to be made in the equipments being evaluated. It is also a task of this unit to plan tests and flight plans that will provide or simultate the necessary dynamic operating conditions to properly evaluate and calibrate range equipment.

Conclusions

The Photogrammetric Triangulation System represents the application of sound design principles to the solution of instrumentation problems defined by the most rigid of performance specifications. The system is completely self-contained and independent in operation. Its performance has already demonstrated a sound instrumentation and engineering approach, and it has become the range standard for all optical and electronic instrumentation and systems. When operated as specified, the system will provide data to an accuracy of 1 part in 200,000, which is 10 times greater than the next most accurate system in use today, the tracking Cinetheodolite.

*A Photogrammetric Radio Telescope Calibration**

DONN L. OCKERT,
Photogrammetry, Inc.,
Silver Spring, Maryland

Introduction

A radio telescope with a reflector 40 feet in diameter is operated by the Ohio State University Department of Electrical Engineering. The reflector was designed to be a paraboloid of revolution with a focus of 18 feet. Although the structure, shown in Figure 1, was designed and built within the Department of Electrical Engineering, the errors accumulated during construction were not known. Once erected, its size prevented any simple direct measurement of the reflector. It became obvious that a photogrammetric method that would measure photographs of the telescope rather than the telescope itself would be desirable, if reasonable accuracy could be obtained with available equipment. Encouraged by some kind advice and ideas from Professor Frederick J. Doyle, and armed with the facilities of the Institute of Geodesy, Photogrammetry, and Cartography which include a Wild A-7 Autograph, the author undertook to calibrate the telescope by a photogrammetric method.

Donn L. Ockert

There are two different photogrammetric approaches to the problem of measuring such a geometric figure: one approach is to employ a system that requires exact knowledge about the position and angular attitude of the camera; the other approach is to employ a system that requires only approximate knowledge of the position and angular attitude of the camera. Of course, both approaches require a precision camera, but less field work is required in the second approach since instrument set-up time is reduced.

The second approach is the subject of this paper, and the purpose is to investigate the

* Presented at the Society's 25th Annual Meeting, Hotel Shoreham, Washington, D. C. March 10, 1959. This paper is a part of the panel on Special Applications of Photogrammetry.

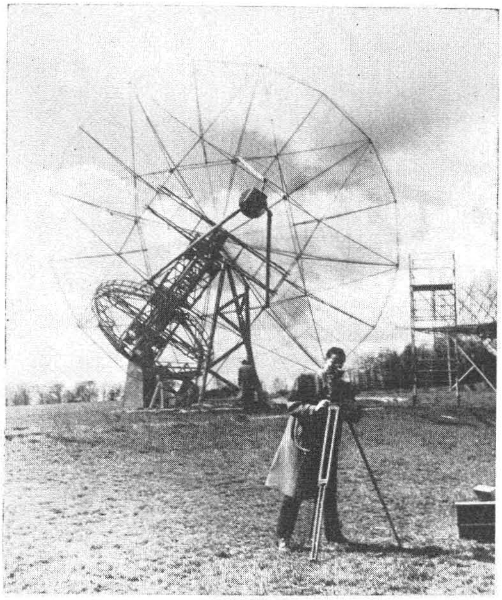

FIG. 1. (Courtesy of Prof. P. Borchers.) The Ohio State University 40-foot radio telescope.

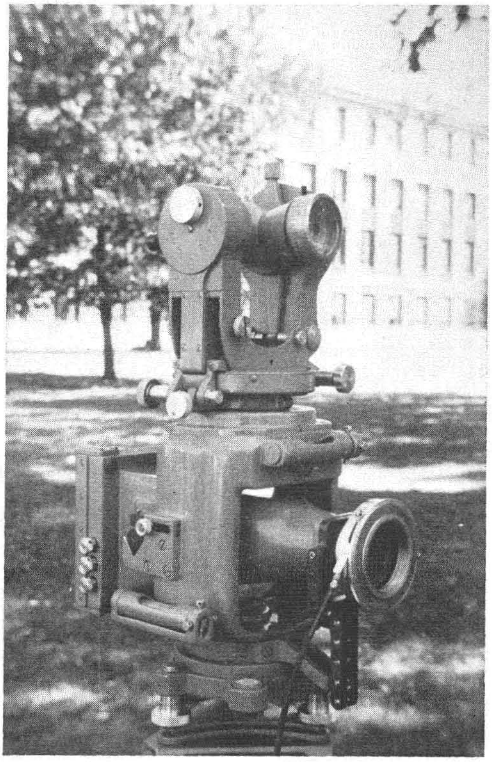

FIG. 2. Wild P-30 Phototheodolite. The shutter was added by the Institute of Geodesy, Photogrammetry, and Cartography.

feasibility of two methods used for the calibration of the Ohio State University 40-foot radio telescope.

