*Proposed Forest Survey Applying Aerial Photographs in Regression Technique**

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ABSTRACT: *A refined method is needed for determining volumes from aerial photographs in forestry work. Formulas are developed for volume, sampling error of volume, and for the number of sample plots necessary for required efficiency of determination.*

FREQUENTLY aerial photo volume-tables are not available or their reliability is questioned when applied to an unfamiliar forest area. If volume strata are not essential prior to doing ground work, then the procedures to be described will eliminate the need for a photo volume-table before ground data are collected.

PLAN IN BRIEF

Basically the plan is a triple sample design. First a large number of photo points are classified as forest and non-forest. These are cheap and hence many are obtained.

Second, a sample of the forest points is interpreted on $\frac{1}{5}$ acre photo-plots for one independent variable, crown closure[1]. This measurement on each plot determines the average crown closure. In addition to this measurement the broad forest type may be determined in order to obtain area statistics of this forest characteristic.

Third, a sample of the photo plots are selected for ground plots. The data collected from these ground plots are the same as those required for Forest Survey purposes. These volumes from ground data and the crown cover from the same photo plot provide the basis for a regression equation for estimating the average net volume per $\frac{1}{5}$ acre of forest land. This average net volume per $\frac{1}{5}$ acre multiplied by the proportion of forest area is an estimate of the mean volume per $\frac{1}{5}$ acre of gross land area. The sampling error of this estimate may be computed. This mean volume times the gross land area in $\frac{1}{5}$ acres is an estimate of total volume.

This plan makes direct photo volume-estimates instead of classifying the land area into volume classes, and fits well into the continuous inventory techniques now under study.

FORMULAS

Formulas were developed for volume, for sampling error of volume, and for number of sample plots. To simplify the exposition the linear equation assumes constant variance. It is possible that the use of logarithms would improve volume estimates from regression and increase the reliability of estimates of sampling errors.

VOLUME FORMULA

$\overline{V}_t = P_f(a + b\overline{X})$ where:

- $\overline{V}_t$ = mean volume per $\frac{1}{5}$ acre of gross land area
- P_f = proportion of forest land to gross land area interpreted from photo points
- a = constant, computed by method of least squares
- b = regression coefficient, also computed by method of least squares
- $\overline{X}$ = average crown cover in forest area measured on photo plots

The statistician will immediately recognize the regression $(a + b\overline{X})$ which is the estimate of mean volume per $\frac{1}{5}$ acre of forest land, V_f. This estimate times the proportion of forest land gives the mean volume per $\frac{1}{5}$ acre for gross land area, i.e., $\overline{V}_t = (P_f)(\overline{V}_f)$. Gross land area is assumed to be free of error. The proportion of forest land (P_f) is determined from sample photo points. Parameters a and b are the constant and regression coefficient

[1] More independent variables will be desirable if they furnish significant improvement in volume estimates. In most timber stands, height is an important variable in this connection.

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computed by the method of least squares using photo plot data and ground plot data. Crown cover in the forest area is measured on all photo plots.

ERROR FORMULA

$$S_{vt}^2 = \frac{P_f^2 S_{yx}^2 + P_f^2 d^2 \left(\frac{S_{yx}^2}{S_x^{2'}}\right)}{N_g} + \frac{P_f^2 b^2 S_x^{2''}}{N_p} + (a + b\bar{X}) P_f Q$$

which may be coded as follows:

$S_{vt}^2 = A/N_g + B/N_p + C/N$

S_{vt}^2 = variance of mean volume per $\frac{1}{5}$ acre

S_{yx}^2 = variance independent of regression

Q = proportion of non-forest area to gross land area interpreted from photo points.

N_g = number of ground plots

N_p = number of photo plots

N = number of photo points

$S_x^{2'}$ = variance of crown cover based on N_g plots; $S_x^{2''}$ is based on N_p plots.

d = difference between mean of crown covers from photo interpretation of the N_p photo plots and the N_g photo plots.

The variance of this mean volume per $\frac{1}{5}$ acre of gross land area includes the contribution to this variance from the regression shown in the first term of the equation, contribution of measurement error of the crown cover shown in the second term, and contribution of forest area measurement shown in the third term. The development of this formula is based upon Deming (3), Chapman and Schumacher (1) and Chapman (2). Coding the numerators for the terms shown is done to simplify procedures.

