A mess of pottage, a glass of wine,
A game and he travels on.
He's going once,—He's going twice,—
He's going, and almost gone.
Then the Master comes, and the foolish crowd
Can never quite understand
The worth of a soul, and the change that is wrought
By the touch of the Master's hand.

In our discussion this morning I have tried to indicate *what* we may develop in photogrammetry and photo interpretation in the years ahead; I have also tried to indicate *how* we might develop it. I believe I would have been remiss in speaking to the topic assigned me, if I had not made this concluding attempt to raise the question,—"*Why?*"

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*The Use of a Medium Size Digital Computer in the Solution of Two-Stage Photo Rectification Problems**

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ABSTRACT: *This paper presents a description of the photo-rectification problems and the requirements of two-stage rectification. A method of computer solution is outlined in which an iterative procedure is used to calculate settings for photo-rectification equipment. The restrictions on the settings are imposed by the projective characteristics of the photography and the mechanical and optical limitations of the rectification equipment.*

INTRODUCTION

THIS paper describes a method for use in computing instrument settings for one- and two-stage photo rectification. The body of the paper is divided into the following topics:

A. Discussion of the Rectification Process.
B. Description of the Specific Rectification Problem.
C. Presentation of the Method of Solution.
D. Discussion of the Limitations of this Method.

It has been found that this method is well suited to application on a medium-size digital computer.

A. THE RECTIFICATION PROCESS

Rectification is defined, photogrammetrically, as the projection of an oblique photograph to a horizontal plane. The utility of rectification is seen in its property of restoration of constant scale to all points imaged on the oblique photograph, thus greatly simplifying the task of obtaining metric information from oblique photography.

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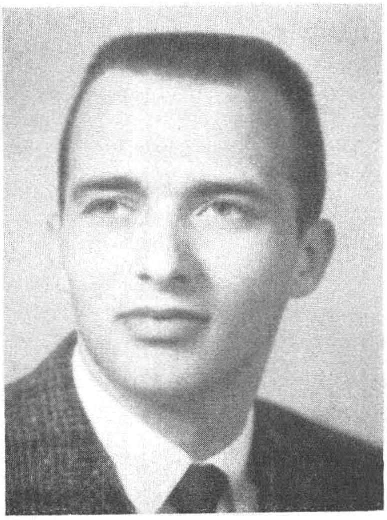
Rectification can be accomplished in several ways; numerically, graphically and photographically. This discussion is concerned with the photographic means; i.e., through using transforming printers and rectifiers.

The range of variables (focal-length, magnification and tilt) associated with an oblique photograph that can be accommodated by a given rectification instrument is limited by the mechanical and optical characteristics of the instrument. However, photographs whose variables fall outside this limited range can be rectified with the given instrument if the rectification is accomplished in several stages, i.e., projection to planes intermediate between the plane of the photograph and the horizontal plane. Thus the limitations of the rectification instrument can be extended by multiple stage rectification.



CHARLES W. HANSON

B. The Specific Problem

The development of the method of solution was motivated by a specific problem—that is, rectification of oblique photography, with a specific range of variables, using a tilting lens type of rectifier. The limitations imposed by this particular rectifier are grouped in three categories:

1. Mechanical
2. Photographic
3. Theoretical

The *mechanical limitations* include the range of tilts of the negative, lens and easel planes, and the maximum and minimum allowable distances between these planes. Also included is a dimensional limitation on the format of the photography due to the sizes of the negative carrier and the easel.

The *photographic limitations* are comprised of (1) the usable field of projection of the rectifier lens system and (2) the minimum angle of incidence of light rays at the surface of the easel.

The *theoretical limitations* are contributed by the focal-length of the projecting lens system of the rectifier. This quantity combined with the projective elements of the photography appears as the argument of an inverse trigonometric function. This function is undefined when the argument exceeds a certain limit; hence, rectification is theoretically impossible in that case.

For this specific problem, involving two-stage rectification, there are 13 limitations or restrictions imposed on each stage of the rectification. Since the limiting values of each restriction may not be the same in both stages, there is a total of 26 restrictions imposed by the rectification equipment.

A further restriction is placed on the projective elements of each stage of the rectification—when combined projectively they must satisfy the projective requirements imposed by the photograph. This means that the projective elements of one stage can be chosen independently, but the elements of the other stage must be chosen in accord with some dependency relationships between the projective elements of the first stage and those of the photography.

