confusion of smoke and debris. (No. 30). This illustrates the really primitive conditions under which we operate during a Pacific test.

You have been shown photographic examples taken from years of tests and experimental work. The scientists and engineers who accept the challenge of these problems in the laboratory and in the field know one reward—the importance of their effort to the national defense. Those who are doing the work know the horror and the chaos with which they are dealing, and we all hope and believe that the knowledge gained—carefully disseminated—may help prevent the use of such weapons in anger.

Precision Photogrammetry in Missile Testing*

DUANE C. BROWN, RCA Missile Test Project, Atlantic Missile Range, Patrick A.F.B., Fla.

ABSTRACT: Ballistic cameras employed at the Atlantic Missile Range are described and the operational procedures and the philosophy underlying their use are outlined. Certain aspects of the data reduction are considered. The accuracies attainable from the ballistic camera system are discussed in relation to those attainable from other missile tracking systems. Some important applications of ballistic cameras are indicated.

INTRODUCTION

THE ballistic camera has been facetiously described as a glorified box camera. In some respects this description is rather appropriate, for one of the paramount virtues of the ballistic camera is its essential simplicity. On the other hand, the extraordinarily precise construction of the ballistic camera makes it a masterpiece of the opticomechanical industry. Because of its extreme precision, the ballistic camera provides a missile tracking system capable of producing position data having an absolute accuracy unattainable by any other system. Consequently, a major function of the ballistic camera system at the Atlantic Missile Range is to provide a standard for the evaluation and calibration of other tracking systems.

Description of the Cameras

The ballistic cameras currently employed at the Atlantic Missile Range, the Wild BC-4's, consist essentially of modified RC-5 aerial cameras mounted on the base of the celebrated Wild T-4 astronomical theodolite. Three different camera cones are used: the 115 mm. Aviogon (f/5.6; field 76° square), the 210 mm. Aviotar (f/4.2; field 47° square),



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and the 300 mm. Astrotar $(f/2.6; field 33^{\circ} square)$. The distortion of each lens is calibrated by means of star recordings to an accuracy of one micron or better. Photographic

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recording is done on ultraflat (6 fringe) glass plates of 6 mm. thickness.

TYPICAL OPERATIONAL PROCEDURE

Experiments have indicated that with suitable shutters, filters and photographic emulsions, it is possible to employ ballistic cameras in daytime operations. At present, however, ballistic cameras are used almost exclusively at night to record missile-borne flashing lights. The nighttime operational procedure at each ballistic camera station is generally as follows. Shortly before the missile flashes are to be recorded, several exposures of the stars are made with the camera securely locked in a stationary orientation. This procedure constitutes the precalibration and consists, more specifically, of successive exposures of 2, 1, $\frac{1}{2}$ and $\frac{1}{4}$ seconds, separated by intervals of 30 seconds. Each recorded star thus gives rise to four point-like images of graduated size. After the precalibration has been completed, the camera shutter remains closed until shortly before the programmed missile flashes are expected to appear in the field of view. It is then opened and remains opened until the termination of the flashing sequence.

A short time later the "postcalibration" is performed. This follows the same procedure as the precalibration, except that the exposures are made in reverse order, namely in the sequence $\frac{1}{4}$, $\frac{1}{2}$, 1 and 2 seconds. Useable stars to 9th magnitude are recorded by the Astrotar, to 8th magnitude by the Aviotar, and to 7th magnitude by the Aviogon. The final plate thus consists of a series of images of the missile-borne flashing light recorded against a background of hundreds of superb reference points provided by the stellar images. Inasmuch as the stellar images vary widely in size and density, one has little difficulty in obtaining a large selection of control consisting exclusively of stellar images whose photographic characteristics closely match those of the flashing light images. When proper care is taken in the selection of the control, the problem of personal bias in measuring is essentially eliminated, for personal bias then tends to be the same for both the flashing light images and the measured control.

