Calculating the Area of a Traverse from Aerial Photographs by Means of the Parallax Wedge, the Parallax Bar, and an Engineer's Scale

ALVARO E. CARDOSO,
Georgia Institute of Technology, Atlanta, Georgia

ABSTRACT: *The parallax wedge is a very economical device for use in photogrammetric work where a high degree of accuracy is not required. The purpose of this paper is to compare the relative accuracies of the parallax wedge and the parallax bar in computing the area of a traverse, the true area of which is known.*

THE parallax wedge is a simple device for measuring parallaxes in a stereoscopic pair of photographs. For the experiment described in this report a parallax wedge was made of a thin sheet of lucite on which two lines were drawn as shown in Figure 1. The lines were 20 cm. long and were drawn so that their midpoints lay on the principal points of the oriented photographs. The distance between the corresponding ends of the pair of lines was 4 mm. less at one end than at the other. Thus the angle of convergence was approximately 2° 30′. The distance between the principal points was measured and found to be 190.8 mm. The midpoint on each line was marked zero on the wedge. For the purpose of measuring difference in parallax, a scale of distance was marked on each line, using the midpoint as the zero index. For convenience, the smallest scale subdivisions marked as dots along the lines, were made equal to 0.1mm.

The four millimeters convergence of the two lines on the parallax wedge was arrived at after finding the parallaxes of the points with the highest and lowest elevations on the photographs. This is actually unnecessary, because the proper distance and angle between the lines could be obtained by using a stereoscope and making the lines fuse at the highest and lowest points of the photographs. These points can be obtained by observation.

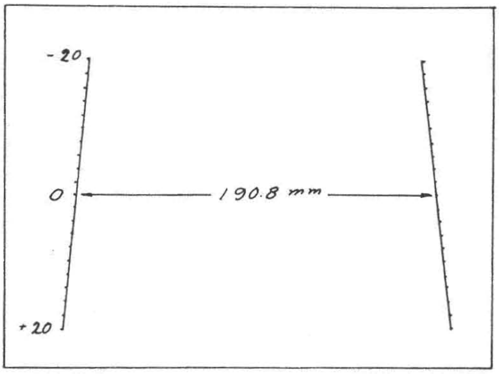

FIG. 1

For this investigation a Wild parallax bar was used.

In this example five points of known coordinates and elevations were selected from a pair of overlapping photographs. The two photographs were oriented under a stereoscope and the parallaxes of the five points were read by means of the parallax wedge and parallax bar. The absolute parallaxes of the five points were also measured by means of a metric engineer's scale. The observed data are shown in Table 1.

CALCULATION OF THE AIR BASE OF THE PHOTOGRAPHS

The air base of the overlapping photographs was found as follows:

Known flight data:

$f = 12$ inches
Elevation of Erskine = 347.0 feet

TABLE 1

Point	*Wedge reading*	*Wedge abs. parallaxes*	*P. bar reading*	*P. bar abs. parallaxes*	*Measured abs. parallaxes*
West	+0.20	80.40	14.58	79.78	79.7
Washington	+0.10	80.24	14.91	80.11	80.1
Backstop	+0.40	80.60	14.36	79.56	80.0
South Base	+0.40	80.60	14.92	80.12	80.1
Erskine	+0.22	80.42	14.79	79.99	80.0

Wedge constant: 110.60
Parallax bar constant: 65.20

Coordinates of:

	X	Y
North Base	10,190.77 ft.	20,206.25 ft.
Erskine	8,806.30 ft.	19,024.19 ft.

The true length of the line North Base–Erskine is found as follows:

$$L = \sqrt{(X_n - X_e)^2 + (Y_n - Y_e)^2}$$

$$L = \sqrt{(10{,}190.77 - 8{,}806.30)^2 + (20{,}206.25 - 19{,}024.19)^2}$$

$$L = 1{,}820.66 \text{ ft.}$$

The length of the line North Base–Erskine is 1,820.66 ft.

The photographic coordinates of North Base and Erskine were measured using the lines joining the fiducial marks as axes.

	Erskine	*North Base*
Left Photograph:		
	$x = +14$ mm.	$x = +90$ mm.
	$y = +20$ mm.	$y = -74$ mm.
Right Photograph:		
	$x = -65.5$ mm.	$x = +8.0$ mm.
	Parallax = 79.5 mm.	Parallax = 82.0 mm.

