Conclusions

From these results it may be concluded that for problems in which great accuracy is not required, the use of either the parallax wedge or the parallax bar would be satisfactory. With experience in the use of the wedge, results obtained are as good if not better than those obtained with the parallax bar.

Aside from economy, one of the advantages of the parallax wedge over the parallax bar is that it is easier to operate because it involves the fusing of intersecting lines, whereas the parallax bar involves the fusing of dots.

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*Determination of Space Coordinates of Photographic Exposures by Semi-Graphic Method**

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Introduction

A COMMON problem which occurs in photogrammetry is the desire or need to know the position in space of a camera lens, at the instant of exposure of a photograph. For many photogrammetric operations it is most convenient to know the position of the exposure station. Problems in photogrammetry which are solved by the analytical method depend almost entirely upon starting with assumed space coordinates of the exposure station, and then computing better positions by the Newton method of successive approximations.[1,2,3] This paper presents a method, thought to be original, whereby the space resection coordinates of a camera station are easily determined using a convenient combination of analytical and graphic principles. The method is universal, that is, it is not limited to near-vertical photography.

For offices which do not have an electronic computer readily available, but where the need arises to determine the tilt, swing, and azimuth and/or the X, Y, and Z coordinates of the exposure station of a photograph, a simple, rapid, and accurate method of obtaining at least the first estimates is necessary. As convergent and other oblique photography becomes more common, and for photography over areas of high relief, it becomes more necessary to have a method of determining an answer to the degree of accuracy commonly associated with graphical solutions, say to three significant figures.

If the assumption of the space coordinates of an exposure station is close to the true coordinates, an analytical solution can easily be made, using a desk calculator with one or two readjustments of the assumed coordinates. But, in cases of high tilt and high relief differences, the first assumption usually is not close to the true coordinates of the exposure station, and calculations then become longer because of many readjustments of the first assumption to converge on the truth.

In the Anderson method,[1] if tilt is high and if the relief differences between the ground points, whose images are the terminals of the scale check lines, are great, the result obtained for tilt usually will be greatly in error. The convergence toward the correct tilt, swing and tilt axis scale will be very slow or the solution even may fail to converge.

* This paper was awarded First Place in the 1958 competition for the Bausch & Lomb Photogrammetric Award for Students. A photo of the author is in the YEARBOOK issue.

In the Herget method,[2] the approximate geocentric coordinates of the exposure station are necessary; in cases of high tilt the solution converges very slowly.

In the Church method, approximate space survey-coordinates of the exposure station are estimated. If the corrections to the assumed coordinates of the exposure station are small, a linear equation with first-order corrections to the space survey-coordinates can be established; otherwise many iterations are required.

In case of high tilt, visual assumption of the space coordinates, might differ greatly from true coordinates of the exposure station. This will cause many successive iterations and will require a long time.

In this paper an approximate determination of the space coordinates of the exposure station by a semi-graphical method is presented in order to shorten the desk calculation time for the determination of the coordinates and of subsequent tilt, swing and azimuth.

Description of Method

The method described in this paper determines the space resection of an exposure station for any case where three complete (horizontal and vertical) ground-control points are imaged on a photograph. For purposes of description, assume that the method presented herein is to be used to determine the approximate coordinates of the exposure station for the first estimate in the Church method for space resection, and space orientation for cases of high tilt and large ground relief. The method involves a graphical solution of the ground pyramid and an analytical solution of the coordinates of the exposure station.

The Church method of determining the space resection of a single photograph requires that the following information shall be known: the space survey-coordinates (X, Y, and Z) of the ground-control points (A, B, and C) whose images (a, b, and c) appear on the photograph; the photographic coordinates (x and y) for each of the three points; the focal-length of the lens (f). The three points should form as strong a triangle pattern on the photograph as possible.

Obviously, when the camera lens occupies its proper resection position, face angles at the lens for the photo pyramid will be identical with face angles at the lens for the ground pyramid. These face angles are independent of the tilt of the photograph and of the coordinate system in which the images are measured.

In the photo pyramid the x and y photo-coordinates and the focal-length of the lens are known. From these, as in the Church method[3], the magnitudes of the face angles I, II, and III (Figure 1), common to both pyramids, are calculated. These angles form the rays of the opened ground pyramid (Figure 2).