FLASHING LIGHT SOURCES

The most widely-used missile-borne flashing light sources consist of pyrotechnic flares and high intensity strobe lamps. The most powerful pyrotechnic flare currently employed in missile work, the "Daisy," weighs about an ounce, and is sufficiently brilliant to be recorded at distances in excess of 5,000 nautical miles, by a camera of 4 inch aperture employing an infra-red sensitive emulsion. An advantage of flares is that they entail but a small weight penalty as long as the number of flashes required is not large (each flare together with its ejection mechanism weighs about one half pound). Their main disadvantages are relatively long flash durations (3 to 5 milliseconds) and the fact that they must be ejected an appreciable distance from the missile before ignition. A strobe lamp, on the other hand, is attached to the missile and has a flash duration of the order of 0.2 milliseconds. Early strobe units weighed as much as 175 pounds but more recent models have been reduced to less than 70 pounds and further drastic reductions in weight are anticipated. The limiting photographic range of existing strobe lamps suitable for missile work is far less than of Daisy flares, being about 250 to 300 nautical miles with a camera of 4 inch aperture.

The telemetered response of a missile-borne photocell establishes the time of occurrence of each flash to an accuracy of the order of one millisecond. Accuracies approaching 0.1 milliseconds are anticipated from ground based photomultiplier systems being developed.

The Calibration of Camera Orientations

Since the precise times of the stellar exposures are recorded on a chronograph, the directions of any selected stars at the various instants of exposure can be computed from the known astronomical coordinates of the stars, as reduced from star catalogue data, and from the known geographical coordinates of the station. While it is possible to compute both the rotational and the interior orientation of a camera from a minimum of three stars, it is the practice in missile photogrammetry to exploit redundancy to minimize the effect of measuring errors. Consequently, the calibration of orientation is based upon a least squares solution carrying as control points a total of 20 to 30 stars, compactly distributed about the flashing light images. Stars from both the precalibration and the postcalibration are employed in the solution. In this way, one readily detects any significant change in the camera orientation which may have occurred during the critical interval between the recording of the calibrations.

The Triangulation of Missile Positions

Since an overwhelming degree of redundancy is exploited in the least squares adjustment, the calibrated orientation of each camera may be regarded as being sensibly error-free insofar as the photogrammetric reconstruction is concerned. Consequently the errors in the computed directions of the missile points are predominantly the result of errors in the measured plate coordinates of the flash images themselves. The spatial position of each missile flash is established by the triangulation of corresponding rays from two or more stations. Often as many as six stations are involved in a triangulation. Because of measuring errors, corresponding rays to a given flash point are slightly skew. The measured plate coordinates of the flash images are therefore subjected to a rigorous least squares adjustment in order to determine the most probable point of intersection of each set of rays. The plate coordinate residuals resulting from the adjustments of both orientation and triangulation provide valuable checks of internal consistency and indicate the presence of any significant degree of systematic error.

EFFECTS OF ATMOSPHERIC REFRACTION

While atmospheric refraction imposes severe limitations upon the absolute accuracies ultimately attainable from all other tracking systems, particularly electronic systems, it causes no sensible degradation of ballistic camera accuracies as long as the points observed are well outside the effective atmosphere. The reason for this is basically that the computed directions of the reference stars in the background are the true, and not the apparent, directions-in other words, the directions one would observe in the absence of the atmosphere. A missile flash above the atmosphere is affected by refraction almost to the same degree as a background star having an identical apparent elevation angle. The small difference which exists is due to the fact that the flash point is at an essentially finite distance from the atmosphere, whereas the star is at an essentially infinite distance. Thus the difference is in the nature of parallax and the appropriate correction can be made with high precision.

By way of illustration one may use the fact that a missile point at an altitude of 100 miles shares about 96% of the refraction of a background star at the same elevation angle. Therefore, to make the necessary allowance for parallax, one needs, in effect, only to apply a differential correction amounting to 4% of the astronomical refraction. If, in turn, the astronomical refraction, say as computed from standard formulas, were in error by 5% (which would be rather extreme), the error in the parallax correction would amount to only

0.2% of the astronomical refraction, or roughly one half second of arc for an elevation angle of ten degrees. If the elevation angle were forty five degrees, the error in the parallax correction would be about one tenth second of arc.

From such considerations one is well assured that atmospheric refraction leads to no sensible dilution of ballistic camera accuracies for points well outside the atmosphere. This is an apparently insurmountable advantage of the ballistic camera system over all other tracking systems, for no other system can utilize extra-terrestrial control in a comparable manner.