The problem is to calibrate the reflector of the telescope, i.e., to determine the deviation between the designed surface and the actual surface in a direction parallel to the axis of rotation, and to determine the focal-length of the actual constructed figure.

FIELD WORK

The most important single item of the field work is, of course, the exposure of the negatives. The negatives of the 40-foot telescope were exposed in the camera of a Wild T-30 phototheodolite. This instrument, shown in Figure 2, consists of a precise camera and a T-2 theodolite mounted together as a single instrument.

The camera has a fixed focus and a fixed aperture ratio of 165 mm. and f/12 respectively. The shutter shown in Figure 2 was added to the camera in order to make it usable for faster emulations. Glass plates mounted in individual plate holders are exposed in the camera, and a special device positions the plates at the focal-plane.

Although the usual tasks where a phototheodolite is employed require the use of both the theodolite and the camera, only the camera was needed and its angular attitude and position were, at best, only approximately known. Proper framing of the antenna required an uptilt of the camera which could not be accomplished with the available vertical settings. Consequently, the whole frame was tilted with the foot screws. This made the level bubbles useless, yet the photographs were not random exposures. The camera was positioned in front of the telescope so that: (1) the photographs would be more or less perpendicular to the base, (2) the distance to the object would be as short as possible to provide full coverage at the largest attainable scale, and (3) the base (distance between exposure stations) would be as large as possible and still achieve stereoscopic coverage of the object.

These criteria differ markedly from those usually used with a terrestrial camera. Dr. Zeller[1] gives a rule of thumb that the base distance should be between $\frac{1}{4}$ and $\frac{1}{20}$ of the object distance. He obtains the lower limit from plotting accuracy considerations, and the upper limit from the practical point that two completely opposite sides of an object may be photographed which makes stereo perception difficult. But if the photographs are made as close as possible to the surface of

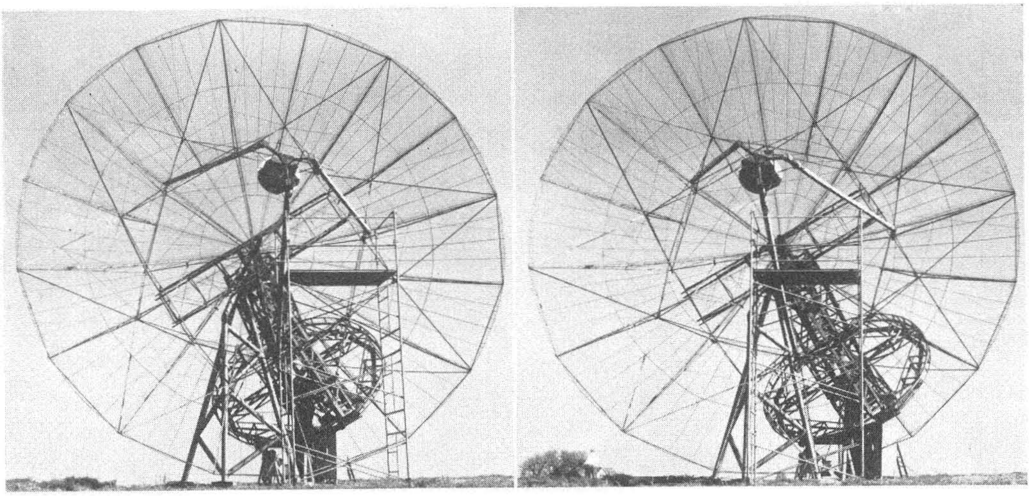

FIG. 3. Stereopair of the radio telescope.

the telescope, and still include the whole surface of the antenna in the field of view, and are perpendicular to the base, then the upper limit will not be critical since only one side of the surface is photographed. The lower limit need not be considered because the base-length will be limited by the field of view of the camera.

From the criteria given here it may be seen that the camera could be hand-held, or if the telescope could not be aimed horizontally as in Figure 3, then the photographs could be made using an aerial camera. However, for this project it was desirable to have the camera mounted because the low-speed emulsion used required a long exposure and because it was convenient to view the image upon a ground glass for camera adjustment.

The field work at the site was completed when a few measurements were made of the accessible parts of the antenna. Some measured distance on the antenna must be known to compute the scale of the stereoscopic model. The distance between two ordinary surveyor's chaining pins placed in the wire mesh was measured. These pins are visible in Figure 3.

Other distances were measured as a check between the center ring and the outer rim at two places to complete the field work required.

THE STEREOSCOPIC MODEL

A first-order stereoscopic plotting machine, a Wild A-7 Autograph, was employed to form and measure the stereoscopic model. Figure 4 shows the Autograph and an automatic coordinate printer (on the left) which records the coordinates of the model space. The coordinates are regular rectangular coordinates of a left-handed coordinate system built into the machine and they are read, or recorded, direct to 0.01 mm. The glass negatives exposed in the field were used directly in the Autograph in order to avoid any loss of detail as a result of further photographic processing.

