An Investigation of Errors in Aerial Triangulation

ARTHUR H. FAULDS,
Assistant Professor of Civil Engineering
Syracuse University, Syracuse, N.Y.

ABSTRACT: *This paper is a report of an investigation to determine the manner in which errors in focal-length and image coordinate measurement propagate and affect the results of an aerial triangulation procedure developed by the late Professor Earl F. Church. A method for evaluating aerial triangulation procedures is indicated.*

EXTENSION of photogrammetric surveys through a strip of aerial photographs without any terrestrial surveying for control, beyond that required for determining the resection and orientation data for the first photograph or pair of photographs, has long been the goal of photogrammetrists. Rigorous analytical methods were developed for solving this problem long before the advent of electronic computers but, because computation time with desk calculators proved prohibitive for all but a few specific applications, these techniques were primarily of academic interest only. Of necessity, then, practicing photogrammetrists turned to complex optical-mechanical devices for attaining this goal, and by far the greatest emphasis in photogrammetric research has been directed to developing precision stereoplotters and associated techniques. Through the years these equipments and procedures have been perfected to the point where some reduction in the amount of ground control needed for mapping has been realized, but it appears evident that any further, major improvements can come only from theoretical studies employing analytical techniques.

Now that electronic digital computers are available, complex photogrammetric computations, formerly requiring weeks and months to complete with desk calculators, can be accomplished in minutes. This tremendous reduction in the time factor has created a marked interest in the practical applications of analytical methods to photogrammetric problems, and studies are underway to assess various existing procedures and, indeed, to develop new ones. Equally important, however, is the fact that the electronic computer has opened the door to a tremendous area of photogrammetric research. For the first time it is both possible and practical, with the aid of theoretically correct fictitious photography, to study all types of errors inherent to photogrammetry, singly and in combination, without introducing personal or instrumental bias. Results of such studies will certainly have a pronounced effect upon the photogrammetric technique, regardless of whether the methods ultimately adopted are instrumental, analytical, or a combination of both.

This paper is a report of studies conducted at Syracuse University to determine the manner in which errors in photographic image location and measurement propagate and affect the results of an analytical triangulation procedure developed by the late Professor Earl F. Church, and described by him in *Syracuse University Bulletin No. 15*, "Revised Geometry of the Aerial Photograph." The research program has not been completed but the results to date seem sufficiently interesting and important to justify this report.

Bulletin No. 15 has received such wide distribution since its publication in 1945 that it is undoubtedly available to most photogrammetrists. Consequently, no attempt will be made herein to discuss the aerial triangulation procedure used in this study. It is sufficient to say that if the orientation and resection data for the first photograph in a flight strip are known, together with the ground survey coordinates of two points appearing in the overlap area of the first two photographs, it is possible to compute the position and orientation of every photograph in the strip, and also the position and elevation of any ground point appearing in two or more photographs without any additional ground survey data.

A flight strip of 10 fictitious photographs was prepared for this investigation in order to have data known to be theoretically correct. The focal-length of this "photography" is 150.000 mm.; the nominal flying height is 20,000 ft. above sea level with variations amounting to ±200 feet; the tilts range from 0° 15′ to 4° 30′, and the line of flight is exactly parallel to the ground survey x-axis. Image coordinate values provided by an IBM-650 electronic digital computer were rounded to the nearest micron.

The initial computation of this study was made to determine the mathematical rigor of the aerial triangulation procedure using theoretically correct data throughout, with the photographic coordinates rounded to the nearest micron. The resulting errors in exposure station and ground pass-point positions are shown in Figure 1. Although the rigor of the procedure is clearly indicated, it is evident that small errors resulting from round-off of the image coordinate values to the nearest micron and, perhaps, from 8-place round-off in the computations, accumulate and cause a gradual deterioration of accuracy as the computations progress. In this strip of 10 photographs, the errors are so small that no attempt was made either to eliminate them or to determine the relative effects of micron accuracy and 8-place computational round-off. However, before triangulation of flight strips up to 25 or more photographs in length is attempted, such an investigation appears mandatory.

In preparation for computations concerning determination of the effects of random errors of coordinate measurement upon the aerial triangulation process, a tabulation of errors was prepared for the x- and y-coordinates of all photographic images with the aid of a table of random normal deviates. A standard error of measurement of ±10 microns in each direction was assumed. Errors in the x-direction ranged from −20 to +24 microns, and in the y-direction from −26 to +25 microns.