Aspects of the Data Reduction Routines

Most photogrammetrists may be surprised to learn that of all reductions performed at the Atlantic Missile Range, the over-all ballistic camera reduction is by far the most complicated and extensive, particularly in terms of the variety of operations performed. In spite of this, a high degree of automation has been achieved on the present FLAC computer and complete automation will be achieved shortly on the IBM-709 computer. By complete automation it is meant that only a single computer run is required for the entire reduction-no intermediate data handling is necessary. A factor which contributes markedly to the efficiency of the present reduction is the recording of the entire Boss Star Catalogue on high speed magnetic tape. The master tape of the star catalogue is of epoch 1950 and is employed only to produce special catalogue tapes having epochs updated to the beginning of each solar year. The updated tapes are much shorter than the master tape, for they are intended to serve only for a period of one year and hence need to include only the catalogue numbers, the right ascensions and declinations, and the annual proper motions. The independent star numbers, taken from the ephemeris for a given year, are recorded on the same tape as the updated star catalogue for the year.

Because of the star catalogue tapes, the formerly tedious and time-consuming operations of identifying the selected reference stars and of compiling the catalogue data are now performed by the electronic computer. The computer makes far fewer mis-identifications than a human, and can establish the identification of a total of 150 stars, consisting of 25 stars from each of six plates, in less than two minutes of computing and tape hunting. Indeed, on the 709 computer an entire six station reduction, involving an average of 25 reference stars per plate and about 100 flash images, can be accomplished in a single computer run of less than one half hour.

It is believed that this routine approaches the near ultimate in refinement and efficiency. By way of contrast, it may be pointed out that only three years ago, with the rather limited computer then in operation, a similar reduction of the Atlantic Missile Range required as many as 12 separate computer runs (not counting reruns). The total computer time ranged from 6 to 10 hours, and considerable data handling was necessary between runs. Moreover, the computer reduction did not include automatic star identification, nor a number of other refinements incorporated into the present routine.

BALLISTIC CAMERA ACCURACIES

The standard deviations currently attained in measuring ballistic camera plates range from 2.5 to 3.5 microns, with 3 microns being typical in a normal reduction. These figures represent the combined effect of setting error and emulsion instability. The plate measuring accuracy can be converted into angular accuracy (in radians) by division by the focal length of the camera. With a camera of 300 mm. focal length, for example, 3 microns amounts to 0.00001 radians or about 2 seconds of arc. It follows that with good geometry, a pair of 300 mm. ballistic cameras are capable of providing positions accurate to the order of 1 part in 100,000 of the range from the cameras. Even higher accuracies are possible when more than two cameras are employed.

Since the plate measuring accuracy is essentially independent of focal-length, it is obvious that ballistic camera accuracies may be increased by the use of longer focal-lengths. It appears from considerations of field of view. stability, atmospheric shimmer, and versatility that the practical upper limit for focallengths for ballistic cameras would lie somewhere between one and two meters. Since improved photoprocessing methods and measuring instruments are expected to increase plate measuring accuracies to about 2 microns, and perhaps ultimately to 1 micron, the upper limit to potential photogrammetric accuracies in missile work prior to adjustment is in the neighborhood of 1 part in 500,000 (0.4) to 1 part in 1,000,000 (0.2).

The accuracy of the ballistic camera is strikingly demonstrated by a comparison with the next most accurate optical tracking instrument on the Atlantic Missile Range, the

tracking cinetheodolite. A typical cinetheodolite provides angular data, accurate to about 30 seconds of arc (combined random and systematic error), and a well adjusted cinetheodolite may produce data good to 15 seconds of arc. By contrast, a 210 mm. ballistic camera gives angles having a typical accuracy of about 3 seconds of arc (combined random and systematic error). This is an improvement over cinetheodolite accuracies by a factor of 5 to 10. However, since the standard deviation of an average is inversely proportional to the square root of the number of observations averaged, it follows that it would be necessary to average the results provided by 25 well adjusted or 100 typical cinetheodolites in order to attain a directional accuracy equal to that provided by a single 210 mm. ballistic camera. In this sense, then, it may be said that one 210 mm. ballistic camera is the equivalent of 25 to 100 cinetheodolites. Similarly, one 300 mm. ballistic camera would be the equivalent of 50 to 200 cinetheodolites.

