*Large-Field Photographic Astrometry**

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ABSTRACT: *A brief review is given on (a) development of large-field astrographs from the Normal Astrograph up to the 20-inch Carnegie Astrograph of Lick Observatory, (b) engines for measurement of plates, including an outline of an automatic engine being planned for Lick Observatory, and (c) reduction of measurements.*

INTRODUCTION

EXPERIMENTS for using photography in astronomy started almost immediately after the invention by Daguerre had been reported to the Paris Academy of Sciences by the French astronomer François Arago, on January 7, 1839. A more successful start, however, was possible only after the dry process had been developed.

From the very beginning the experiments were carried out in two directions: spectroscopy and direct photography. The importance of photography in astronomy rapidly increased, and at present most astronomical observations are made using photography. In astrometry, i.e. determination of positions and proper motions of celestial objects, visual observations with cross-hairs are still carried out for determination of absolute equatorial coordinates; all the differential work, however, is done almost exclusively by photography.

Although reflectors occupy the most prominent position in astronomy as a whole, they find almost no use in astrometry. A parabolic mirror has a very small field of good definition; although a Schmidt type reflector gives excellent definition over large fields, there is a distortion of the field which cannot be assumed to be stable. Recent experiments initiated by the Hamburg-Bergedorf Observatory[1] have, however, indicated that Schmidt telescopes can be used in astrometry. This review will be limited to refractors only.

Photographic astrometry can be subdivided in two areas: (a) long-focus or small-field, and (b) large-field astrometry. Long-focus astrometry employs photographic or visual refractors with long focus, of the order of 30 feet and more[2]; correspondingly the angular field is small, of the order of one degree or less. Long-focus instruments are used for determination of parallaxes, of relative proper motions, for observations of double stars, and in other problems where the highest angular precision is required without a necessity of a large angular field.

Large-field astrometry employs photographically corrected refractors, with focal-length of the order of 10 feet. The angular field is at least two degrees square. These telescopes are called astrographs. Large-field astrometry deals mostly with positions and absolute proper motions of stars. For this purpose a number of reference points is necessary on each plate, and the field has to be large enough to contain a sufficient number of these references. The present review is limited to large-field photographic astrometry, which is more related to photogrammetry than other branches of astrometry.

ASTROGRAPH

Towards the end of the 19th century the two French astronomers, the brothers Henry, were so successful in astronomical photography that an international conference to consider the field was organized. It took place in Paris in April, 1887, and resulted in a plan for photographic mapping of the whole sky and for determination of stellar positions and magnitudes from the photographs.[3] The procedure of the undertaking was worked out and a uniform type of instrument was adopted. The instrument is equipped with a two-component lens of 33 cm. aperture and 343 cm. focal length. It is called the Normal Astrograph; it covers a field two degrees square with a scale of 60″ per mm. The whole sky was divided into 18 zones and assigned to the same number of observatories.[4] Later some observatories withdrew and the corre-

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sponding zones were reassigned to other observatories.[5] In spite of an early start and of active international cooperation, the program, known as Astrographic Catalogue, is still not completed.[6] One of the main reasons for this delay is the relatively small field.

In 1914 F. Schlesinger and C. J. Hudson proposed to use a larger field instrument for position determination, in order to decrease the number of plates necessary to cover an area and to increase the number of reference stars available on each plate.[7] Positions of reference stars are observed individually by visual meridian circles requiring much effort and time; therefore the number of reference stars available is not very large.

Schlesinger and his collaborators repeated photographically a few so-called AG zones,[8] observed earlier visually by international cooperation under leadership of Astronomische Gesellschaft. Photographs were taken by a four-component lens with effective aperture of 7.55 cm. and focal length of 163. 5 cm. (scale: 126″ per mm.), covering a field of 5° square. The experiment was so successful, that it was decided to continue the work on a larger scale; the result was the re-observation of many AG zones by Schlesinger and Ida Barney, published in several volumes of *Transactions of Yale University Observatory*.