In order to understand the procedure followed with the Autograph, some of the problems of axis definition should be explored. Different operations are performed with the machine to recover the axes of reference for the antenna. The equation for the designed surface of the antenna,

$$x^2 + y^2 = 72z, \tag{1}$$

is in standard position; i.e., the surface is generated about the z axis and it is tangent to the xy plane. If the coordinates of points on the constructed surface are to be compared to the designed surface, the constructed surface

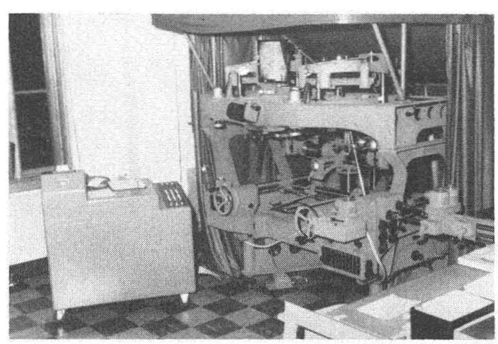

FIG. 4. Wild A-7 Autograph and coordinate printer of the Institute of Geodesy, Photogrammetry, and Cartography.

must also be in standard position. Thus, a reference axis system for the antenna surface must be defined in such a way that it may be recovered either directly or analytically. Two definitions are considered here.

The first definition attempts to reestablish the axis system used during construction to locate the various points on the surface. The xy plane is defined as being tangent to the apparent center of the surface and parallel to the plane of the outer rim with the z axis perpendicular to the plane of the outer rim and passing through its center. This defines exactly the position of the axes for equation (1) in terms of observable features on the antenna proper. An approximation of this axis system, or one parallel to it, can be obtained directly in the Autograph provided the outer rim of the antenna is accurately constructed.

A second reference axis system may be defined in such a way that it is independent of any single feature of the antenna or its original axes. This system is taken as the system that is in standard position with respect to the second-degree surface that best fits, in the sense of least squares, the measured points on the antenna. In this way the position of the axis system is defined by the shape of the whole antenna surface rather than any particular feature such as the outer rim.

Although the first definition may appear adequate to the problem, measurements were made of the model with respect to both axes, to determine whether or not the axes defined by a fitted surface would lead to a more accurate calibration. A further aim was to determine the requirement for an absolute orientation of the model.

With these reference systems in mind, two different models were measured in the Autograph: the first was absolutely oriented, and the second was not. After the original glass negatives were placed in the machine and a relative orientation completed, the resulting model was carefully scaled and "leveled"; i.e., the outer rim was made as parallel as possible to the xy plane of the machine. Small errors in the outer rim make it not quite planar so the operator had to make a visual adjustment to the average plane of the rim. Arbitrary values were assigned to the center to avoid negative coordinates, and the xyz coordinates of some 62 points on the model surface were recorded on the printer.

Then a second model was formed in the Autograph with the same negatives. But no absolute orientation was attempted, and the model, after a careful relative orientation, was left at an unknown scale and angular attitude in the machine. Again coordinates of points on the surface were recorded to complete measurements of the telescope model.

The coordinates from the scaled and leveled model were directly compared to the design equation, and the coordinates from the first and second models were used in the computation procedure outline in the next section.

Computation

The model formed in the Wild Autograph was a true three-dimensional scale model that differed geometrically from the actual antenna only in size and a restricted point of view. Since the size of an object has no bearing on its shape, the shape of the model can be considered as that of the antenna. Further, the shape of the antenna is independent of its position, therefore, the shape of the antenna model is used to establish the second reference axis system defined above.

Although one of the models was positioned and the coordinate origin known, the only data used in this reduction were the coordinates of points on the surface. The points on the surface of the antenna can be thought of as vectors from an unknown origin. The problem is to establish the surface that best fits the vectors, to recover the origin, to put the surface in standard position, and, finally, to establish the actual surface on the axis system of the surface of best fit, also in standard position, for a comparison with the designed surface of Equation (1).

The first problem is to find the equation of a second degree surface that fits the antenna in terms of the machine axis system. Since the origin and angular attitude of the surface are presumed to be unknown, the general quadratic equation,

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Fx + Hy + Iz + 1 = 0, \tag{2}$$

must be used. The Autograph coordinates of points on the surface give the x, y, and z values. Then with any nine points on the surface, the coefficients can be computed in a system of nine equations in nine unknowns. However, many points on the surface were measured and they all should be used to compute the coefficients of Equation (2). The method of least squares provides a solution utilizing the high redundancy of observations on the surface.