An air base of 2,000 ft is assumed. Using this value and the measured photographic coordinates, a length of the line *North Base–Erskine* is found by using the following formulae:

$$X = \frac{Bx}{p} \qquad Y = \frac{By}{p}$$

$$X_E = \frac{(2{,}000)(+14.0)}{(79.5)} \qquad X_N = \frac{(2{,}000)(+90.0)}{(82.0)}$$

$$X_E = 352.2 \text{ ft.} \qquad X_N = 2{,}195.1 \text{ ft.}$$

$$Y_E = \frac{(2{,}000)(+20.0)}{(79.5)} \qquad Y_N = \frac{(2{,}000)(-74.0)}{(82.0)}$$

$$Y_E = 503.1 \text{ ft.} \qquad Y_N = -1{,}804.9 \text{ ft.}$$

$$L = \sqrt{(X_E - X_N)^2 + (Y_E - Y_N)^2}$$

$$L = \sqrt{(352.2 - 2{,}195.1)^2 + (503.1 - -1{,}804.9)^2}$$

$$L = 2{,}953.53 \text{ ft.}$$

By direct proportion B is corrected:

$$\frac{1{,}820}{2{,}953} = \frac{\text{Corrected B}}{2{,}000}$$

$$\text{Corrected } B = 1{,}232.4 \text{ ft.}$$

This value is used in calculating the length of the given line in order to check the accuracy of the corrected air base B.

Erskine

$$X = \frac{(1{,}232.4)(+14.0)}{(79.5)}$$

$$X = 217.3 \text{ ft.}$$

$$Y = \frac{(1{,}232.4)(+20.0)}{(79.5)}$$

$$Y = 310.4 \text{ ft.}$$

North Base

$$X = \frac{(1{,}232.4)(+90.0)}{(82.0)}$$

$$X = 1{,}352.6 \text{ ft.}$$

$$Y = \frac{(1{,}232.4)(-74.0)}{(82.0)}$$

$$Y = -\ 1{,}122.1 \text{ ft.}$$

$$L = \sqrt{(X_E - X_N)^2 + (Y_E - Y_N)^2}$$

$$L = \sqrt{(217.3 - 1{,}352.6)^2 + (310.4 - -\ 1{,}112.1)^2}$$

$$L = 1{,}819.9 \text{ ft.}$$

The length of the line *Erskine–North Base* is found to be 1,819.9 ft. This is within a foot of the true length (1,820.66 ft.). The air base of 1,232.4 ft. may be used now.

CALCULATION OF COORDINATES BY MEANS OF THE PARALLAX FORMULAE

The ground coordinates of the five points based on the ground plumb point were calculated as follows:

$$X = \frac{Bx}{p} \qquad Y = \frac{By}{p}$$

See Table 2.

TABLE 2

TABULATION OF COORDINATES

Points	*Wedge Coordinates* X	Y	*Parallax Bar Coordinates* X	Y
West	+272.84	+656.02	+274.95	+661.12
Washington	+737.22	+688.06	+738.42	+689.19
Backstop	+654.39	− 53.51	+662.87	− 54.21
South Base	+ 46.63	−533.75	+ 46.91	−536.98
Erskine	+214.23	+306.94	+215.38	+308.60

Points	*Measured Abs. Coordinates* X	Y	*True Survey Coordinates* X	Y
West	+275.22	+661.77	+8,462.9	+19,072.7
Washington	+738.52	+689.29	+8,425.8	+19,521.9
Backstop	+659.33	− 53.91	+9,151.5	+19,467.5
South Base	+ 46.91	−536.91	+9,640.7	+18,869.9
Erskine	+215.25	+308.56	+8,806.3	+19,024.2

CALCULATION OF AREAS

The area of the traverse was calculated using the three sets of coordinates found above as well as the true coordinates. The areas calculated by the "coordinate method" were:

West	1
Washington	2
Backstop	3
South Base	4
Erskine	5

$$\frac{X_1}{Y_1} \quad \frac{X_2}{Y_2} \quad \frac{X_3}{Y_3} \quad \frac{X_4}{Y_4} \quad \frac{X_5}{Y_5} \quad \frac{X_1}{Y_1}$$

Parallax Wedge

(+272.84)(+688.06) =	+187,730.29	(+656.02)(+737.22) =	+483,631.06
(+737.22)(− 53.51) =	− 39,448.64	(+688.06)(+654.39) =	+450,259.58
(+654.39)(−533.75) =	−349,280.66	(− 53.51)(+ 46.63) =	− 2,485.70
(+ 46.63)(+306.94) =	+ 14,312.61	(−533.75)(+214.23) =	−114,345.26
(+214.23)(+656.02) =	+140,539.16	(+306.94)(+272.84) =	+ 83,745.50
	+342,582.06		+1,017,636.14
	−383,729.30		− 116,840.97
	− 46,147.24		+ 900,795.17

$$\frac{(-46{,}147.24) - (+900{,}795.17)}{2} = 473{,}471.2 \text{ sq. ft.}$$

$$\frac{473{,}471.2}{43{,}560} = \underline{10.8694 \text{ acres}}$$

Parallax Bar

(+274.95)(+689.19) =	+189,492.79	(+661.12)(+738.42) =	+488,184.23
(+738.42)(− 54.21) =	− 40,029.74	(+689.85)(+622.87) =	+457,280.86
(+662.87)(−536.98) =	−355,947.93	(− 54.21)(+ 46.91) =	− 2,542.99
(+ 46.91)(+308.60) =	+ 14.420.56	(−536.98)(+215.38) =	−115,654.75
(+215.38)(+661.12) =	+142,392.02	(+308.60)(+274.95) =	+ 84,849.57
	+346,361.23		+1,030,314.66
	−395,977.67		− 118,197.74
	− 49,616.44		+ 912,116.92

