedier operations.

The third advantage is greater adaptability for use with convergent photography without attendant calibration problems, and greater adaptability for use with airborne orientation and position data.

These advantages can be realized only by effective implementation of the four necessary operations of selection, marking, measurement and corrections. At the present time, facilities designed expressly to accomplish these operations are nonexistent. Regarding each of these deceptively simple operations, here are a few general comments:

REGARDING PASS-POINT SELECTION:

The stereocomparator, the traditional approach to the selection and measurement operation, is objectionable for military use when it approaches the size, complexity and cost of the heavy-base map-plotting instruments. Precision screws and complex optical trains introduce manufacturing, calibration and maintenance problems that must be avoided. What is needed is simple, light instrumentation of sufficient accuracy that will provide for analytical triangulation, the effective tool provided for map compilation by the Kelsh Plotter.

REGARDING MARKING OF PASS-POINTS:

The main consideration is that the pass-point must be quickly and precisely marked on at least one, but preferably on all three, consecutive photographs. Numerous potential means are available: a photographic record, marking or etching using dies or ultra-violet light, and various mechanical means.

REGARDING PASS-POINT MEASUREMENT:

Measurement of the pass-point coordinates should be reliable to within five microns. Besides precise lead screws, the design possiblities under consideration incorporate glass scales, glass grids, and a system based on the interferometer principle. A solution that readily lends itself to automatic read-out is preferable.

REGARDING CORRECTION OF PASS-POINT COORDINATES:

All of the potential systematic errors, such as those caused by lens distortion, dimensional changes in the aerial film, and refraction, can be readily eliminated by correcting the pass-point coordinates. This constitutes one of the main advantages of analytical triangulation over map-plotting instrument techniques. The required computations can be programmed for solution, together with the triangulation computations, in the electronic computer. The problem is, of course, in determining the corrections to be made. Aerial camera calibration data of higher accuracy are now needed, together with more data concerning the shrinkage effects in each photograph. Until these data are available, it is questionable whether substantial improvements in accuracy over conventional methods can be made.

The foregoing operations of selection, marking, measurement and correction of pass-points provide the data that are supplied to the electronic computer, which solves for the ground position and elevation of the desired points. A wide variety of general purpose computers are available to meet the storage and speed requirements imposed by the analytical computations.

This concludes my brief survey of some of the operational aspects involved in analytical triangulation.*

* This is not the end of the discussion by Mr. Bodnar. The balance consisted mainly of a brief outline of the instrumentation in experimental use at ERDL. Omission as a part of the printed panel discussion was considered advisable by the Publications Committee and Mr. Bodnar because of the much longer, greater detail and illustrated description appearing in his paper in the December 1957 issue of PHOTOGRAMMETRIC ENGINEERING, pages 957–962—EDITOR.

Discussion by Mr. V. A. van Praag

WHAT you have heard discussed up to the present time is heavy equipment and complexities of equipment required to handle this triangulation problem, either with stereo comparators or planigraphs or equipment of that nature. Accordingly the Moderator has asked me—the exhibitor of the small G15D Electronic Computer of the Bendix Computer Division—to describe from the small computer viewpoint what I believe that equipment could contribute and how it could make possible getting away from this heavy equipment. I am delighted to discuss the triangulation problem from that viewpoint. As a matter of fact, after listening to everybody beating the tom-toms for the IBM 650, I felt much like the hero of a poem written some hundred years ago by the Chaplain of an Irish Regiment at the Siege of Lucknow. It goes:

"Cut off from the land before me,
Betrayed by the land that I find
The brightest have gone before me
Leaving the dull one here behind."

But be that as it may, one matter I should like to straighten out immediately is that our position in this whole field has been of complete ignorance, but one which we are trying to remedy. When I was originally asked to speak on this panel, I rushed out and read a bunch of technical papers and the like. Then I talked to a few people around the country and we even spent a few dollars and hired a few consultants, because I've got a little motto that says, "If you don't know anything, find a guy who does and ask him, then listen." And it works pretty well.