In the least squares method, Equation (2) is considered to be an observation equation. The x, y, z coordinates of each point measured should satisfy this equation. Small errors in the surface and in the measurement cause

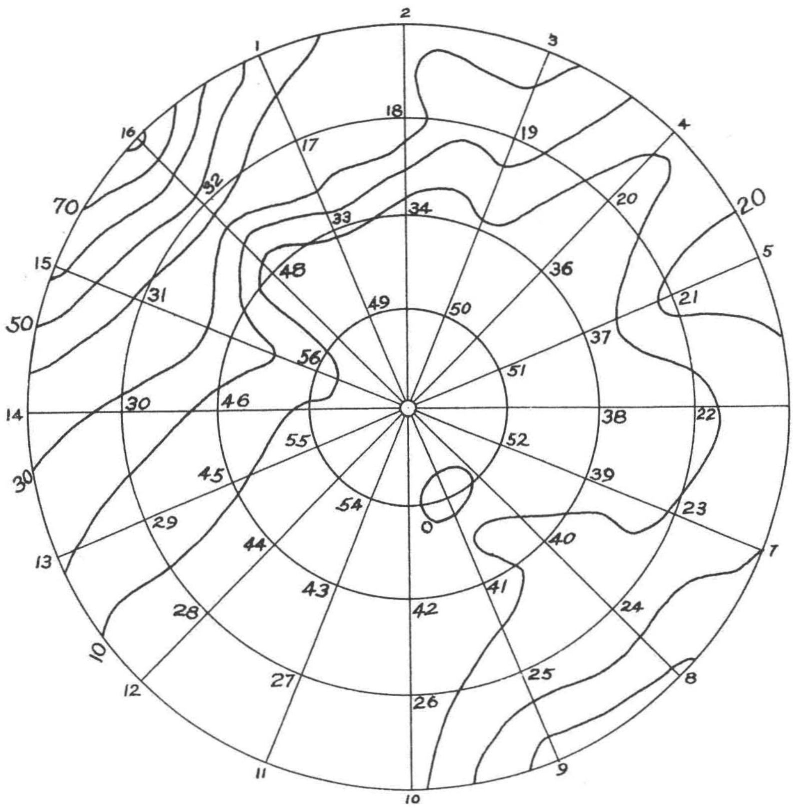

FIG. 5. Error map of the reflecting surface. Isoerror line interval: 10 mm.

small deviations from zero in Equation (2). Let V_i be the residual at point i, and the form of the error equations is

$$V_i = Ax_i^2 + By_i^2 + Cz_i^2 + Dx_iy_i + Ex_iz_i + Fy_iz_i + Gx_i + Hy_i + Iz_i + 1. \tag{3}$$

From the error equations, a system of normal equations, nine linear equations in nine unknowns, is formed such that the sum of the squares of the residuals will be a minimum. The general form of the normal equations is given in Table I.

In the normal equations of Table 1

$$[x^2x^2] = \sum x_i^4, \qquad [x^2(xy)] = \sum x_i^3y_i. \qquad (0 \le i \le 62)$$

The products are purposely not completed within the brackets to show the general formation of the equations. The matrix of coefficients is symmetrical, and the coefficients are written only in the upper half for brevity.

An example computation is given here for the oriented model where the coordinates of 62 points were recorded. The bulk of the time

TABLE I

$$\begin{array}{rrrrrrrrrl}
[x^2x^2] + & [x^2y^2] + & [x^2z^2] + & [x^2(xy)] + & [x^2(xz)] + & [x^2(yz)] + & [x^2x] + & [x^2y] + & [x^2z] + & [x^2] = 0, \\
 & [y^2y^2] + & [y^2z^2] + & [y^2(xy)] + & [y^2(xz)] + & [y^2(yz)] + & [y^2x] + & [y^2y] + & [y^2z] + & [y^2] = 0, \\
 & + & [z^2z^2] + & [z^2(xy)] + & [z^2(xz)] + & [z^2(yz)] + & [z^2x] + & [z^2y] + & [z^2z] + & [z^2] = 0 \\
 & & & ([xy)(xy)] + & [(xy)(xz)] + & [(xy)(yz)] + & [(xy)(x)] + & [(xy)y] + & [(xy)z] + & [xy] = 0, \\
 & & & & [(xz)(xz)] + & [(xz)(yz)] + & [(xz)x] + & [(xz)y] + & [(xz)z] + & [xz] = 0, \quad (4) \\
 & & & & + & [(yz)(yz)] + & [(yz)x] + & [(yz)y] + & [(yz)z] + & [yz] = 0, \\
 & & & & & & [xx] + & [xy] + & [xz] + & [x] = 0, \\
 & & & & & & & [yy] + & [yz] + & [y] = 0, \\
 & & & & & & & & [zz] + & [z] = 0,
\end{array}$$

TABLE II

MODEL COORDINATES FROM AUTOGRAPH

(Values in Millimeters)