In the first aerial triangulation computation of the series concerned with coordinate measurement errors, errors were assumed *only* for the two images measured on the first photograph. Otherwise, theoretically correct data were used throughout. Effects of these errors on the positions of the exposure stations and ground positions of pass-points along the bottom and top edges of the flight strip are shown in Figure 2. To maintain the proper perspective when comparing this and subsequent figures to Figure 1, it should be noted that the vertical scale of that figure

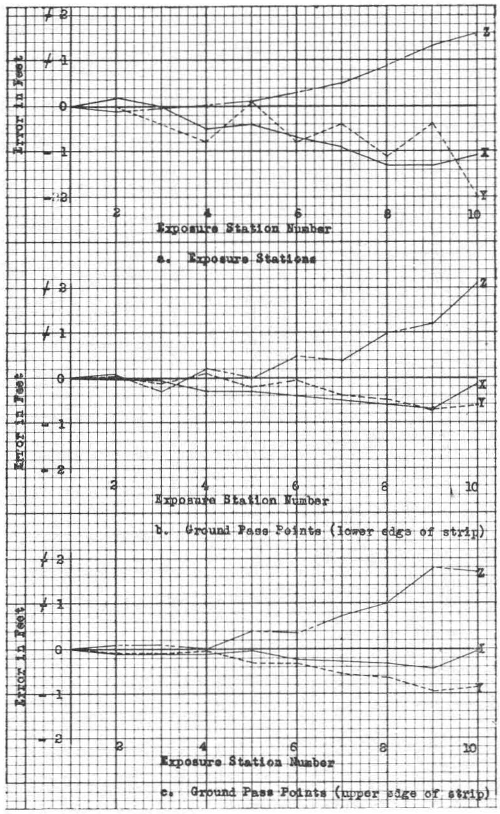

FIG. 1. Errors in exposure station and ground pass-point positions resulting from use of theoretically correct image-coordinates rounded to the nearest micron.

is larger by a factor of 25. In the second triangulation computation of this series, coordinate measurement errors were assumed only for images on the first two photographs. Resulting errors in exposure station and ground pass-point positions are shown in Figure 3. Figures 4 and 5 show the errors produced by altering the image coordinate values on the first three and the first four photographs, respectively, and Figure 6 shows the errors obtained when errors were applied to the image coordinate values of all ten photographs.

As previously mentioned, the aerial triangulation procedure used in this study requires that the position and orientation data for the first photograph be known. In all previous computations theoretically correct data were assumed for this photograph, but normally, of course, these values would be determined from separate orientation and resection computations using a minimum of three ground-control points. Errors in measuring photographic coordinates of the con-

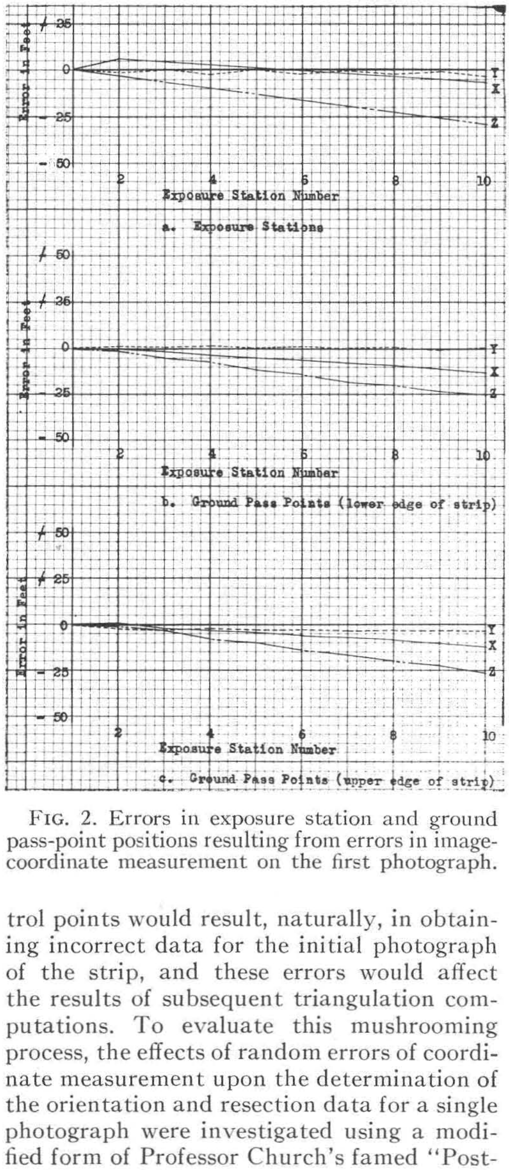

FIG. 2. Errors in exposure station and ground pass-point positions resulting from errors in image-coordinate measurement on the first photograph.