In the ground pyramid (Figure 2) the lengths of CA, AB, and BC, which are the distances between three ground-control points, are known (calculated from the coordinate system of the three points). With these six elements known the problem is to draw polygon $OCABC$ Figure 2. The vertex of polygon O will lie at the intersection of the circles which can be drawn with Angles I, II and III.

This makes use of two relationships from plane geometry: (1) that the locus of all vertices of a given angle subtended by a line lies on a great circle through the ends of the line (this is the condition avoided in plane three-point resection); (2) that from a vertex on a circle the angle subtended by a chord is one-half the central angle subtended by the same chord.

Thus in Figure 3 (a) the radius of the arc required with line CA is determined by the magnitude of the face angle I. That is, the two radii to the ends of the line cut off two

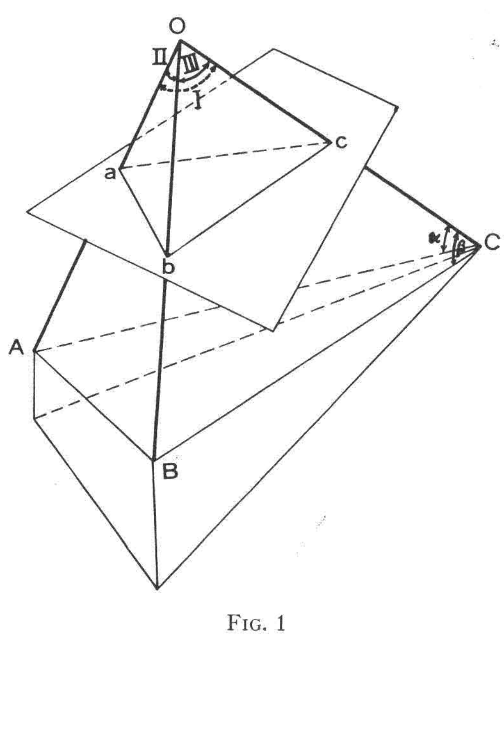

FIG. 1

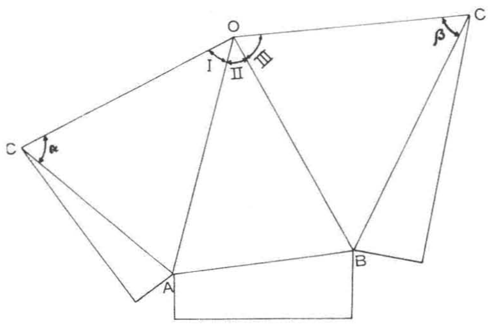

FIG. 2

times the face angle.

Now draw the three lines CA, AB, and BC on separate sheets of tracing paper with the circles for the corresponding face angles. Next place these papers on each other in such a way that line CA adjoins with line AB and AB adjoins with line BC, and place pins to join ends of the lines A, and of B. Now, we can turn the two tracing papers (right one and left one) around B and A respectively. But, the edge OC, of the ground pyramid is the same line in both sheets, so try intersecting the three circles at a point such that the distances between O and C on both papers are equal. Four or five trials are usually more than sufficient to make the two distances equal.

When above mentioned distances are equal, the ground pyramid will have been constructed. The distance OC is scaled and angles α and β are measured with a protractor. We can then proceed to find coordinates of the exposure station.

PROCEDURE

1. Determine the face angles (I, II, and III) from images of ground-control points, (a, b, c). If the Church method is used, the face angles are already computed in the usual manner using analytical geometry.[1]
2. Construct the ground pyramid by graphical method.