Pt.	*X*	*Y*	*Z*	*Pt.*	*X*	*Y*	*Z*
1	186.16	99.87	195.58	32	162.51	135.78	182.68
2	225.45	102.13	195.28	33	193.16	150.38	174.05
3	260.79	119.13	195.37	34	212.55	151.15	173.92
4	286.92	148.65	194.88	35	230.31	159.78	173.93
5	300.35	186.69	195.27	36	243.66	174.47	182.90
6	298.16	226.07	195.28	37	250.06	193.60	182.80
7	280.03	262.12	195.23	38	248.50	213.46	182.77
8	251.86	287.21	195.50	39	239.86	230.85	182.78
9	215.53	300.11	195.53	40	225.51	243.36	174.00
10	175.74	298.56	195.15	41	207.07	249.83	173.80
11	140.29	281.90	195.02	42	188.30	248.96	182.87
12	112.63	251.59	194.98	43	169.74	240.42	182.75
13	99.55	214.39	195.37	44	156.50	225.61	182.85
14	102.00	174.88	195.33	45	150.19	207.04	182.93
15	119.09	139.51	195.81	46	151.41	187.37	182.98
16	148.98	112.97	196.03	47	159.77	169.86	174.20
17	189.92	126.04	182.63	48	174.49	156.47	173.80
18	218.68	127.81	182.53	49	187.14	178.25	168.65
19	244.85	140.41	182.52	50	206.61	175.40	168.80
20	264.46	162.23	182.20	51	222.05	187.26	168.67
21	274.00	190.45	182.42	52	224.39	206.61	168.62
22	272.23	219.60	182.28	53	212.49	221.76	168.30
23	258.88	246.05	182.28	54	193.76	224.63	168.57
24	238.06	264.31	182.32	55	177.61	212.80	168.82
25	211.09	273.94	182.37	56	176.01	193.61	168.72
26	182.06	272.76	182.32	57	198.44	197.59	166.82
27	155.58	260.13	182.23	58	200.88	197.34	166.80
28	135.48	237.68	182.18	59	202.48	198.50	166.78
29	126.04	210.54	182.35	60	202.81	200.91	166.87
30	127.70	181.36	182.55	61	201.49	202.62	166.87
31	140.45	155.20	182.67	62	199.39	202.94	166.88

required for the computation is needed to form the normal equations. The author used an IBM 650 Data Processing Machine for the most of the actual computations.

Table II lists the Autograph coordinates for the 62 points, and Table III gives the normal equations with the Gauss-Doolittle forward solution. Following the format of the United States Coast and Geodetic Survey and Rainsford[2], Table III shows the normal equation in the first row of each horizontal set, the reduced equations in the second row, and the divided equation in the third row. The solution of the normal equations gives the coefficients of Equation (2). This equation of the surface of best fit multiplied by 10^6 is

$$\begin{aligned}&7.135058x^2+7.199703y^2-.366048z^2+.144419xy\\&+.086025xz-.057182yz-2901.83x-2905.015y\\&-2448.98z+1\ 000\ 000=0.\end{aligned} \tag{5}$$

The surface defined by this equation is not in standard position. It must be translated to the origin to remove the linear terms, and rotated to remove the product terms. The translation constants are computed by completing the square. From the general form

$$\begin{aligned}ax^2+bx &= a\left(x^2+\frac{bx}{a}+\frac{b^2}{4a^2}\right)-\frac{b^2}{4a},\\ &= a\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a},\end{aligned}$$

completing the square for x in (5) yields

$$\frac{b}{2a}=-\frac{2901.83}{14.270116}=-203.35,$$

and similarly for y and z, $b/2a=-201.74$ and $+3345.16$, respectively. The correction to the constant term is $b^2/4a$ which when evaluated for x, y, and z makes the constant term, K, equal 45080353. If we define a new axis system such that

$$\begin{aligned}X &= x-203.35,\\ Y &= y-201.74,\\ Z &= z+3345.16,\end{aligned} \tag{6}$$