In summary, the specific problem is to choose projective elements for a two-stage rectification process so that these elements meet the projective requirements of the photography and do not exceed the limitations of the rectifying equipment.

C. The Method of Solution

The method of solution used is basically an iterative trial and error procedure. A means of improving estimates of the final solution has been developed on the basis of a linear model. In this section this means of improvement is described. Also presented herein is a description of a digital computer program which automates the method of solution. The first step in the solution procedure for two-stage rectification is to make an initial estimate of the final solution, i.e., a choice of the independent projective elements for one stage. This is made on the basis of the projective elements of the photography, and some empirical rules to aid in formulating this first guess can be established.

The next step is to calculate the dependent projective elements, and to check this trial solution to determine which, if any, of the limitations described above are exceeded. If none of these are violated, then an acceptable solution has been found. If not, then a refined estimate must be made.

To refine an estimate, a measure of the total amount by which the various restrictions are exceeded is defined. The effects of changes of the independent projective elements on this "system error" are investigated, and a refined estimate is made in such manner that the anticipated system error decreases. The procedure is then repeated, the estimates becoming more and more refined until an acceptable solution is found; that is, until the system error has finally decreased to zero.

To mechanize this method of improving estimates, the restrictions are measured with respect to a normalized scale; that is, a restriction is exceeded if its restriction measure is greater than unity. Besides creating a common reference for comparisons among the various restrictions, this normalized method of measurement is convenient for determining the amount by which a limitation is exeeded. The system error is defined as the sum of those restriction measures greater than unity, thus giving an indication of how far away the estimated solution is from an acceptable solution.

The effect of changes in the independent projective elements on the system error gives an indication of the direction in which the estimate must be refined, the amount of refinement depending upon how large a change can be made before some new limitation is exceeded.

The mechanics of this method of improving estimates are illustrated in the following example.

Figure 1(a) shows a plot of restriction measures of eight restrictions for a particular estimate of the projective elements. In this example, the second-stage projective elements (focal-length—f_2, magnification—m_2, tilt—t_2) are chosen as the independent variables. The restriction measures for incremental changes (Δf, Δm, Δt) in these projective elements are presented in Figures 1(b), 1(c) and 1(d) respectively. The system error for each of the four cases illustrated is:

(a) $E_0 = 5.8 \qquad (f_2, m_2, t_2)$

(b) $E_{\Delta f} = 5.5 \qquad (f_2 + \Delta f, m_2, t_2)$

(c) $E\Delta_m = 4.5 \qquad (f_2, m_2 + \Delta m, t_2)$

(d) $E_{\Delta t} = 4.1 \qquad (f_2, m_2, t_2 + \Delta t_2)$

In all cases the system error is seen to decrease for an incremental increase of each projective element—thus the direction of refining the estimate is clearly indicated.

The number (n) of incremental changes that can be made before a new restriction becomes violated is calculated for each variable by dividing the distance from each restriction point to the boundary by the incremental distance that restriction point moves toward that boundary for the incremental change of the variable, the minimum of these ratios being the maximum number of incremental changes allowable. Thus:

$$n_{\Delta f} = \min\left[\frac{1-.5}{.7-.5}, \frac{1-.4}{.6-.4}, \frac{1-.8}{1.3-.8}\right] = \underline{0.4}$$

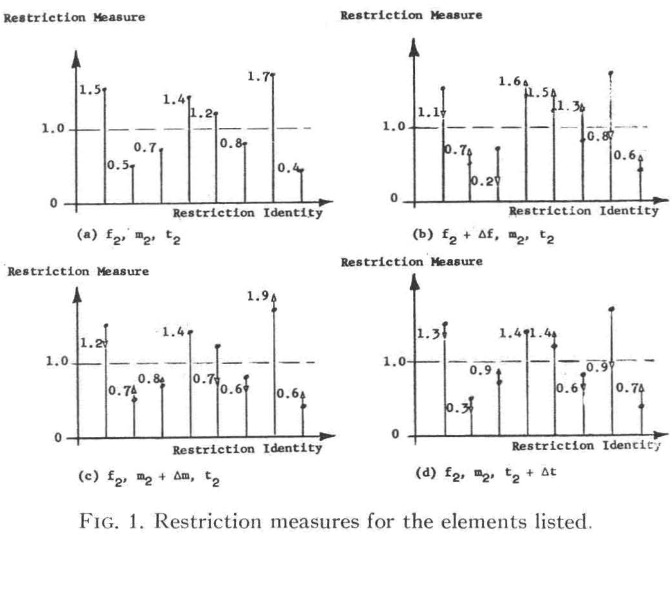


FIG. 1. Restriction measures for the elements listed.