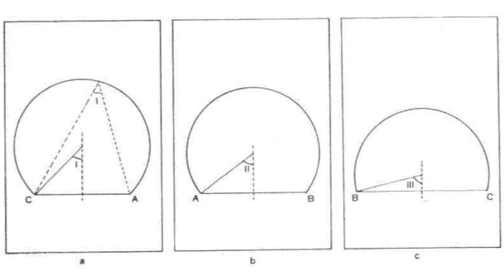

FIG. 3

   a) Calculate the lengths of AB, BC, and CA from known X, Y, and Z coordinates of ground-control points.
   b) Draw each calculated length on a separate tracing paper.
   c) Erect a perpendicular bisector on each line. Lay off on each respective bisector a length such that from the point located an angle equal to the face angle will be subtended by half the length of the ground line. For example, in Figure 3 (a) on the perpendicular bisector of line AC lay off a distance from line AC equal to $AC/2$ tan. I.
   d) Using the points on the perpendicular bisectors as centers, draw the corresponding arcs connecting the ends of the ground lines.
   e) Place the tracing papers on each other in such a way that the ends of the lines with the same letters will be on each other. That is, B of Figure 3b and B of Figure 3c; A of Figure 3b and A of Figure 3a, will be on each other. Place pins at points A and B.
   f) Rotate the circles about A and B respectively so as to obtain an intersection of the three circles at a point. Measure the two distances OC on each tracing paper. Revise point of intersection of the three circles until the two measured distances become equal. That is, move the intersection point to right or left according to which distance is larger or smaller. (Figure 4). The developed shape of the ground pyramid will have been constructed when the OC distances on both tracing papers are equal to each other.
   g) Measure the distance OC and the angles α and β
3. Calculate the space coordinates of exposure station.
   a) Choose one of the ground-control points as origin of an auxilliary coordinate system. (This point will be C in our example because we measured distance OC and the angles α and β).
   b) Calculate direction cosines of lines CA and CB and find space coordinates of O (exposure station) as follows: Let cosine directions of OC be L, M, and N. Then

$$L^2 + M^2 + N^2 = 1 \qquad (1)$$

Let cosine directions of CB be

$$L_1, M_1, \text{ and } N_1.$$

Let cosine directions of CA be

$$L_2, M_2, \text{ and } N_2.$$

TABLE 1

SAMPLE PROBLEM*

Computation Form for Space Resection

Ground Control Points	*X (ft.)*	*Y (ft.)*	*Z (ft.)*
A	54,204	40,103	2,734
B	31,378	30,476	107
C	57,934	20,972	612
Image Coordinates	*x (mm.)*	*y (mm.)*	*z (mm.)*
a	−101.53	− 22.69	−152.40
b	+ 75.91	−105.47	−152.40
c	+ 10.74	+ 98.28	−152.40

By calculation from given coordinates

Face Angles:	$\cos AOB=0.48468$	$\cos BOC=0.37587$	$\cos COA=0.59384$
	$AOB=61°00'$	$BOC=67°55'$	$COA=53°34'$
Distances:	$AB=24,912$	$BC=28,210$	$CA=19,606$

By graphic solution

Distance	$OC=22,900$	$\alpha=56^\circ.5$	$\beta=63^\circ.0$

Direction cosines:

CB	$L_1=0.94137$	$M_1=0.33690$	$N_1=-0.01790$
CA	$L_2=0.19025$	$M_2=0.97577$	$N_2=0.10823$

$\cos 63^\circ.0=W=0.45399$

$\cos 56^\circ.5=Q=0.55194$

$$S=\frac{L_2N_1-L_1N_2}{L_2M_1-L_1M_2}=\frac{-0.10515}{-0.85446}=0.12306$$

$$R=\frac{L_2W-L_1Q}{L_2M_1-L_1M_2}=\frac{-0.43321}{-0.85446}=0.50699$$

$$T=\frac{W-RM_1}{L_1}=\frac{0.28319}{0.94137}=0.30083$$

$$U=\frac{SM_1-N_1}{L_1}=\frac{0.05159}{0.94137}=0.05480$$

U^2.........	0.00300	$2TU$.......	0.03296	T^2.........	0.09050
S^2.........	0.01514	$2RS$.......	0.12477	R^2.........	0.25704
	1				−1
		Diff........	−0.09181		
Sum.......	1.01814			Sum.......	−0.65246

$$N=\frac{+0.09181\pm\sqrt{(0.09181)^2+4(1.01814)(0.65246)}}{2(1.01814}$$

$N=0.84687$

$M=R-SN=0.40278$

$L=T+UN=0.34724$

$(OC)_x=OC\cdot L=22,900\cdot 0.34724=7951$

$(OC)_y=OC\cdot M=22,900\cdot 0.40278=9223$

$(OC)_z=OC\cdot N=22,900\cdot 0.84687=19,393$

Survey Coordinates of Exposure Station	*True Values*
$X=57,934-\ 7,951=49,983$	50,000
$Y=20,972+\ 9,223=30,195$	30,000
$Z=\ \ 612+19,393=20,005$	20,000

* Problem taken from MANUAL OF PHOTOGRAMMETRY, by American Society of Photogrammetry, 1952 Edition, p. 376.