TABLE III

SOLUTION OF NORMAL EQUATIONS

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
136206.77	111992.10	86269.505	117583.13	106472.02	95328.29	586872.14	527594.00	475905.28	2633.892
136206.77	111992.10	86269.505	117583.13	106472.02	95328.289	586872.14	527594.00	475905.28	2633.892
1.0000000	.92222124	.63337164	.86326935	.78169404	.69987923	4.3086855	3.8734785	3.4939914	.01933745
	136553.20	86314.525	117742.80	95375.519	106621.92	527836.50	587650.57	476146.43	2635.171
	44470.920	15381.906	21063.460	7831.9620	28240.980	45297.760	153851.58	84847.000	469.5286
	1.0000000	.34588684	.47364570	.17611423	.63504375	1.0185928	3.4595997	1.9079210	.01055810
		66765.456	81018.320	73406.759	73407.923	404600.38	404614.86	366758.22	2021.323
		6804.399	−741.074	3261.429	3261.505	17224.310	17236.540	35985.850	190.6868
		1.0000000	−.10891101	.47931184	.47932301	2.5331466	2.5331466	5.2886155	.02802405
			111992.10	95328.289	95375.519	527594.00	527836.50	448033.35	2485.648
			428.860	59.889	60.523	1386.100	1386.880	930.750	10.2669
			1.0000000	.13965697	.14112531	3.2320571	3.2338759	2.1702887	0.2393998
				86268.505	81018.320	475905.28	448033.35	404600.38	2237.355
				90.037	−44.603	723.870	65.460	266.88	2.9340
				1.0000000	−.49538523	8.0396948	.72703444	2.9641148	.03258660
					86314.525	448033.35	476146.43	404614.86	2237.468
					68.026	434.640	766.710	409.09	4.4943
					1.0000000	6.3893217	11.270838	6.0137300	.06606739
						2633892.16	2485647.63	2237354.61	12412.15
						2427.20	2159.60	1543.50	17.0970
						1.0000000	.88974951	.63591793	.00704392
							2635170.73	2237467.89	12412.90
							523.50	176.90	1.9540
							1.0000000	.33791786	.00373257
								2021323.17	11176.64
								4.90	.012
								1.0000000	.00244898
									62.00
									49.259
									1.0000000

the equation of the surface of best fit is

$$7.135058X^2 + 7.199703Y^2 - .366048Z^2 + .14419XY + .086025XZ - .057182YZ = K. \tag{7}$$

The translation constants listed in Equation (6) are given in millimeters at model scale, 1:60. The values assigned to the center of the antenna in the Autograph are (200, 200, 166.70). The differences between the assigned values of the center and the translations, Equation (6), represent the relative positions of the centers of the constructed surface and the surface of best fit. The large z translation is not unreasonable since the surface of best fit is a hyperboloid of two sheets.

Equation (7) expressed in matrix notation is

$$(xyz)\begin{pmatrix} 7.135058 & .0722095 & .0430125 \\ .0722095 & 7.199703 & -.028591 \\ .0430125 & -.028591 & -.366048 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = K \tag{8}$$

The center matrix is the coefficient matrix for the quadratic Equation (7). Let A be the coefficient matrix, then

$$f(\lambda) = |A - \lambda I| = 0 \tag{9}$$

is called by Dresden[3] the discriminating equation. Jacobson[4] and Perlis[5] call this the characteristic equation of A. Writing Equation (9) out we have

$$f(\lambda)\begin{vmatrix} 7.135058-\lambda & .0722095 & .0430125 \\ .0722095 & 7.199703-\lambda & -.028591 \\ .0430125 & -.028591 & -.366048-\lambda \end{vmatrix} = 0 \tag{10}$$

Expansion of (10) leads to a cubic equation in λ. If

$$f(\lambda) = -(\lambda^3 + a\lambda^2 + b\lambda + c) = 0 \tag{11}$$

is the result of such an expansion, the coefficients as given by Dresden[3] are

$$\begin{aligned} a &= f''(0) = -(a_{11} + a_{22} + a_{33}) \\ &= -13.968713, \\ b &= f'(0) = \begin{vmatrix} a_{22} & a_{23} \\ a_{22} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \\ &= 46.115206 \\ &= f(o) = -|A| \\ &= 18.821417. \end{aligned}$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}.$$

Then

$$f(\lambda) = \lambda^3 - 13.968713\lambda^2 + 46.115206\lambda + 18.821417 = 0 \tag{12}$$

and the roots are

$$\lambda_1 = 7.0886242,\ \lambda_2 = 7.2464946,\ \lambda_3 = -.3664058.$$

The roots resemble the diagonal elements of the coefficient matrix because the model was oriented in the Autograph.

These roots of the characteristic equation are used to compute a transformation matrix, P, transposed such that

$$\check{P}\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{13}$$

which gives a new axis system with elements u, v, and w in terms of the present translated system. The surface of best fit should be in standard position in this new axis system. Therefore the matrix equation of the surface of best fit in the new system is

$$(u\ v\ w)\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = K \tag{14}$$

The matrix P is required to transform the translated coordinates into the new $u\ v\ w$ coordinates. The column vectors of P are sometimes called characteristic vectors, but they may be better known as Eigenvectors. Let P_i indicate a column of P, then

$$(A - \lambda_i I)P_i = 0 \tag{15}$$

is used to compute the Eigenvectors. For λ_3 (15) becomes

$$\begin{bmatrix} 7.5014638 & .0722095 & .0430125 \\ .0722095 & 7.5661088 & -.028591 \\ .0430125 & -.028591 & -.000378 \end{bmatrix}\begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = 0 \tag{16}$$

which is a homogeneous system of equations that has a non-trivial solution because $(A - \lambda I)$ is theoretically singular. Practically, however, the singularity of $(A - \lambda I)$ depends upon the value of λ. λ must be determined with sufficient accuracy to determine P.