trol points would result, naturally, in obtaining incorrect data for the initial photograph of the strip, and these errors would affect the results of subsequent triangulation computations. To evaluate this mushrooming process, the effects of random errors of coordinate measurement upon the determination of the orientation and resection data for a single photograph were investigated using a modified form of Professor Church's famed "Post-card Method." Four computations were performed for each of the 10 fictitious photographs, using a different group of three control points each time, and assuming a standard error of coordinate measurement of ± 10 microns. In each case, the triangle formed by the control points covered nearly one-half of the picture format. As a check on the photogrammetric method, the computations were also performed using theoretically correct coordinate values rounded to the nearest micron. The 40 computations were repeated a second time using incorrect coordinate values and a focal-length of 150.010 mm. rather than the correct value of 150.000 mm. The results of these three groups of computations are summarized in Table 1.

In the final series of aerial triangulation computations performed to date, the calculations were made using incorrect values for all image coordinates and with the position and orientation of the first photograph altered, first, by amounts equal to the standard errors of Table 1 (b) and, next, with the data for the first photograph altered by amounts equal to the standard errors of Table 1 (c), with the mean error in Z_L added to the flying height of the first photograph, and with the focal-lengths of all photographs changed to 150.010 mm. The resulting errors obtained from these two sets of computations are shown in Figure 7. Curves labeled X, Y, and Z show the error patterns obtained in the first computation while those labeled X_1 and Z_1, show those error curves which were changed as the result of the change in focal-length.

Figures 1 through 7 are largely self-explanatory and require little comment, as careful examination and comparison of the error curves provide much interesting and provocative information about the aerial trian-

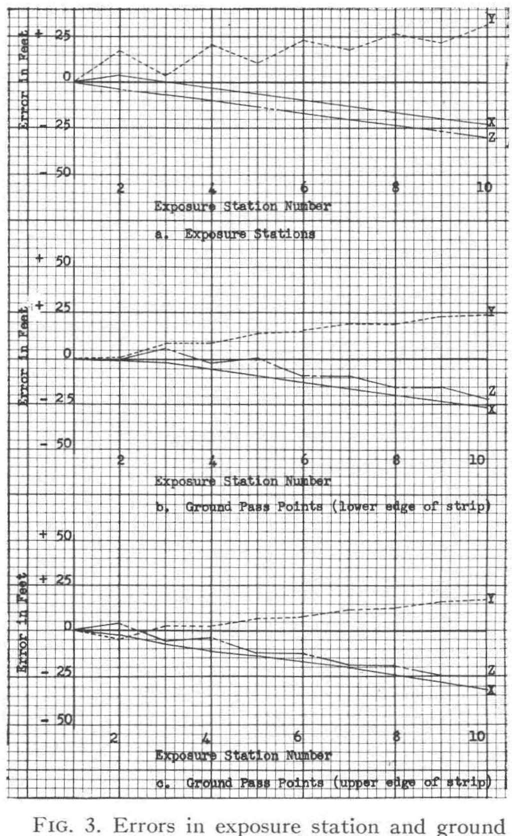

FIG. 3. Errors in exposure station and ground pass-point positions resulting from errors in image-coordinate measurement on the first two photographs.