So we are presently in the process of learning more about your problems. When we get into new areas—and photogrammetry for us represents a relatively new area— we have a tremendous vocabulary problem. You people know your problems, —you have lived with them—and you use words that to us have very little meaning. About the only solution for that from our point of view is first, to hire a couple of people who know something about your problems, secondly to train them so they can understand something about computers, and thirdly to try to find a couple of people with lots of courage and money, who want us to explore their problems for them. We have done all of the three. In this process of learning, we have had a great deal of assistance from such as the course at M.I.T. that Charlie Miller ran. We also have had much cooperation from some of our customers, one of whom is sitting here —Don Lewis from Pacific Air Industries— and from consultants such as Claus Aschenbrenner, and a few others we have hired.

Where do we go from here? In the first place, let's get one thing quite definitely established. When you talk about small computers, about all you really mean is small in physical size, or small in purchase price. The small computer will do anything the big computer will do, and about the only thing you are getting for the additional money is speed. The big computer will solve the same problems but faster.

Depending upon the nature of the problem, this speed is sometimes a pretty nebulous thing to tie down. For example, we have one customer who has a big Univac; he ran a series of problems on it and compared them to a similar series run on our small computer. They can beat our time by a factor of seven, but when you stop and think of what his computer cost with a factor of twenty or more in money, there is a good economic reason for the use of small computers. As a matter of fact, the tendency of the private photogrammetrist to go to the smaller computer manufacturers for his computer is quite obviously shown by the number of them who have either purchased computers, or purchased time from programming services.

Another condition that I am delighted to note is that mathematics have some bearing on the subject in which you are dealing; we get very little of the old, "This is judgment—or experience," which is quite prevalent in other fields.

It is a pleasure to note that you are thinking about the basic problems to be done, and then about what basic hardware is available, because we are finally in a stage in computers where it is possible to solve the problem and warp the machinery to the problem, instead of warp the problem to the available facilities. I think that is a pretty healthy situation.

The Mathematical Formulation of Analytical Aerial Triangulation

G. H. SCHUT,
National Research Council of Canada, Ottawa, Canada

ABSTRACT: *The present methods of analytical aerial triangulation are discussed. The three current orientation procedures are described. The simplest and most economical formulation of condition and correction equations is given. The storage space required on an electronic computer is derived, including, for the correction equations, either general or special solutions.*

INTRODUCTION

IN RECENT years several methods of analytical aerial triangulation have made their debut in photogrammetric literature. Each of these methods employs the procedure of determining the orientation of the photographs of a strip in a rectangular spatial coordinate system, after which spatial coordinates of measured points are computed by intersection of corresponding rays. Comparison of the methods is complicated by the fact that many different notations and auxiliary symbols are used, and also different branches of mathematics. It is the purpose of this paper to point out their main characteristics and to consider the most advantageous mathematical formulation.

ORIENTATION PROCEDURES

Three fundamentally different orientation procedures are employed. In two of these, each photograph in succession is oriented with respect to the preceding one. In the first, initiated by the British Ordnance Survey, this orientation is performed by separate relative orientation and scaling of the resulting model. Relative orientation is accomplished by making corresponding rays intersect; scaling by making heights or distances of points in the model equal to those of the identical points in the preceding model. This procedure is also employed in the method developed by the author at the National Research Council of Canada. A result of this procedure is that an error in the relative orientation of a photograph causes a positioning error of the following part of the strip with respect to the preceding part. An error in scaling causes a difference in scale of the two parts.

In the second procedure the orientation of each photograph is performed in such a manner that the position of points common with the preceding model is enforced in the new model. This procedure is employed in the method developed by Church and by Bartorelli, and also in a modification of Herget's method, developed by McNair. As a result of this procedure, deformation of a model caused by an orientation error causes deformation of the next and all subsequent models. Consequently the propagation of errors through a strip is different from what it is in the first procedure. Also, accurate identification of identical points in each two consecutive models is essential. Furthermore, since only in the initial model of a strip is deformation independent of errors in other models, the outcome of a strip triangulation depends on the choice of initial model. This does not mean that this procedure is necessarily inferior to the first. The relative merits of the two procedures should be determined either by a theoretical study of the propagation of errors or by an empirical treatment of a great number of strips.

The third procedure is employed in a method developed by Schmid, and in the original Herget method. Here the orientation of all photographs is computed simultaneously from all available data. In the case of redundant observations a simultaneous adjustment of the orientation elements of all photographs is performed. That makes this procedure the best from a theoretical point of view. In practice it has the disadvantage that it is by far the most complicated. As yet it has been coded only by Schmid and only for separate models.