In 1933 Schlesinger and Barney increased the field considerably by using a four-component lens designed by F. E. Ross.[9] With an aperture of 14.3 cm. and focal-length of 202.5 cm. a field of 10°×14° was photographed on 48 cm.×58 cm. plates.[10] This is the largest field ever used for precise astrometric purposes, and the same astronomers found it advisable to reduce the field. Their next lens, also by Ross, was made with 5-inch aperture and 206.7 cm. focal-length (scale 99″.77 per mm.), covering a field 10° square on plates 17 inches square.[11]

Schlesinger's success stimulated the Astronomische Gesellschaft to repeat photographically all the AG zones from the North pole to declination −2°, and at the 1921 meeting of this society it was decided to work out a program.[12] The final program was adopted in 1927[13]; six German observatories and Pulkovo observatory determined positions of reference stars[14], and only three observatories carried out the photographic work.[15] The Zeiss Astrovierlinser was adopted as objective for the astrograph; its four-component lens of 14 cm. aperture and 206 cm. focal length defines well a 5° square field at the scale of 100″ per mm. This type of instrument is known as AG Astrograph.

Shortly before the war W. H. Wright, then Director of Lick Observatory, initiated a program for determination of stellar proper motions with reference to faint external galaxies.[16] None of the existing astrographs was suited for this purpose because of small apertures and large focal-ratios. A 20-inch four-component lens was designed by F. E. Ross especially for the Lick Proper Motion Program, and the construction of the astrograph was supported by Carnegie Corporation. The lens has a focal length of 146 inches (scale 55″.1 per mm.) and it defines well a 6° square field on plates 17 inches square; the astrograph is depicted in Figure 1.

For determination of stellar proper motions two sets of photographs, taken at sufficiently distant times (epochs) have to be compared. The first-epoch set was taken in the years 1947 to 1954.[17] The second set will be started in about 1967.[18]

Measurement of Plates

Only glass plates are used in precise photographic astrometry, and rectangular coordinates of objects are measured. When the Astrographic Catalogue was initiated, there was no previous experience on precision obtainable from photographs. Since a shift of emulsion during development was considered to be one of the most dangerous sources of errors, a precise rectangular grid, called a reseau, was printed on each photograph. The lines on the reseau were 5 mm. apart, and positions of star images were referred to these lines by means of an eyepiece-micrometer or by a precise glass scale placed in the focal-plane of the microscope eyepiece.

Later investigations showed that there is no need to use a reseau and instead of a printed reseau, a long scale built into engine was used for position reference.[19] F. Schlesinger introduced measuring engines with a long screw,[20] and since then this type is mostly used for precise measurement of plates. Long-scale as well as long-screw engines measure one coordinate at a time. After one coordinate of all stars on a plate is measured, the plate is rotated by 90° and the second coordinate is measured. In order to eliminate some systematic errors, mainly personal ones, each coordinate is measured in direct and reverse positions of the plate, or an eyepiece reversing prism is used for reversing the field,

Before the war Carl Zeiss developed a measuring engine with two glass scales perpendicular to each other and placed so that directions of both scales intersect on the axis of the microscope.[21] This arrangement of scales, according to Abbe's principle,[22] elimi-

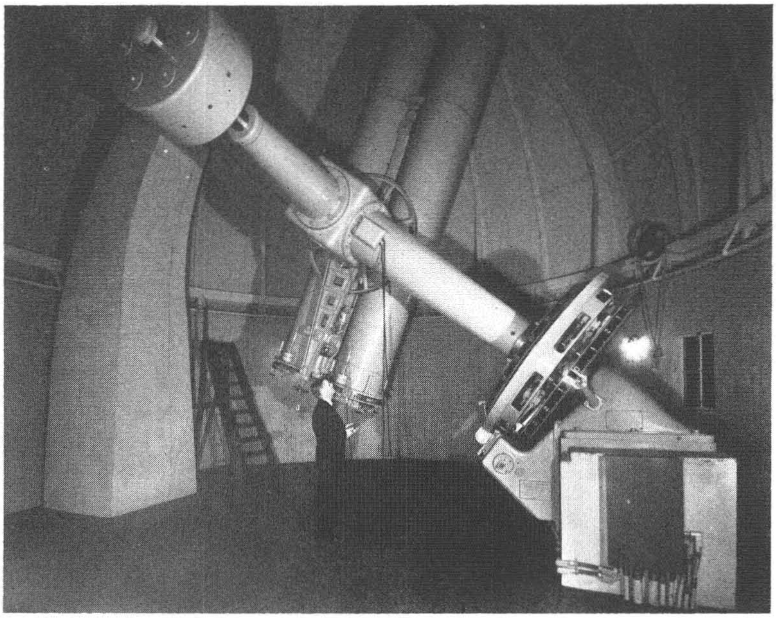

FIG. 1. 20-inch Carnegie Astrograph of Lick Observatory. (Lick Observatory photograph.)

nates errors of ways, and both coordinates are measured simultaneously.