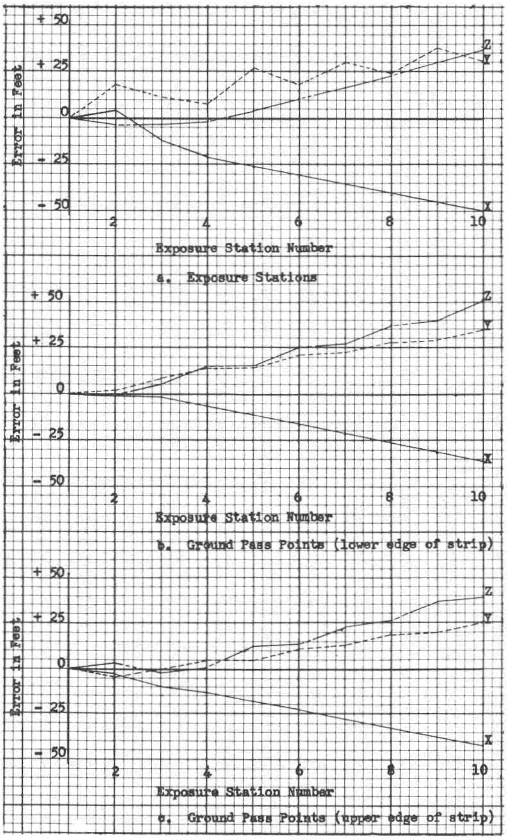

FIG. 4. Errors in exposure station and ground pass-point positions resulting from errors in image-coordinate measurement on the first three photographs.

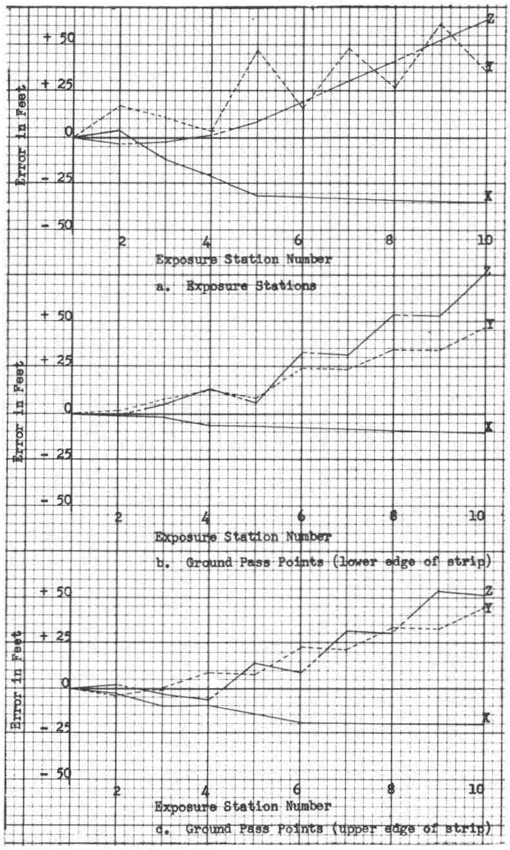

FIG. 5. Errors in exposure station and ground pass-point positions resulting from errors in image-coordinate measurement on the first four photographs.

gulation procedure used in this study. However, a few items seem worthy of special attention:

1. The striking similarity in pattern of corresponding error curves in Figures 1 and 7 suggests expectation that these patterns will prevail regardless of the magnitude of the standard error of coordinate measurement.

2. The orientation and resection data for the first photograph are extremely critical, as indicated by comparison of Figure 6 and 7. Clearly, every effort should be made, includ-

TABLE 1. ERRORS IN DETERMINATION OF PHOTO ORIENTATION AND RESECTION RESULTING FROM ERRORS IN IMAGE COORDINATES AND FOCAL LENGTH

Value	(*a*) *Correct Data Throughout*		(*b*) $f=150.000$ mm. *Image Coords. in Error*		(*c*) $f=150.010$ mm. *Image Coords. in Error*	
	Mean Error	Std. Error	Mean Error	Std. Error	Mean Error	Std. Error
X_L	0 ft.	$+0.1$ ft	$+1.1$ ft.	± 4.9 ft.	$+1.2$ ft.	± 5.0 ft.
Y_L	0 ft.	0.2 ft.	-0.3 ft.	± 3.9 ft.	-0.3 ft.	± 3.9 ft.
Z_L	0 ft.	0 ft.	$+0.3$ ft.	± 1.6 ft.	± 1.5 ft.	± 1.6 ft.
Tilt	0 ft.	$\pm 0° 00' 01''$	0	$\pm 0° 00' 42''$	0	$\pm 0° 00' 42''$
Swing	$+0° 00' 05''$	$\pm 0° 01' 01''$	$-0° 01' 34''$	$\pm 0° 36' 34''$	$-0° 01' 17''$	$\pm 0° 36' 22''$