Condition and Correction Equations; their Solution

The mathematical formulation of analytical aerial triangulation requires the use of a condition of intersection of corresponding rays. The published methods state this condition in five different forms. The Ordnance Survey method minimizes the y-parallax of corresponding rays. The method developed at the National Research Council makes use of co-planarity of corresponding rays. In this case the contents of a tetrahedron, the vertices of which are the two projection centers and the two corresponding image points, is minimized. Church's method employs the condition of intersection in a modified form: angles at the projection center in the object space are made equal to the corresponding angles in the image space. Subsequently the camera is oriented in such a way that rays in the image space coincide with the corresponding rays in the object space. Herget's method minimizes the minimum distance of corresponding rays. A fifth method makes use of y-parallaxes in the photographs.

The second and third procedures require, besides the condition of intersection, the condition that certain points must lie on the corresponding rays. Those are points common with the preceding model, and ground-control points, respectively.

Each condition gives rise to a condition equation in which the various orientation elements and measured photograph coordinates occur. From a sufficient number of condition equations the orientation elements can be computed. Since the condition equations are non-linear with respect to the orientation elements, they are replaced by correction equations, which are approximations, linear with respect to the orientation elements and to the want of intersection. The solution is obtained by an iterative process. In the case of redundant observations, an adjustment procedure is necessary. Weight and correlation factors, to be applied to the linear equations, must be computed from weight and correlation of the measured coordinates.

The solution of the correction equations can be obtained in two different ways. If the points are chosen in or near special pre-determined positions, it is possible to pre-solve the correction equations by desk calculator and to use the solution for each iteration. The solution gives the orientation corrections as simple linear functions of the want of intersection. This principle is employed by the Ordnance Survey, who choose their points in the six positions of numerical relative orientation. The other methods which have been mentioned by name do not impose this restriction. These general solutions require solving the correction equations at least once for each model. In practice the equations are solved for each iteration. The general solution has the advantage of being more flexible and of requiring fewer iterations than special solutions. The special solutions, on the other hand, require less computation per iteration.

Selection of a Condition of Intersection

The result of a triangulation is independent of the choice of a condition of intersection and virtually independent of the use of a general or a special solution, provided that weight and correlation are derived as before mentioned and that the interation procedure is pursued until convergence is reached. This makes economy of computation the only consideration in this selection.

Evaluation of the five conditions of intersection shows that the choice does not affect the convergence of the iteration. However, the condition of co-planarity of corresponding rays leads to the simplest correction equations for use in a general solution and to the simplest formulation of the want of intersection for use in a special solution. If division is a built-in feature of the electronic computer on which the triangulation is performed, the y-parallax condition is virtually equivalent as far as the formulation of the want of intersection is concerned.

The condition of co-planarity states that the want of intersection, as expressed by a third order determinant, must be equal to zero:

$$\begin{vmatrix} b_X & b_Y & b_Z \\ X_i & Y_i & Z_i \\ X_{i+1} & Y_{i+1} & Z_{i+1} \end{vmatrix} = 0$$

The elements of this determinant are the base components and the spatial coordinates, with respect to origins in the corresponding projection centers, of the two image points. The spatial coordinates are linear transformations of the photograph coordinates. In the coefficients of the transformation formulas, the sines and cosines of the camera rotations occur. Rotations ω, ϕ and κ about orthogonal axes lead to the simplest formulas. The correction equation is obtained in its simplest form if the MacLaurin series is used to differentiate the condition equation. In this case its coefficients contain only base components and transformed photograph coordinates. For fast convergence of the iteration process computed corrections should not be added to the approximate values; rather, new approximations should be computed by matrix multiplication of the coefficients of the transformation formulas.

A detailed account of this formulation, applied to the procedure of separate relative orientation and scaling, is given in a report, "Analytical Aerial Triangulation," by the National Research Council of Canada, together with complete results of the triangulation of two strips.

Of the alternative conditions of intersection, one, the y-parallax condition, gives a correction equation which contains in its coefficients the same nine elements, but which requires more computation. The condition of equality of angles and the condition of minimum distance both necessitate computation of direction cosines, which makes the computation more laborious, even in the exceptional case where extraction of a square root is a built-in feature of the electronic computer. A detailed comparison of the various conditions of intersection has been given in a paper recently read before an international meeting on aerial triangulation in Ottawa. This will be published in an early issue of *Photogrammetria*.