In order to increase the accuracy, usually two settings on an image are made in each position of the plate. The errors of measurement are of the order of two microns, depending on the quality of images. Good images are of the order of 50 microns in diameter; if they are much larger, the precision decreases. If the magnitude range of stars to be measured is large, a coarse grating is placed in front of the objective. The grating produces short spectra symmetrically arranged on each side of the image, and these spectra are measured instead of the central image.

A new approach to measurement of plates has been pioneered by Watson Scientific Computing Laboratory of IBM; a conventional long-screw engine has been converted into an automatic machine.[23] Approximate coordinates of images are punched on IBM cards, and they serve as input data for automatic guiding to the image selected. The visual microscope is replaced by a spinning-sector photoelectric scanner. When the image reaches the field of the scanner, the latter centers automatically on the image; then the precise coordinate is read and recorded automatically on the same IBM card. The accuracy of measurement is considerably higher than with a visual machine.[24]

The successful experiment by IBM stimulated the planning of an automatic engine for the Lick Proper Motion Program, and the machine is in process of development in accordance with the following outline.

The measurement will consist of two separate operations: (a) survey of plates, selection of objects to be measured, and recording of approximate coordinates of the objects selected for automatic setting in the process of final measurement; (b) automatic measurement of positions and photographic magnitudes. Each of these operations will require a separate machine: (a) a surveying machine, and (b) a measuring engine.

The function of the surveying machine is to select objects for measurement and to obtain their approximate coordinates for automatic positioning in the measuring engine. The number of images measured on each plate will be of the order of 200 to 300. This number is a very small fraction of all the images on plate. Two plates of the same field, taken at different epochs are put into the surveying machine, and both fields are superimposed optically for simultaneous or alternating view of corresponding fields on both plates. After an object is selected, a crosswire is centered on the image, and its approximate coordinates are read and recorded, with an accuracy of 0.1 mm. or better. Instead of a visual reading of scales and recording by hand, a reader and recorder is operated by a single push-button, after the selection is made.

After a plate-pair has been surveyed, one plate of the pair is put into the measuring engine, and properly centered and oriented

From then on the measurement proceeds automatically with the aid of the records produced in the surveying process. The input of the approximate coordinates of an image operates servo systems and guides a photoelectric scanner to the image. Then the scanner centers itself upon the image, and precise rectangular coordinates, as well as the opacity of the image, are read and recorded automatically. After the measurement of one image is recorded, the machine proceeds automatically to the next image and repeats the operation described until all the images selected in the survey of the plate are measured. Then the second plate of the pair is measured with the same input data.

Normally, this process should continue automatically until the measurement of the plate is completed, and no continuous attention by an operator should be necessary. In case of any failure in the automatic operation a signal is given to the operator at the surveying machine in an adjacent room.

Two calibrated scales will be used as the measuring standards in the engine, and a variable-iris type photometer[25] will be incorporated for measurements of opacity. The average interval between two successive measurements, including the guiding, will not exceed 30 seconds, i.e. the speed will be at least 10 times of that with a conventional long-screw measuring engine. It is expected that the automatic measuring engine will be manufactured and installed at Lick Observatory within three years.

Reduction of Measurements

The most general problem in photographic astrometry is to calculate the equatorial coordinates, δ of celestial objects from the measured rectangular coordinates x, y of their photographic images. This reduction is done in two steps, using as a bridge so-called standard coordinates[26] X, Y. Standard coordinates are purely geometrical rectangular coordinates with the origin at the tangential point of gnomonic projection, and with X and Y axes correctly oriented in E and N directions respectively. If A and D are right ascension and declination of the origin of standard coordinates, the relationship between the standard and the equatorial coordinates of an object is[27]

$$\tan(\alpha - A) = \frac{X \sec D}{1 - Y \tan D}$$
$$\tan \delta = \frac{Y + \tan D}{1 - Y \tan D} \cos(\alpha - A) \qquad (1)$$

Similar formulae can be written for the inverse problem, i.e. for computation of standard coordinates from equatorial ones. The relationship can be expressed in many different forms depending on computational means, but the problem is always a straightforward geometrical one of the gnomonic projection. The relationship between the measured and standard coordinates is much more complicated and less definite.

Many errors and corrections have to be taken into account for reduction of measured coordinates to standard ones. The principal factors to be considered are:

Instrumental errors: centering, orientation, scale, tilt of the plate, optical distortion, curvature of the plate, distortion of the photographic emulsion, magnitude equation, and color magnification.