$$\frac{(-49{,}616.44) - (+912{,}116.92)}{2} = 480{,}866.68 \text{ sq. ft.}$$

$$\frac{480{,}866.68}{43{,}560} = \underline{11.0391 \text{ acres}}$$

Measured Absolute Parallax

(+275.22)(+689.29) =	+189,706.39	(+661.77)(+738.52) =	+488,73 .38
(+738.52)(− 53.91) =	− 39,813.61	(+689.29)(+659.33) =	+454,469.57
(+659.33)(−536.91) =	−354,000.87	(− 53.91)(+ 46.91) =	− 2,528.91
(+ 46.91)(+308.56) =	+ 14,474.54	(−536.91)(+215.35) =	−115,623.56
(+215.35)(+661.77) =	+142,512.16	(+308.56)(+275.22) =	+ 84,921.88
	+345,693.09		+1,028.121.83
	−393,814.48		− 118,152.47
	− 48,121.39		+ 909,969.36

$$\frac{(-48{,}121.39) - (+909{,}969.36)}{2} = 479{,}045.37 \text{ sq. ft.}$$

$$\frac{479{,}045.37}{43{,}560} = \underline{10.9973 \text{ acres}}$$

True Survey Coordinates

(+8,462.9)(+19,521.9) =	+165,211,887.5
(+8,425.8)(+19,467.5) =	+164,029,261.5
(+9,151.5)(+18,879.9) =	+172,779,404.8
(+9,640.7)(+19,024.2) =	+183,406,604.9
(+8,806.3)(+19,072.7) =	+167,959,918.0
	+853,387,076.7
(+19,072.7)(+8,425.8) =	+160,702,755.7
(+19,521.9)(+9,151.5) =	+178,654,667.8
(+19,467.5)(+9,640.7) =	+187,680,327.3
(+18,879.9)(+8,806.3) =	+166,262,063.4
(+19,024.2)(+8,462.9 =	+160,999,902.2
	+854,299,716.4

$$\frac{(854{,}299{,}716.4) - (+853{,}387{,}076.7)}{2} = 456{,}319.8 \text{ sq. ft.}$$

$$\frac{456{,}319.8}{43{,}560} = \underline{10.4756 \text{ acres}}$$

The percentage of error between the true area and the method used are as follows:

Parallax Wedge	3.3%
Parallax Bar	5.0%
Measured Abs. Parallax	4.5%

The main source of error in the measured absolute parallaxes was in the reading of the engineer's scale. The last digit of these readings had to be estimated.

Conclusions

From these results it may be concluded that for problems in which great accuracy is not required, the use of either the parallax wedge or the parallax bar would be satisfactory. With experience in the use of the wedge, results obtained are as good if not better than those obtained with the parallax bar.

Aside from economy, one of the advantages of the parallax wedge over the parallax bar is that it is easier to operate because it involves the fusing of intersecting lines, whereas the parallax bar involves the fusing of dots.

References

Class Notes by Professor J. O. Eichler, Georgia Tech.
"Surveying," by Rayner and Schmidt.

*Determination of Space Coordinates of Photographic Exposures by Semi-Graphic Method**

NEDRET BERKAY,
Cornell Univ., Ithaca, N. Y.

Introduction

A common problem which occurs in photogrammetry is the desire or need to know the position in space of a camera lens, at the instant of exposure of a photograph. For many photogrammetric operations it is most convenient to know the position of the exposure station. Problems in photogrammetry which are solved by the analytical method depend almost entirely upon starting with assumed space coordinates of the exposure station, and then computing better positions by the Newton method of successive approximations.[1,2,3] This paper presents a method, thought to be original, whereby the space resection coordinates of a camera station are easily determined using a convenient combination of analytical and graphic principles. The method is universal, that is, it is not limited to near-vertical photography.

For offices which do not have an electronic computer readily available, but where the need arises to determine the tilt, swing, and azimuth and/or the X, Y, and Z coordinates of the exposure station of a photograph, a simple, rapid, and accurate method of obtaining at least the first estimates is necessary. As convergent and other oblique photography becomes more common, and for photography over areas of high relief, it becomes more necessary to have a method of determining an answer to the degree of accuracy commonly associated with graphical solutions, say to three significant figures.

If the assumption of the space coordinates of an exposure station is close to the true coordinates, an analytical solution can easily be made, using a desk calculator with one or two readjustments of the assumed coordinates. But, in cases of high tilt and high relief differences, the first assumption usually is not close to the true coordinates of the exposure station, and calculations then become longer because of many readjustments of the first assumption to converge on the truth.

In the Anderson method,[1] if tilt is high and if the relief differences between the ground points, whose images are the terminals of the scale check lines, are great, the result obtained for tilt usually will be greatly in error. The convergence toward the correct tilt, swing and tilt axis scale will be very slow or the solution even may fail to converge.

* This paper was awarded First Place in the 1958 competition for the Bausch & Lomb Photogrammetric Award for Students. A photo of the author is in the YEARBOOK issue.