If the procedure of separate relative orientation and scaling is used, the condition of co-planarity leads to the following correction equation:

$$\begin{vmatrix} b_X & b_Y & b_Z \\ X_i & Y_i & Z_i \\ 0 & -Z_{i+1} & Y_{i+1} \end{vmatrix} d\omega + \begin{vmatrix} b_X & b_Y & b_Z \\ X_i & Y_i & Z_i \\ Z_{i+1} & 0 & -X_{i+1} \end{vmatrix} d\varphi$$

$$+ \begin{vmatrix} b_X & b_Y & b_Z \\ X_i & Y_i & Z_i \\ -Y_{i+1} & X_{i+1} & 0 \end{vmatrix} d\kappa + \begin{vmatrix} Z_i & X_i \\ Z_{i+1} & X_{i+1} \end{vmatrix} (b_Y + db_Y)$$

$$+ \begin{vmatrix} X_i & Y_i \\ X_{i+1} & Y_{i+1} \end{vmatrix} (b_Z + db_Z) + \begin{vmatrix} Y_i & Z_i \\ Y_{i+1} & Z_{i+1} \end{vmatrix} b_X = 0$$

If one of the two alternative procedures is used, the base component b_X in the last term must be replaced by $b_X + db_X$, because then b_X is also a variable. The condition that certain points must lie on the corresponding rays, which is required here, can be expressed by two conditions of intersection, namely, the conditions that a ray must intersect both of two vectors through a given point. If these vectors are chosen parallel to the X- and Y- axes, and the vector from the point to the projection centre is called β, the two conditions are

$$\begin{vmatrix} \beta_X & \beta_Y & \beta_Z \\ 1 & 0 & 0 \\ X_{i+1} & Y_{i+1} & Z_{i+1} \end{vmatrix} = 0$$

and

$$\begin{vmatrix} \beta_X & \beta_Y & \beta_Z \\ 0 & 1 & 0 \\ X_{i+1} & Y_{i+1} & Z_{i+1} \end{vmatrix} = 0$$

The corresponding correction equations are

$$\begin{vmatrix} \beta_Y & \beta_Z \\ -Z_{i+1} & Y_{i+1} \end{vmatrix} d\omega - \beta_Y X_{i+1} d\phi - \beta_Z X_{i+1} d\kappa$$

$$+ Z_{i+1}(\beta_Y + db_Y) - Y_{i+1}(\beta_Z + db_Z) = 0$$

$$-\beta_X Y_{i+1} d\omega + \begin{vmatrix} \beta_Z & \beta_X \\ -X_{i+1} & Z_{i+1} \end{vmatrix} d\phi - \beta_Z Y_{i+1} d\kappa$$

$$- Z_{i+1}(\beta_X + db_X) + X_{i+1}(\beta_Z + db_Z) = 0$$

For use in the third procedure, additional equally simple equations can be set up for three corresponding rays and for ground control points of which either only the plane position or only the height is known.

The above formulas are the simplest which have yet been developed for analytical aerial triangulation.

The Choice Between A General Solution and A Special Solution

A comparison of a general solution with special solution must include, besides the difference in flexibility already mentioned, an evaluation of the economy of computation. This economy is governed by the required computation time and storage space on an electronic computer. The comparison is rather difficult, since the computation time required for the various mathematical and logical operations is not the same on different computers. Also, the storage space required for the program depends on the instruction code of the computer. Only the procedure of separate relative orientation and scaling and the condition of co-planarity of corresponding rays will be considered in detail.

A rough comparison between computation times can be obtained from the number of multiplications and divisions which is required for relative orientation of a photograph. On most computers these operations are considerably more time-consuming than additions, subtractions and logical operations. Extraction of square roots need not be considered, since in the described method it is not required.

The number of multiplications and divisions depends to a large degree upon the required number of iterations. Experience at the National Research Council has shown that the general solution requires three iterations of the relative orientation only in exceptional cases. Generally one iteration using five points, followed by one iteration using all available points, is sufficient. According to Ordnance Survey reports, their special six-point solution may in exceptional cases require up to eight iterations. The resulting number of multiplications and divisions is listed in Table 1. The first part gives the number for each step in a general solution and in the special six-point solution; the second part gives the totals required for the necessary number of iterations. It follows from the table that, as far as computation time is concerned, the general solution is not at a pronounced disadvantage.