Since the matrix in (16) is singular, one of the variables is arbitrary. Eliminating p_1 from the first two equations yields

$$7.5654137\ p_2 = .029005\ p_3.$$

Let $p_3 = 1$ and

$$p_1 = .0057708,$$
$$p_2 = .0038339,$$
$$p_3 = 1.$$

which check in the third equation. Dividing each element by $\sqrt{p_1^2 + p_2^2 + p_3^2}$ we normalize the vector, and

$$p_3 = \begin{bmatrix} -.0057707 \\ .0038339 \\ .9999760 \end{bmatrix}.$$

The other vectors are found in the same way to obtain

$$P = \begin{bmatrix} .8392083 & .5437830 & -.0057707 \\ -.5437662 & .8392259 & .0038338 \\ .0069223 & -.0000749 & .9999760 \end{bmatrix}.$$

An evaluation of P to check on a possible reflection of the surface shows that

$$|P| = +1.000000$$

which indicates no reflection.

The vectors translated to the computed center of Equation (6) are rotated by (13). Substituting P into (13),

$$\begin{aligned} u &= .8392083X - .5437662Y + .0069223Z, \\ v &= .5437830X - .8392259Y - .0000749Z, \\ w &= -.0057707X + .0038338Y + .9999760Z, \end{aligned} \tag{17}$$

gives the specific relation between the translated Autograph axis system and the axis system that is in standard position with respect to the surface of best fit, the $u\ v\ w$ system.

The w coordinates are parallel to the direction of focus and they represent the ordinates of each measured point on the constructed surface from the uv plane. Substituting the uv coordinates from (17) into the equation of the designed surface,

$$u^2 + v^2 = 365.76w', \tag{18}$$

yields the designed w value for each point. Equation (18) is given in millimeters at model scale, 1:60. The difference between w from (17) and w from (18) are the deviations between the rotated model of the actual surface and designed surface.

Similarly, the original model coordinates translated to the apparent center point (200, 200, 166.70), were substituted into (18), and, again, the differences of the z ordinate for the model surface and the designed surface were noted. Assuming that the model represented the actual telescope surface, these ordinate differences represent the error at each measured point on the antenna.

Examination of these errors computed both ways indicated that the axes determined solely by the shape of the antenna were not parallel to nor coincident with the original axes employed to lay out and construct the original antenna surface. However, the error computed directly from the oriented model coordinates where the model was visually adjusted by the Autograph operator indicated that the model was very nearly in the correct position. To further check on the misalignment of the fitted axes, the second model that was neither leveled nor scaled was measured at 213 points, and the data were used in a calculation outlined above. In addition, the method outlined above was modified so that the fitted surface would be a paraboloid rather than a general second degree surface.

These two additional computations closely agreed with the first results, and showed that the axes could not be determined with the desired accuracy from the shape alone in this case and that the initial model coordinates were sufficient to calibrate the antenna. The errors determined from the initial coordinates are shown in Table IV. The primary reason why the fitted surfaces failed to adequately define reference axes is that there is not enough of the paraboloid existing. Any one of the other second-degree surfaces would nearly fit the measured coordinates without being exactly centered. It is uneconomical to build much more of a paraboloid for radio purposes, but there are other areas of engineering where second degree surfaces are employed; e.g., plexiglass domes and balloons. The shapes of these surfaces may well serve to establish reference axes for measurement.

TABLE IV

Z ERRORS DIRECT FROM LEVEL MODEL

(Scale 1:1)

Pt.	*mm.*	*ft.*	*Pt.*	*mm.*	*ft.*
1	49	.16	32	44	.14
2	30	.10	33	11	.04
3	33	.11	34	8	.03
4	11	.04	35	10	.03
5	25	.08	36	4	.01
6	15	.05	37	0	.00
7	20	.06	38	0	.00
8	31	.10	39	0	.00
9	38	.12	40	15	.05
10	9	.03	41	2	.01
11	6	.02	42	6	.02
12	0	.00	43	−3	−.01
13	23	.08	44	3	.01
14	31	.10	45	11	.04
15	65	.21	46	16	.05
16	83	.27	47	28	.09
17	34	.11	48	0	.00
18	30	.10	49	4	.01
19	29	.09	50	12	.04
20	6	.02	51	4	.01
21	22	.07	52	2	.01
22	8	.03	53	−15	−.05
23	10	.03	54	−2	−.01
24	13	.04	55	0	.00
25	15	.05	56	12	.04
26	8	.03	57	−2	−.01
27	7	.02	58	−3	−.01
28	5	.02	59	−4	−.01
29	15	.05	60	1	.00
30	29	.09	61	0	.00
31	39	.13	62	1	.00

The focal length of the dish can be obtained from Equation (1) rewritten as

$$p = \frac{x^2 + y^2}{z} \tag{19}$$

so that a value of p, which is 4 times the focal length, is obtained for each point measured. An average value for the focal-length may be obtained for the whole surface, or for particular zones of the surface. The average focal-length determined in this way for the Ohio State 40 foot dish is 17.70 feet. It is interesting to note that the focal-lengths for all of the surfaces fitted by least squares were 17.70 feet $\pm$.02 feet.