Spherical corrections: atmospheric refraction, aberration of light, precession and and nutation.

The number and the size of corrections increases with the field, and for a small field 2 degrees square most of the corrections mentioned can be neglected, and usually the following relationship, derived first by Turner,[28] is adequate

$$X = a + bx + cy$$
$$Y = d + ex + fy \qquad (2)$$

These equations, known as the six-constants formulae, include not only centering, scale and orientation errors, but also the linear terms of spherical corrections. Before (2) can be used, the plate constants a, b, c, d, e, and f have to be determined, and this is done as follows:

On each plate there must be a number of reference stars with known equatorial coordinates. Standard coordinates of these stars are computed by formulae inverse to (1), and then they are compared with the measured coordinates of the same stars, rewriting (2) in a form

$$a + bx + cy = X - x$$
$$d + ex + fy = Y - y \qquad (2')$$

Each reference star provides the above two equations with six unknowns, and a minimum of three stars permits one to compute these unknowns. Usually more than the necessary three reference stars are used, and the plate constants are found from the least squares solution.

After the plate constants a, b, etc, are computed, their values are inserted into (2) for each star measured, and the standard coordinates are calculated. Then the equatorial coordinates are found from (1).

For larger fields other corrections have also to be applied. The first of the most thorough developments, of formulae for larger fields was carried out by Zurhellen[29]; many discussions of measurements on large fields were made by Schlesinger and Barney,[30] and one of the more recent comprehensive reviews of reduction on large fields is given by A. König.[31]

Each new correction introduces additional plate constants and requires additional reference stars, if the least squares method is used for computation of constants. The scarcity of reference stars and a rapid increase in labor of computation, if old computing means are used for least squares solution, has led to a conventional procedure of independent determination of additional constants. After they are found and applied to the measured coordinates, formulae (2′) can be used.

Tilt of a plate is expressed as

$$\begin{aligned} \Delta x &= (px + qy)x \\ \Delta y &= (px + qy)y \end{aligned} \tag{3}$$

where p and q are rectangular coordinates of the tangential point. Even the most careful adjustment of the astrograph does not guarantee that the tangential point will coincide with and remain for a longer interval at the center of plates. A number of methods has been proposed and used for determination of p and q, each of them involving the weak assumption that the tilt repeats itself from one plate to another. One of the simplest and most accurate methods requires a dummy-plate with a small centering telescope.[32]

Optical distortion usually can be expressed as

$$\begin{aligned} \Delta x &= V(x^2 + y^2)x \\ \Delta y &= V(x^2 + y^2)y \end{aligned} \tag{4}$$

where the constant V is positive or negative depending on whether the distortion is of a "pin-cushion" or "barrel" type. A method for an independent determination has been developed by König and Heckmann[33] and modified by the author.[34] Schlesinger has derived formulae for determination of distortion from residuals of least squares solution (2′) and has shown that most of the distortion is absorbed by the scale coefficient.[35] Schlesinger's conclusions, however, are too optimistic because of two errors in his derivation: (a) in development of formulae for one coordinate, the mean of the other coordinate was taken for all stars; actually distortion is very sensitive to other coordinate; and (b) integration of a continuous function was used to demonstrate the absorption of distortion in the process of least squares solution; actually the least square solution requires a summation of coordinates of discrete points. If these errors are taken into account, the conclusion is significantly different.

Curvature of plates is usually small if proper care is exercised in selection of glass.[36] Schlesinger has shown[37] that the effect of spherical curvature has the form of (4), and a cylindrical one can be expressed as

$$\begin{aligned} \Delta x &= C_1xy^2 + C_2x^2y + C_3x^3 \\ \Delta y &= C_1'y^3 + C_2'xy^2 + C_3'x^2y \end{aligned} \tag{5}$$

The type and the amount of the curvature can be measured by laboratory means, but there still is a degree of uncertainty whether the curvature during the exposure is the same as after the plate has been processed and dried.

Distortion of the photographic emulsion is one of the least known sources of errors, in spite of many investigations, and there is no general rule or formula for corrections. Some recent investigations indicate on one hand that there are some large-scale distortions depending on the particular area of the plate[38]; on the other hand, there is indication of random shifts as reported by Toulouse Observatory.[39] Toulouse statistical results show that 64% of star images are shifted less than two microns, 18% of displacements are between two and three microns, and 2% of shifts are larger than eight microns. Large-scale distortion can be partly absorbed by plate constants, particularly if higher order terms are used in addition to the linear ones, but the effect of the random shifts can be minimized only by repeated coverage of the same field.