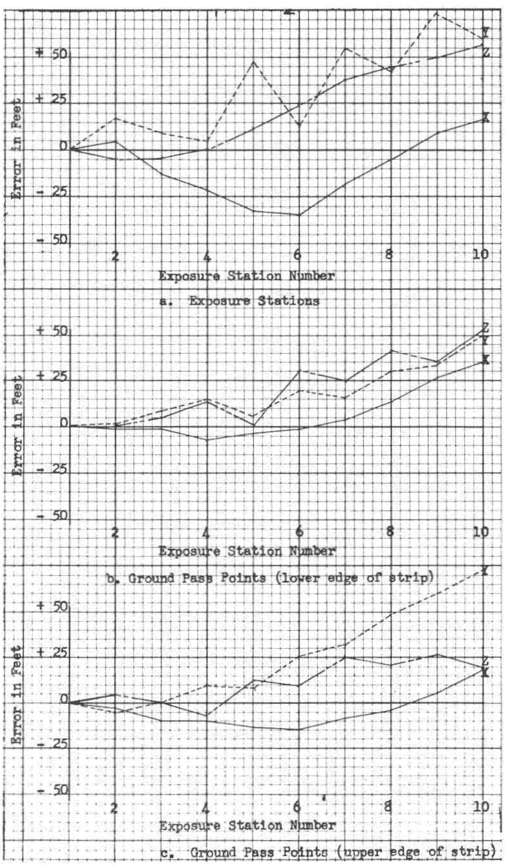

FIG. 6. Errors in exposure station and ground pass-point positions resulting from errors in image-coordinate measurement on all ten photographs.

ing use of redundant measurements, to secure the best possible results in determining these quantities.

3. The value assumed for the focal-length of the aerial photography is of relatively minor importance when the computations are used to bridge between bands of control. As shown by the curves denoted as X_1 and Z_1 in Figure 7, any error in focal-length simply rotates the curves about the starting point and can easily be corrected by a straight line adjustment.

4. In Figure 7 there is little to indicate that random errors in coordinate measurement produce error curves which follow a simple curvilinear form.

5. It seems quite evident that the method for analytical aerial triangulation used in this study is particularly weak in establishing the Y-coordinates of the exposure stations and the Z-coordinates of the ground pass-points. Undoubtedly use of redundant data throughout the computations would reduce the magnitudes of the oscillations of these curves, but also undoubtedly true the oscillations would remain, thus making it very difficult to develop a satisfactory adjustment procedure. Two solutions appear possible: 1) modify the computations to strengthen these weaknesses; or 2) abandon the procedure entirely in favor of another method which does not contain weaknesses. Here, perhaps is a clue to a system for evaluating aerial triangulation procedures which is sounder than a simple analysis of ability to incorporate redundant data.

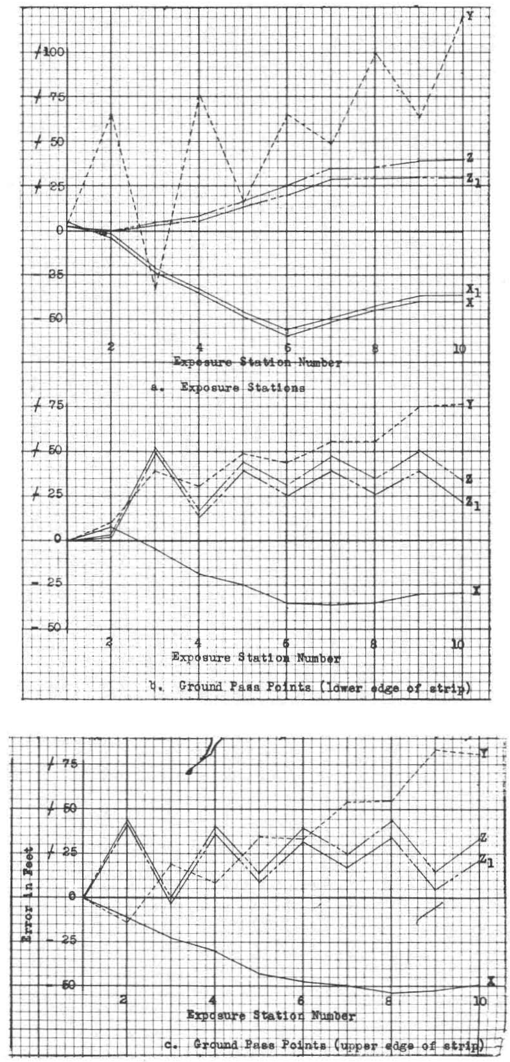

FIG. 7. Errors in exposure station and ground pass-point position resulting from image-coordinate errors on all ten photographs and from orientation and resection errors for the first photograph.