In closing this section a few remarks on the accuracy achieved in the calibration are presented. The reading accuracy of the operator was determined from repeated observation of several points. The standard reading error of the operator was ± 0.02 mm. for all three coordinate directions at model scale which corresponds to ± 1.2 mm. on the antenna itself. Investigation into the errors introduced by relative and absolute orientation of the circular model indicated that these errors were larger, and that the standard error of any one point including reading errors was ± 10 mm. on the antenna. This is a large error; large enough that it is known that it can be reduced more than 50% by increasing the accuracy of the relative orientation with a control system in the foreground as done by Hallert,[6] yet small enough to indicate that a photogrammetric calibration is worthwhile and small enough to be of use to the electrical engineers since the construction tolerances were much larger.

CONCLUSION

Photogrammetric calibration of large radio telescope antennas is not only feasible, but very practical as well. The large error experienced in this particular example does not indicate the limitations of such a procedure, because two distinct things could be done to materially reduce this error: (1) a wide-angle camera with a larger format would allow larger scale photography with a corresponding decrease in error, and (2) special control not on the antenna would decrease model errors caused by the relative orientation. It is not unreasonable to expect a standard error at any point of ± 3 mm. (full scale) for a 40 foot dish if the above two changes were adopted.

The old and standard method of directly measuring a stereoscopic model referenced to visually oriented axes proved to be more reliable than measuring the model referenced to axes determined from the shape of the model. Since only a small part of the paraboloid is represented in a radio antenna and since this part of the surface differs very little from other second degree surfaces including the sphere, the shape method of defining reference axes is not practical. However, the method may be of value in determining the orientation of a vector with respect to other surfaces.

References

1. Zeller, Max. *Text Book of Photogrammetry.* Translated by E. A. Miskin and R. Powell, London: H. K. Lewis and Co., Ltd., 1952, p. 46.
2. Rainsford, Hume F. *Survey Adjustments and Least Squares.* London: Constable and Co., Ltd., 1957, p. 54.
3. Dresden, Arnold. *Solid Analytical Geometry and Determinants.* New York: John Wiley and Sons, Inc., 1948, pp. 188–191.
4. Jacobsen, Nathan. *Lectures in Abstract Algebra,* Volume II. New York: D. Van Nostrand Co., Inc., 1953.
5. Perlis, Sam. *Theory of Matrices,* Cambridge, Mass.: Addision Wesley Publishing Co., Inc., 1956.
6. Hallert, Bertil, "*Determination of the Accuracy of Terrestrial Stereophotogrammetric Procedures,*" PHOTOGRAMMETRIC ENGINEERING, 1955, Volume XXI. No. 1.

*Determining Small Deflections in Aerodynamic Models**

JOSEPH P. SHERBY,
President, Stereofoto, Inc.,
3064 Eaton Rapids Road, Lansing, Mich

ABSTRACT: *Through the use of photogrammetry this paper presents an approach to the problem of the determination of small deflections in aerodynamic models. The results of experiments in both the static and dynamic conditions are presented.*

THIS paper presents the results of experiments to determine the feasibility of utilizing photogrammetry for determining small deflections of an aerodynamic model in both static and dynamic conditions. Since the consistent determination of these deflections is at present exceedingly difficult, or impossible, it is hoped that photogrammetry may provide the solution, provided sufficient accuracy can be obtained. These then are the results of the first such steps in that direction.

Photographs for the project were taken with a pair of Santoni photo-theodolites, equipped with a pair of auxiliary lenses of 100 cm. focal-length. The cameras utilized glass plate negatives 10×15 cm. in size. The theodolites were mounted on a bar to obtain a minimum of motion.

The model to be photographed was about 14″ wide by 10″ high. It was constructed of balsa wood, with an aluminum spar through its center, so that in all respects it would react as a full-sized section would respond when loads of different amounts and with various conditions were applied.

For horizontal and vertical control an aluminum sheet, with a one-inch grid scribed upon it, was mounted upon a sheet of three-quarter inch plywood. The outline of the model was traced upon the surface of the plywood, and this portion was cut out so that the surface of the model was flush to the surface of the plywood.

To check the calibration of the plotting instrument, three machined aluminum plugs were mounted upon the surface of the aluminum sheet. The heights of these plugs were .250″, .500″ and .880″.

* Presented at the Society's 25th Annual Meeting, Hotel Shoreham, Washington, D. C. March 10, 1959. This paper is a part of the panel on Special Applications of Photogrammetry.