Magnitude error is a function of stellar magnitude (brightness), i.e. of the size of photographic image. If a plate is measured only in one position without reversing the field, the personal error of the measurer enters into measurements as a magnitude effect. By reversing the field this error is eliminated, but there are still other sources of magnitude effect.

The most common is the guiding error during long exposures, which produces slight deformation of images, depending on their size and distribution of density within the image. Another source is a coma of the optical system, and instruments with an appreciable coma have never been considered adequate for photographic astrometry. Recently, however, Eichhorn has shown that the coma effect may be present even with instruments considered as free of coma.[40] It is impossible to give a general formula valid in every case; one of the simplest ways to express the combined effect of guiding and coma is

$$\begin{aligned} \Delta x &= (m - m_0)(g + hx) \\ \Delta y &= (m - m_0)(g' + h'x) \end{aligned} \tag{6}$$

where m is the stellar magnitude, m^0 is an adopted constant, usually close to the mean magnitude of all stars on the plate, and g, g', h, h' are the plate constants to be determined for each plate. A coarse objective grating, mentioned in the previous section, eliminates the magnitude error to large extent.

Color magnification is caused by different scale values of the objective for different colors and it produces a shift of red stars relatively to blue ones. If k is a value related to the color of a star, e.g. the ratio of light intensities in two selected regions of the spectrum, then the simplest case of the color effect is

$$\begin{aligned} \Delta x &= s(k - k_0)x \\ \Delta y &= s(k - k_0)y \end{aligned} \tag{7}$$

where s is a plate constant, which usually depends on the type of a lens and on color sensitivity of the plate emulsion. The maximum color effect of the 20-inch Carnegie Astrograph with Eastman Emulsion 103a0 is of the order of 0.″2 at the edges of 17-inch square plate.[41]

Atmospheric refraction enters into the measured coordinates only as a difference between the refraction for a star and for the plate center. In spite of this differential character, the formulae for refraction are rather complicated, and they have been a subject of controversy for several decades, until the differences in various proposals were explained and reconciled by C. Vick.[42] The differential effect of refraction is expressed in power series of coordinates, usually terminating with the third power. The latter is significant only on large fields at low altitudes.

Since refraction is a function of color, the color effect of the atmospheric dispersion may become significant at lower altitudes. This color effect is not symmetrical towards the plate center, but it is oriented with respect to direction of zenith.

The differential effect of aberration is smaller than that of refraction, and even the terms of the second order are significant only in extreme cases. Precession and nutation do not cause relative displacement of stars to each other, and their effect is absorbed by the linear plate constants of (2′).

The procedure of reduction described above is used when precise positions of many stars on a plate have to be determined. When coordinates of a single object, e.g. a comet or an asteroid, have to be measured, simpler reduction methods can be used. Simplified reductions can also be made if differences of coordinates on two plates have to be determined, as for proper motion programs. Then the effects, which are equal on both plates, are eliminated.

Finally, the traditional reduction methods, as reviewed above, can be modified if (a) a large number of position references is available, and (b) high-speed computers are used for computation. Then instead of (2′) a series of powers up to the third power can be written and all the plate constants, including those of higher powers, can be found from the least squares solution. One may object that in this way effects cannot be separated because some of them have the same mathematical form, and the solution will lump them together; formal attempts at separation of constants do not lead to reliable results because of intercorrelation in the least squares solution. This objection is not valid, if the aim is to determine the positions of stars, but not the instantaneous instrumental errors, which are of no interest as long as they are small. The latter condition is always achieved by customary instrumental adjustments. The least squares solution with many constants means a mathematical stretching of the plate so as to have the best fit with the fixed reference points.

Conclusion

In order to show the advantages of the photographic method, let us compare it with the visual one for determination of stellar positions. The comparison is not meant to suggest that visual observations should be abandoned; at present both methods do not overlap, but supplement each other.

Since the equatorial coordinates are referred to planes determined by the rotation of earth and its revolution around the sun, these coordinates have to be determined by instruments referred to the earth. Stars are observed individually during their meridian transit, hence the corresponding instrument is called a meridian or transit circle. Although photographic and photoelectric methods have been tried in these observation, they do not offer significant advantages because of the individual star observations; one of the important features of photography is to cover simultaneously a large field. Therefore practically all the meridian work is done visually.