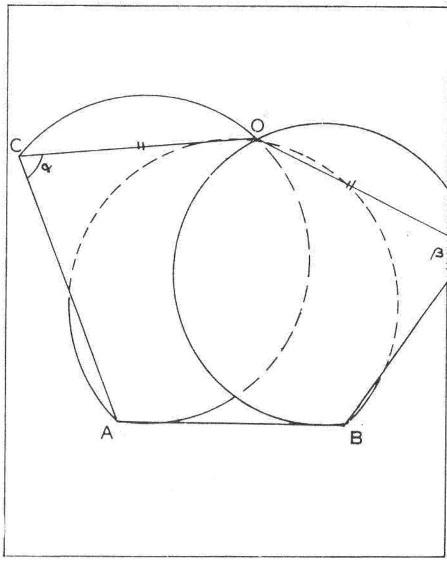

FIG. 4

$$LL_1 + MM_1 + NN_1 = \cos B \qquad (2)$$

$$LL_2 + MM_2 + NN_2 = \cos a \qquad (3)$$

Solving these three equations determine L, M, and N. Multiplying both sides of equation (2) by L_2 and letting $\cos\beta$ be W.

$$L_2(LL_1 + MM_1 + NN_1) = \cos\beta L_2 = WL_2$$

Multiplying both sides of equation (3) by L_1 and letting $\cos\alpha$ be Q.

$$L_1(LL_2 + MM_2 + NN_2) = \cos\alpha\, L_1 = QL_1$$

Subtracting and gathering terms

$$(L_2M_1 - L_1M_2)M + (L_2N_1 - L_1N_2)N = L_2W - L_1Q$$

or

$$M = \frac{L_2W - L_1Q}{L_2M_1 - L_1M_2} - \frac{L_2N_1 - L_1N_2}{L_2M_1 - L_1M_2} N.$$

or

$$M = R - SN. \qquad (a)$$

Similarly,

$$L = \frac{W}{L_1} - \frac{M_1}{L_1} M - \frac{N_1}{L_1} N$$

$$= \frac{W}{L_1} - \frac{M_1(R - SN)}{L_1} - \frac{N_1}{L_1} N$$

$$L = \frac{W - RM_1}{L_1} + \frac{SM_1 - N_1}{L_1} N$$

$$L = T + UN \qquad (b)$$

Substituting (a) and (b) into equation (1)

$$(T + UN)^2 + (R - SN)^2 + N^2 = 1$$

$$(U^2 + S^2 + 1)N^2 + 2(TU - RS)N + T^2 + R^2 - 1 = 0 \qquad (c)$$

Solving this second order equation (of the form $aN^2+bN+c=0$), determine N, and substituting N into equations (a) and (b) determine M and L.

After finding direction cosines of line OC, obtain the coordinates of O with respect to the chosen auxilliary coordinate-system as follows:

$$(OC)_x = OC\cdot L$$
$$(OC)_y = OC\cdot M$$
$$(OC)_z = OC\cdot N$$

These coordinates can easily be translated to any desired survey coordinate-system.

CONCLUSION

For the sample problem the graphic scale of the construction is 1 in. = 2,500 ft. If the accuracy of measuring angles is within a half degree, the results are within 200 ft. If the angles are measured with a three-armed protractor or are calculated by the sine theorem to within 0°05′ the results would be within 45 ft.

Another problem was solved with this method and the following results were obtained:

X: 38,755	True X: 38,867
Y: 220,800	Y: 220,888
Z: 40,110	Z: 40,019

This method may help the assumption of coordinates of the exposure station in case of high tilt and greater relief differences.

The time required for solving the problem was about one hour. If calculation forms were available the time could be reduced. With experience it is probable that these space resection problems could be solved in a half hour.

The method thus provides results rapidly and without the cumulative drafting errors of many graphical methods. The time spent on a solution by this method will more than pay for itself in the time saved in iterations of an analytical solution using desk calculators.

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3. "Revised Geometry of the Aerial Photograph," Bulletin of Aerial Photogrammetry No. 15, by Professor Earl Church, Syracuse University, Syracuse, N. Y.