FORMULAS FOR NUMBER OF PLOTS

1. *Ground plots*

$$N_g = \sqrt{\frac{A}{S_{vt}^2\sqrt{C_g}}}\left[\sqrt{AC_g} + \sqrt{BC_p} + \sqrt{CC_f}\right]$$

2. *Photo plots*

$$N_p = N_g\sqrt{\frac{BC_g}{AC_p}}$$

3. *Photo points*

$$N = N_g\sqrt{\frac{CC_g}{AC_f}}$$

where:

C_g = cost of ground plot

C_p = cost of photo plot

C_f = cost of photo point

For *assumed* forest area, regression, variance and costs the number of plots determined are the most efficient. These formulas are taken from Chapman (2).

Application

Assume that it is desired to survey an area which has similar characteristics to Unit 5 in Maine.

BASIC DATA[1]

Item	*Description*	*Value*
1	Variance of crown cover (S_x^2)	770
2	Variance independent of regression per $\frac{1}{5}$ acre (S_{yx}^2)	9,984
3	Variance of forest area (P_fQ) $=(.85)(.15)$	.1275
4	Variance of mean volume per $\frac{1}{5}$ acre (S_{vt}^2)	54
5	Mean crown cover per $\frac{1}{5}$ acre in per cent ($\bar{X}$)	65
6	Difference between two sample means of crown cover per cent (d)	1
7	Regression coefficient (b)	4.91
8	Constant in regression (a)	−152.0
9	Cost of ground plot (C_g)	$40.00
10	Cost of photo plot (C_p)	$0.20
11	Cost of photo point (C_f)	$0.02

[1] Forest survey experience Unit 5, Maine.

COMPUTATION OF TERMS IN ERROR FORMULA

Code	*Term*	*Value*
A	$P_f^2S_{yx}^2 + \frac{P_f^2S_{yx}^2d^2}{S_x^2} = 7{,}213 + \frac{7{,}213(1)}{770}$	= 7,222
B	$P_f^2b^2S_x^2 = .7225(24.1081)(770)$	= 13,412
C	$(a+b\bar{X})^2P_fQ = (167)^2(.1275)$	= 3,556

COMPUTATION OF NUMBER OF PLOTS

1. *Ground plots*

$$N_g = \sqrt{\frac{7{,}222}{54\sqrt{40}}}\left[\sqrt{7{,}222(40)} + \sqrt{13{,}412(.20)} + \sqrt{3{,}556(.02)}\right]$$

$$= \frac{84.98}{341.28}(537.48 + 51.79 + 8.43) = \underline{149}$$

2. *Photo plots*

$$N_p = 149\sqrt{\frac{13{,}412(40)}{7{,}222(.20)}} = 149(19.27) = \underline{2{,}871}$$

3. *Photo points*

$$N = 149\sqrt{\frac{3{,}556(40)}{7{,}222(.02)}} = 149(31.38) = \underline{4{,}676}$$

ESTIMATED COSTS OF PLOTS

Kind	*Number of Plots*	*Cost per plot Dollars*	*Total cost Dollars*
Ground plots	149	40.00	5,960.00
Photo plots	2,871	.20	574.20
Photo points	4,676	.02	93.52
Total			6,627.72

DISCUSSION

ADVANTAGES OF THIS PLAN:

1. Stand photo volume-tables are not required before the photo interpretation.
2. Photo interpretation work is clear-cut and straightforward.
3. This plan is believed more efficient than present survey techniques.
4. No serious changes are made in the over-all compilation procedures.
5. Plan will fit in with continuous inventory designs now under study.
6. This plan should be very useful in unexplored areas.

DISADVANTAGES OF THIS PLAN:

1. The variable or photo measurement used may not be the best for the area, although present experience shows that it is the most promising in the spruce-fir-hardwood forests in question.
2. Extra work in compilation is required for the construction of the regression formula. But this replaces the work required for the photo volume-table construction.

RECOMMENDATIONS

It is recommended that this plan be tried for forest survey purposes to test the application, soundness, and stated advantages.

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*Precision Measurements of Bubble Chamber Film**

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ABSTRACT: *The accuracy and speed of measurement of the motions and interactions of fundamental particles during the study of high energy nuclear physics is of increasing importance. The use of the bubble chamber has increased the amount of data available. Faster and more accurate methods of 3-dimensional measurement of data are being developed.*

IN THIS paper will be described the solution to a recent problem in reconstructing three dimensional images from two or more stereoscopic photographs. In high-energy nuclear physics we are concerned with the interactions between fundamental particles. We must study the scatters, the fragments, and the new particles which are produced when a high-energy particle collides with an atomic nucleus. When the incident particle has an energy of hundreds of MEV or greater, it no longer can be counted on to bounce off of the nucleus like a billard ball. Instead, it frequently creates new particles such as π-mesons, K-mesons, and at the very high energies, anti-protons. One of the most satisfactory ways for many years of studying these interactions has been by means of the Wilson

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