The telescope of a meridian circle is kept in meridian by a perpendicular axis placed horizontally in East-West direction. Inclination and azimuth of the axis, errors of pivots, collimation and flexure of the telescope, full amount of refraction, aberration, precession, nutation, and many other errors have to be taken into account in order to determine precise equatorial coordinates. In spite of the extreme care exercised and the great labor involved, probable errors of best modern single observations are of the order of 0.″3. Actually errors may be larger because of systematic effects different for different observatories, as becomes evident if observations of many observatories are combined into so-called fundamental catalogues. These individual observations, however, are necessary to provide a reference frame for much more efficient and accurate photographic method.

Since each photographic plate has to contain a number of reference stars, the error of the reference frame is smaller than the individual errors of reference stars. A frame of 25 reference stars with individuals errors of the order of 0.″2 will be accurate to approximately 0.″05. The final error of a photographic position will be a combination of the errors of the reference, the measurement of the image, and of residual systematic errors described in the previous section.

As was mentioned before, the error of a single bisection is of the order of two microns; the customary four bisections will yield the position with probable error of the order of one micron, which corresponds to 0.″1 for an AG astrograph with scale 100″ per mm. This error combined with that of the reference gives 0.″11. Actual probable error of a position from one plate of the Photographic AG Catalogue is of the order of 0.″14. The slight difference may be explained by slightly larger errors mentioned, and by some residual instrumental errors. These errors are still considerably smaller than those of the meridian circle. It should also be added that in the process of reduction the photographic positions of reference stars are computed and improved.

Astrographs with a longer focus yield more precise positions. Experiments with the 20-inch Carnegie Astrograph at Lick Observatory (scale 55″ per mm.) show that position of an image on a plate can be measured with a probable error of the order of 0.″07. Long-focus astrometry yields still higher precision,[2] but this field is outside the scope of this review. It should be mentioned, however, that the precision is not quite proportional to the scale on an instrument; many effects, like refraction anomalies, atmospheric dispersion, seeing and others, do not depend on scale.

Finally it should be mentioned that since the measurements of photographic plates are done in the laboratory, photographic methods are more adaptable to automation and other improvements of efficiency and accuracy than are measurements made directly at the telescope.

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*About the Character of Errors in Spatial Aerotriangulation**

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"We must take into account the errors as they are, not as one would like them to be."
—M. ZELLER

ABSTRACT: *This paper deals with the character of errors in spatial aerotriangulation, and investigates the possibility of separating the different categories of errors. A practical example is worked out in some detail; it shows how the systematic, quasi-systematic, accidental and pseudo-accidental errors are related to each other. A proposal is made to classify the errors in aerotriangulation according to their effect on the results, without regard to their origin or nature. Thus the errors could be classified as errors with systematic effect and errors with accidental effect, and could be treated accordingly.*

FOR about 15 years there has been increasing uneasiness and endless debating among the photogrammetrists dealing with aerotriangulation, because of a big and still unanswered question, brought up by Professor Bachmann, Lausanne, Switzerland, about the character of errors in spatial aerotriangulation. He was soon joined by Professor Roelofs, Netherlands, who gave a paper at the 1948 Congress of the International Society of Photogrammetry, showing that the error propagation in an aerotriangulation is such that the influence of purely accidental errors might give the impression that a systematic error exists. Some of Roelofs' examples show also that the result of accidental errors has a similar effect, as if breaks (jumps or cassures) are present.

Many photogrammetrists have seemed unconvinced by either the Bachmann's or Roelofs' articles and publications and have retained the opinion that a considerable part of the discrepancies and irregularities that they found in their practical work were due to local influences of the photographs—such as lack of flatness of film, local distortions, instrumental errors, etc.—or to changing operators during the triangulation on the stereoplotter.

There still exists a great variety of opinions on this subject. Many photogrammetrists consider this problem of vital importance because one cannot expect to obtain a fruitful development of aerotriangulation if there is lacking a deep and correct insight in the character of errors with which one deals.

In aerotriangulation, as in any other measurement, one expects to have accidental as well as systematic errors. The ideal solution of the problem of the adjustment of aerotriangulation would therefore be based on a separate treatment of each of these categories. The whole difficulty is how to separate the systematic from the accidental errors. Unfortunately, the theory of errors in photogrammetry does not give enough information for solving this problem.

Nowadays, the adjustment of aerotriangulation is made by adopting one of two principles: (1) either one must neglect the acci-

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