The storage space occupied in the general method employed at the National Research Council can be given for Ferut, the Ferranti electronic computer at the University of Toronto, for which machine the method has been coded. This storage space is listed in Table 2. The second column gives the storage space required for the actual computation instructions, and the third column the storage space occupied by tests, constants and transfer instructions. The program contains many loops: in the average computation time of 15 seconds per model for strip triangulation and transformation, about 12,500 instructions are obeyed, against 2,300 stored.

Ferut has the following characteristics: one-address instructions, no instruction for division, the possibility of direct addition of products in the accumulator, seven

TABLE 1

NUMBER OF MULTIPLICATIONS AND DIVISIONS IN RELATIVE ORIENTATION

PART I:	
For the separate steps in both general solution and special solution:	
X_i, Y_i, Z_i, X_{i+1}, Y_{i+1}, Z_{i+1}	18 per point
X_{i+1}, Y_{i+1}, Z_{i+1}, additionally	9 per point per iteration
transformation matrix, from three sines	20 per iteration
matrix multiplication	27 per iteration
Additional, in a general solution:	
correction equations	16 per point per iteration
normal equations	20 per point
scaling of normal equations	30 per iteration
solution of five linear equations	65 per iteration
Additional, in a special solution:	
want of intersection	7 per point per iteration
five corrections	5 per iteration
PART II: TOTALS	
In a general solution:	
6 points, 2 iterations	757
6 points, 3 iterations	994
9 points, 2 iterations	946
9 points, 3 iterations	1,183
In a special solution:	
6 points, 4 iterations	748
6 points, 8 iterations	1,388

TABLE 2

REQUIRED STORAGE SPACE OF THE PROGRAM FOR THE NATIONAL RESEARCH COUNCIL METHOD ON FERUT

	Instruction Lines	Miscellaneous Use
Correction of photograph coordinates	38	40
Relative orientation on five points	622	189
Additional for adjustment on up to 15 points	279	77
Scaling	51	5
Strip coordinates	122	178
Total for strip triangulation	1,112	489
Strip transformation	609	137
Working space for data: 300 numbers		

registers for address modification and conditional transfers, direct access to 512 instruction lines of which each two may serve as storage for a number, and a large store with indirect access. Most medium-sized computers have an instruction for division, but no facility for direct addition of products. They also have fewer, if any, registers for address modification and conditional transfers. They have either one-address or two-address instructions.

From these characteristics it is possible to estimate the storage space which the general method would require on an average medium-sized computer. The program for strip triangulation can first be shortened by 150 instructions through a more extensive use of subroutines. The presence of an instruction for division and the absence of a facility for direct addition of products together make 350 additional instruction lines necessary. This makes the total number of instruction lines for strip triangulation about 1,800. To these must be added an uncertain number required because of fewer facilities for address modification, and 300 storage spaces for data.

Evidently, counting the storage space required by an instruction and by a number each as one "word," it is not possible to contain the program for strip triangulation within 2,000 words. On the other hand, it is unsatisfactory to be obliged to read in the instructions before the orientation of each model. It should be possible to perform a strip triangulation as one continuous operation, without human intervention. This means that the general method requires a storage space of more than 2,000 words. 4,000 words is quite adequate. Of these, less than 500 need be directly accessible.

The program for a special six-point method which employs the procedure of separate relative orientation and scaling and which expresses the want of intersection by means of the condition of coplanarity would seem to require the following number of words on the average medium-sized computer: for radial correction of photograph coordinates, 40 instruction words; for relative orientation, 250 words; for scaling of the model and for strip coordinates, 175 words; for data and results, 175 words, and an additional number for tests. Consequently 1,000 words is quite adequate for strip triangulation.

If the procedure of enforcing the positions of points obtained in the preceding model is employed, the required storage space is of the same order.

It can be concluded from the above that, if an electronic computer is available that has a storage space of 4,000 words or more, a general method is to be preferred because of its flexibility in the choice of the position of the points. For an electronic computer that has a storage space of 1,000 or 2,000 words, a special method is more practical because of its more limited storage requirements.