Normal Applications of Ballistic Cameras

As indicated at the outset, one of the major functions of the ballistic camera system at the Atlantic Missile Range is to serve as a standard for the evaluation and calibration of tracking and guidance systems. The data required to accomplish this mission are obtained not only from missile tests, but also from specially designed aircraft tests. In fact, several hundred plates are reduced each year on aircraft tests alone.

In addition to its function as a range standard, the ballistic camera is employed on missile tests whenever positional data of extreme absolute accuracy are required. With the existing range survey, the five-station ballistic camera net employed for post-burnout coverage of intercontinental ballistic missiles is capable of providing positional data, referenced to Cape Canaveral, to an absolute accuracy of 10 to 20 feet. In this net, the distances from the various camera stations to the missile points range from 200 to 350 miles, and the baselines between consecutive stations average somewhat over 100 miles in length.

Other applications of ballistic cameras include satellite observations, the recording of re-entry phenomena, flame chopping with synchronized shutters, and the recording of continuous flame traces, particularly to provide points of burnout and ignition of multistage rockets.

GEODETIC APPLICATIONS

An application of ballistic cameras which will assume greater and greater importance as missile testing requirements become increasingly stringent, has to do with the improvement of the range survey, which is currently regarded as being accurate to about 1 part in 100,000. Newly developed photogrammetric methods, which underwent a successful field test last year, are expected ultimately to improve the range survey to about 1 part in 300,000 or better. The observations required to accomplish this will, for the most part, be acquired purely as by-products of missile tests already programmed for the next few years. It is believed that as the satellite age unfolds, geodetic photogrammetry will find important applications elsewhere.

CONCLUSIONS

The importance of ballistic cameras as a calibration standard is derived from the fact that somewhat conflicting trajectories are obtained from the various tracking systems observing a given missile flight. Such inconsistencies reflect the systematic or bias errors made by the systems. The "bias factor" of a system may be defined as the ratio of the bias in a typical observation to the standard deviation of the observation (the standard deviation, of course, is a measure of the purely random or accidental error). With cinetheodolite data, for example, the standard deviations of the measured angles range typically from 5 to 10 seconds of arc, while the bias errors range typically from 20 to 40 seconds of arc. Hence the bias factor for cinetheodolite data may range from 2 to 8. Again, one of the more precise of the electronic tracking systems is capable of measuring direction cosines having standard deviations of only 2 to 5 parts per million. On the other hand, the biases in the direction cosines usually range from 50 to 150 parts per million giving the system a bias factor of 10 to 75.

The ballistic camera is the only missile tracking system of high precision which can consistently produce data having a bias factor of substantially less than unity. Consequently, it is also the only system of high precision for which statistically meaningful and precise statements can be made concerning the absolute accuracy of the final product. High precision coupled with a lowbias factor thus makes the ballistic camera uniquely qualified to serve as a calibration standard for other instrumentation. It is therefore highly desirable in missile testing to reconcile conflicting trajectories by "hanging" them upon a ballistic camera trajectory which in turn, is "hung" upon the stars.

An Application of Photogrammetry to Radar Research Studies*

EARL S. LEONARDO,

Development Engineer, Aerophysics Dept., Goodyear Aircraft Corp., Litchfield Park, Ariz.

(Abstract is on next page)

DURING the past year, Goodyear Aircraft Corporation, in conjunction with the Applied Physics Laboratory of The Johns Hopkins University, carried out a radar return research program sponsored by the Navy Bureau of Ordnance. Its purpose was to accurately identify and measure radar return signals at various depression angles from different types of terrain: cultivated areas, forest, lakes, desert areas, and grasslands or meadows. To a lesser degree, identification of return from cultural targets was also included in the program. Relatively flat land was desired, as the many variations in slope and elevation of mountainous or broken terrain would have required intricate, if not impossible, data-reduction procedures.

For this study, Goodyear developed an accurately calibrated side-looking, non-scanning, strip-mapping radar, which was installed in the nose of a P2V-3W "Neptune" aircraft. The radar antenna was pre-set to look perpendicular to the aircraft's desired heading. Since the antenna was gyro-stabi-

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