$$n_{\Delta m} = \min\left[\frac{1-.5}{.7-.5}, \frac{1-.7}{.8-.7}, \frac{1-.4}{.6-.4}\right] = \underline{2.5}$$

$$n_{\Delta t} = \min\left]\frac{1-.7}{.9-.7}, \frac{1-.4}{.7-.4}\right] = \underline{1.5}$$

The anticipated change in system error ($\overline{\Delta}E$) is the product of n and the incremental change (ΔE). Thus:

$$\overline{\Delta}E_{\Delta f} = n_{\Delta f}\Delta E_{\Delta f} = 0.4[5.5 - 5.8] = -\underline{0.12}$$

$$\overline{\Delta}E_{\Delta m} = n_{\Delta m}\Delta E_{\Delta m} = 2.5[4.5 - 5.8] = -\underline{3.25}$$

$$\overline{\Delta}E_{\Delta t} = n_{\Delta t}\Delta E_{\Delta t} = 1.5[4.1 - 5.8] = -\underline{2.55}$$

The variable which yields the greatest decrease in system error is changed to form a new estimate of the projective elements (f_2', m_2', t_2'). For this example:

$$f_2' = f_2$$

$$m_2' = m_2 + n_{\Delta m}\Delta m$$

$$t_2' = t_2$$

For a large number of restrictions the above method becomes cumbersome for manual use because of the numerous calculations involved. However, because of its simplicity in concept and the repetitive nature of the required calculations, it is well suited to automatic computing machinery.

This method has been programmed for a medium-size digital computer. Figure 2 presents the flow diagram of the solution procedure used by the computer. After the data have been input, the first effort of the program is to determine whether or not single-stage rectification is feasible. If not, then the program proceeds with the iterative method of solution, automatically making the initial estimate, and refining it with each iteration until all the restrictions are satisfied, or no further improvement can be made; i.e., more than two stages are necessary for the rectification.

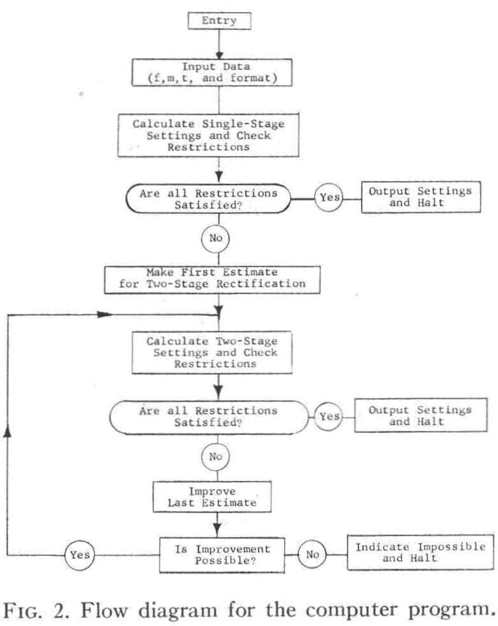


FIG. 2. Flow diagram for the computer program.

D. Limitations of This Method

The application of this method for the solution of two-stage rectification problems has two limitations—one theoretical, the other practical. The first limitation is due to there being no theoretical guarantee that the iterative method will converge universally. This is because the restrictions are very complicated functions of the projective elements, and tests for theoretical convergence become extremely complex. To account for this unknown, several checks were built into the computer program described above to guard against "aimless wanderings" of the method. In practice, use of this method has always yielded an acceptable solution or an indication that no acceptable solution existed.

The second limitation is due to the fact that any number of acceptable solutions may exist for a given problem (in theory there may be a three-fold infinity of solutions; in practice there are only a limited number of realizable, distinct solutions). The solution obtained through the method described is only one of a family, and it depends heavily on the initial estimate which this method has refined.

Conclusion

A method for solving two-stage photographic rectification problems has been presented, following a brief discussion of the rectification process and a description of the specific problem which motivated the development of the method. The method, though cumbersome for manual use, is well suited to automatic computing machinery. The application of a digital computer to this method was presented in flow chart form. The extension of this method to more than two stages of rectification is limited by the fact that the method, although simple in essence, becomes compounded with details as the number of stages is increased.